A Naive Model on the Rise and Decline of Nations and Powers

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I would annex the planets if I could.

Cecil Rhodes (from Hannah Arendt: The Origins of Totalitarianism)

And do you know what "the world" is to me? This world is the will to power – and nothing besides.

Friedrich Nietzsche: The Will to Power.

ABSTRACT

In this paper, I offer a Schumpeterian model on the rise and decline of nations and empires. The model sheds light on why different nations can reach very different equilibrium states in terms of economic strength and military prowess, how the initial condition, the chance of military campaign, personal charisma or leadership, the national spirit, the will to power, the Schumpeterian innovations, and the effective desire of accumulation affect the rise and decline of nations.

Key words: Powers, technology, arms, personal charisma, nations.

JEL classification #:

Introduction

The rise and fall of nations and great powers have been a fascinating

topic. Great early studies include Edward Gibbon (1948), The Decline and Fall

of the Roman Empire\1; Leopole von Ranke (1834), On Great Powers; Oswald

Spengler (1928), The Decline of the West; and Arnold Toynbee, A Study of

History. More recently, we have seen Mancur Olson's (1984) The Rise and Decline of Nations and Paul Kennedy's (1987) The Rise and Fall of the Great Powers. It is indeed as Olson (1982) puts it:

"Many have been puzzled by the mysterious decline or collapse of great

empires or civilizations and by the remarkable rise to wealth, power, or cultural achievement of previously peripheral or obscure peoples. The collapse of the Roman Empire in the West and its defeat by scattered tribes that would otherwise have been of no account is only one or many puzzling examples. ON repeated occasions the imposing empires of China have decayed to the point where they could fall prey to far less numerous or sophisticated peoples like Mongols or to uprisings by poor peasants in remote provinces." (p.1)

In this paper, I offer a Schumpeterian model on the rise and decline of nations. The model sheds light on why different nations can reach very different equilibrium states in terms of economic strength and military prowess, how the initial condition, the chance of military campaign, the personal charisma or leadership, the national spirit, the will to power, Schumpeterian innovations, and the effective desire of accumulation affect the rise and decline of nations. The model is extremely simple and its complication can be easily introduced when all parameters are defined in an endogenous fashion. The model can also be viewed as a preliminary synthesis of the analysis on the rise and decline of nations put forward by some political scientists, economists, historians, sociologists.

I organize this paper as follows. First of all, in the spirit of J. Schumpeter (1974), L. Robbins (1968), A. Pigou (1941), H. Morgenthau (1985), and many others, I define the objective function of a nation or a great power on both material welfare in terms of consumption and the power wielded by a nation in an international community. The will to power or the desire for power is the crucial part of the model, which leads to multiple equilibria across nations. Then, I use a simple figure and a simple example to illustrate the

existence of a powerful and a weak equilibrium in this model, and to show the importance of war and chance, the will to power, personal charisma, technology innovations, internal disorder and conflict, time preference in the process of the rise and decline of a nation. I then speculate on the possibility of convergence of wealth accumulation and power across nations in the spirit of Schumpeter, M. Weber and T. Veblen. Finally, I comment on the possible extension of the simplest model in this paper to a more complicated version.

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Model 1

A Formal Model of Nations and Great Powers

Here I first introduce a model of nations and great powers, then I make detailed justifications for this approach. Let c be consumption and w wealth, the latter is a broad index including capital stock, the size of a nation, military build-ups, human capital, etc. The utility or the objective function of a typical nation is defined on consumption, c, and its power, which in turn is a function of the wealth possessed by a nation:

where p(w) represents power; \7b\1p(w)\ is the enjoyment of power. Parameter characterizes a typical nation's intensity of desire for power and dominion. equals zero, the objective function is identical to the standard Cass (1965) model in the theory of optimal economic growth.

The dynamics of wealth accumulation of a typical nation is

where w is the net increase in the wealth of a nation. Af(w,\7q\1) is the economic

benefit of the wealth, and it can be regarded as a kind of production function or benefit function in a broad sense; indeed, when w includes military power, w is the basis for colonization, conquest, expansion and plunder in the whole human history; when w is physical and human capital, its productivity is what has been dealt with in economics; when w is size and territory of a nation, it indicates natural wealth and fertility by endowments. Parameter A is a productivity measure and I will interpret it as the Schumpeterian innovations later. Parameter \7q \1represents the personal charisma of a nation's leaders, and, naturally, a high value of \7q \1represents the personal and administrative ability to lead a nation to conquer, to mobilize resources, and to build up national power and wealth. The term \7d\1w can be regarded as the rate of depreciation" of a nation's wealth due to physical depreciation of capital and arms, or the adverse shocks to a nation such as the break away and independence of colonies, the natural rate of dissolution of the empire, the internal conflict and civil war of a nation, etc. In this paper, I give a general term, adverse shocks, for parameter

In the existing studies on great powers, wars and imperialism, three approaches have emerged. The first one is the power-oriented approach espoused in Joseph A. Schumpeter (1919, 1974); R. G. Hawtrey (1929, 1952); Hans Morgenthau (1948, 1985). The second one is the economic-oriented

approach taken by many Marxists and others. The third approach is the combination of these two approaches above and has been taken by A. Pigou (1941); L. Robbins (1968). Now I want to justify in very broad sense why the model presented in (1) and (2) reflects the main ideas in all these existing studies.

Schumpeter (1974) emphasizes the role of power-thirsty and desire for dominion in his theory of great powers and imperialism. For Schumpeter, "instead of asking, what is imperialism? he asks, who are imperialists?" (p.vii), as pointed out by Bert Hoselitz in his introduction to Schumpeter's essays. For those founders of many great empires and nations in world history, material interest in terms of gains such as production f(w,\7q\1) and utility u(c) in the model is a by-product of conquest and expansion. The main objective of these empire-builders, their nations and their classes is to seek for power and dominion per se:

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Schumpeter compares this war-like psychology to the miser's saving habit and modern businessman's working habit. Just as Hoselitz has observed, Schumpeter's analysis of imperialists resembles his analysis of entrepreneurs: "The carrier of economic development is the innovating entrepreneur, The carrier of imperialist ventures is the machine of warriors" (p.vii, Schumpeter, 1974). In Schumpeter's own words: "Warriors by environment, the Persians very probably reached the regions where history first finds them with sword in hand. And the psychological dispositions and organization forms gained from such a model of life persisted, continuing in an 'objectless' manner. This is in accord with psychological developments that can be verified everywhere. The miser originally saves for good reasons, but beyond a certain point his hoarding ceases to be rational. The modern businessman acquires work habits far beyond the limits where acquisition still has rational meaning in the hedonist sense" (pp.26-27). Therefore, aggression, expansion, and conquest become an habit and end. This is why we can define their objective function directly on the size of the empire as \7b\1v(w) in (1).

The Assyrian Empire fits Schumpeter's theorization nicely. During their dominion, "year after year king and people took the field to conquer, lay waste, pillage, and murder, with pretext or without. The vanquished were crucified, impaled, flayed, immured alive by the thousands, or had their eyes pull out or limbs struck off. Conquered cities were usually destroyed, the inhabitants often burned with them. Expression like 'grind into the dust' or 'tinge the mountains with the blood of the foe' recur time and again in the annals of the kings" (p.31). Why did this conquest for its own sake happen? Schumpeter has offered the following answer:

"Lust for blood and booty, avarice and the craving for power, sexual impulses, commercial interests (more prominent with the Assyrians than the Persians) — all these, blended to varying degrees, may have played their part in motivating individuals and groups; also operative was the unrestrained will to gratify instincts — precisely those instincts to which a warlike past had

given predominance in the mentality. Such real motives are powerful allies of official motives (whether religious or otherwise), increase their striking power, or usurp their guise". (p.33). "it is evident that the king and his associates regarded war and the chase from the same aspect of sport — if that expression is permissible. In their lives, war occupied the same role as sports and games do in present-day life. ... Foreign people were the favorite game and toward them the hunter's zeal assumed the forms of bitter national hatred and religious fanaticism. War and conquest were no means but ends. They were brutal, stark naked imperialism" (p.33).

Schumpeter extends his analysis to the Arabian Empire, the creators of great powers like Alexander, Julius Caesar, Louis XIV, and Napoleon. In these personalities, we find the impulse to conquer and the will to power. Their objective function can be best described as the maximization of power as defined in expression (1).

Schumpeter's theory can be found both before and after him. Thomas Hobbes has already listed three objectives of man: "wealth, the love of reputation and the love of power" (Viner, 1991, p.76). R. G. Hawtrey (1925) regards the power as the ultimate one in explaining expansion, war and conflict. Hans Morgenthau (1985)\3, \1in \3Politics Among Nations, \1has espoused that power is man's control over the minds and actions of other men. Drives to live, propagate, and dominate are common to all men. Similarly, the struggle for power is universal in time and space and an undeniable fact of experience. Aspiration for power is the distinguishing element of all politics, and hence international politics: "International politics, like all politics, is a struggle for power. Whatever the ultimate aims of international politics, power is always the immediate aim. Statesmen and peoples may ultimately seek freedom, security, prosperity, or power itself... But whatever they strive for realize their goal by means of international politics, they do so by striving for power." (p.31.) In the context of German unification under Bismarck, Heinrich von Treitschke wrote that "the essence of the state is in the first place power, in the second place power, and in the third place once more, power." (cited from Dietze, 1979, p.19)

While the Schumpeterian theory on the formation of the great powers and empires only focuses on the will to power as the objective of a hero or a nation or a class, the Marxian theory has taken the approach to the great power and empire or imperialism in the modern era totally based on the economic interest of the capitalist class . The Marxian theory can be best characterized by the maximization of the capitalist interest as follows

Of course, the importance of economic motives in the formation of great powers and the expansion of empires through war was pointed out long time ago by Plato in his book \3Republic \1(book II). This broadly defined economic gains can be represented by the benefit function f(w,\7q\1): investment opportunity, foreign market, raw materials, etc. through the expansion of the empire or, in the Marxian terminology, imperialist aggression and expansion. This point-view of the Marxian analysis has been suggested by Jacob Viner (1991): "It would argued that 'in the last analysis' the end of foreign policy had been not power, and not power and plenty, but plenty alone, and plenty for the privileged classes only, and it would charge that members of these classes would always be there in every diplomatic episode, pulling the string of foreign policy-making for their special benefit" (p.151).

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The approach taken by Pigou (1941) and Robbins (1968) is the combination of economic motives in the Marxian theory and the will to power in the Schumpeterian theory. Robbins (1968) provides some persuasive evidences why we should integrate both economic motives with the psychological or the

will-to-power arguments: For economic considerations, "Japanese aggression was perhaps a classic case of this type of causation. But as already noted in my text, the motive inspiring Hitler and the Nazis were of a different order and so infused with purely psychological material as largely escape any classification as economic. And the shadows under which we live today (Robbins wrote in 1968, added), the tensions between East and West, are certainly ideological than economic in origin. When Mr. Krushchev caused to be placed on Cuba installations for the discharge of nuclear weapons, he was not thinking of a possible gain or loss of markets. The determination of the government of the United States to prevent the extension of communism is not based on fear of impoverishment" (pp.5-6).

Pigou (1941) explains expansion and war in terms of both economics and power:

"The desire for domination for its own sake, apart from any economic advantages it may confer, is a real and effective motive for action. The English schoolboy who asserts himself by bullying small boys, the self-styled Aryan who bullies Jews, the white man in tropical countries who exacts special marks of respect and subordination from his 'inferior', all at the bottom are displaying a desire for domination. there is no doubt at all the average Englishman or Frenchman does have this sort of desire; and moreover, feels that it is somehow gratified by the fact he is a citizen of a 'Great Power', not of Holland or Switzerland or Spain. In 1914 and again in 1939 many Germans have really desired their country to hold Europe in a thrall of fear, and to hear their country's sabre rattling behind its diplomacy. ... Furthermore, the desire for domination makes nations extraordinarily unwilling to relax their hold on any territory they have come once to 'possess'. They are humiliated, outraged, dishonoured if a subject population passes out from their yoke" (pp.19-20).

3. Rise and Decline of Nations

In this section, I intend to offer some insights from the theoretical model on the questions why great powers and nations have risen and declined and why new great powers and new nations have replaced old ones. The advantage of the model is to help us pin down many existing explanations in a very simple but essential way.

Recall that I have the following dynamic model describing a typical nation

In its abstract form, the model specified in (1) and (2) is essentially the same as the Kurz (1968) model. In the technical aspect, I do not claim any originality here. My contribution is to extend the Kurz model into a model

useful for the explanation of the rise and decline of nations. I have also used the Kurz model to explain economic growth, savings, capital accumulation; see Zou (1991, 1994a, 1994c). The most important part of the model is the existence of multiple equilibria. If \7b \1is set to be zero in (1), we go back to the Ramsey-Koopmans-Cass model where a unique long-run equilibrium is determined by the time preference \7r\1, the depreciation rate \7d \1and the technology f(w,\7q\1). But if \7b \1is not zero, multiple equilibria appear; see Kurz (1968).

History has furnished us countless examples which shed light on how this expansion spirit and national spirit have impacted on the rise and decline of nations and powers. The role of the rising expansions spirit in the Great Britain and its decline has been emphasized by Sombart (1915) and Weiner (1981) in their studies on the rise and fall of the British empire. In this connection, I cite Veblen (1939): "Elizabethan England, and its soaring imperialistic aspirations, was exposed to the slow corrosion of peace and isolation, with the common interest converging more and more on the industrial arts and the fortune of trade; and it took a hundred years and more to displace dynastic statecraft and eliminate imperialist politics ... and it took another two hundred years to reach the farthest point along the line of liberal policy and peaceable ideals eventually attained by the English community" (p.173).

Similarly, many historians have the following observation about the effect of a rising German spirit on the German state. According to Veblen (1939), "The fashion among historian of the period, particularly among patriotic historians, has been to construct this complex movement of forces, material and immaterial, that makes German history through the middle half of the century, as a movement of the German spirit, initiated by the exuberant national genius of the race"(p.71). Indeed, to many Germans in the 19th century, the rising Germany was just a realization of a kind of German spirit or a realization of Hegel's (1956) abstract World Spirit. This is why many authors like N. J. Spykman, Hans Morgenthau and Rudolph Steinmetz have pointed out the role of national character and national moral as a key element in determining national power (Aron, 1966, p.52). Morgenthau (1985) endorses the following description by Samuel Taylor Coleridge:

"But that there is an invisible spirit that breathes through the whole people, and is participated by all, though not by all alike; a spirit which gives a color and character both to their virtues and vices, so that the same action such I mean as are expressed by the same words, are yet not the same in a Spaniard as they would be a Frenchman, I hold for an undeniable truth,

without the admission of which all history would be a riddle. I hold likewise that the difference of nations, their relative grandeur and meanness, all, in short, which they are or do – (not indeed at one particular time, under the accidental influence of a single great man, as the Carthaginians under the great Xantippus, and afterwards under their own Hannibal,) but all in which they preserver, as a nation, through successions of changing individuals, are the result of this spirit;..." (p.147)

In Morgenthau's opinion, this national spirit sets one nation apart from others. He uses the Russian and Germany national character to illustrate the relation between national character and national power (pp.148-153).

Clausewitz (1984) emphasizes the psychological roots of initiative and other kinds of military creativity and makes the psychology of the army and the commander and the society as essential part of the theory of war. In this model, it might be as well to interpret parameter \7b \1as this military psychology if w is viewed mainly as military expansion. Parameter \7b \1not only reflects the expansion spirit in the political and military senses, it also captures the spirit of enterprise and the desire for gain in the economic sense, or the capitalist spirit in the sense of Max Weber (1958) and Sombart (1915) — accumulation for the sake of accumulation.

Personal Charisma

The will to power and expansion is not enough, equally important is the administrative talent of a nation's leaders to realize their inspiration and great ambition. In evaluating Napoleon, Fuller (1954) pays full attention to these two aspects:

"Among the world's greatest autocrats and conquerors, Napoleon has but two compeers - Alexander the Great and Augustus. The warrior spirit of the one he shared to the full, as he did the administrative abilities of the other... Such a man was Napoleon, the supreme egoist and architect, the entirely isolated and self-centered man who relied on himself and centralized everything... 'his genius, superhuman in its activity, carried him away: he felt he possessed the \3means \1and \3time \1to manage everything... in reality it was he who did everything."' (vol.2, p.410)

Following Max Weber (1968), I call this leadership and administrative talent as personal charisma: "The term 'charisma' will be applied to a certain quality of an individual personality by virtue of which he is set apart from ordinary men and treated as endowed with supernatural, superhuman, or at least specifically exceptional powers or qualities. These are such as are not accessible to the ordinary person, but are regarded as of divine origin or as

exemplary, and on the basis of them the individual concerned is treated as a leader" (p.48).

The rise and decline of nations in history have always been associated with the appearance and disappearance of greater military leaders and statesmen. In the Schumpeterian theory on the formation of empires, the role of those exceptional persons with personal charisma are the key to the understanding of the rise and decline of the great powers in history. That is to say, to understand the great powers, it is necessary to understand the leaders or founders of the great powers. This is also the main concept of genius in Clausewitz's (1984) theory of war. Likewise, Morgenthau (1985) emphasizes the decisive influence of the quality of leadership upon national power (pp.141-142): "The power of Prussia in the eighteenth century was primarily a reflection of the military genius of Frederick the Great and of the strategic and tactical innovations introduced by him." In addition, authors like Rudolph Steinmetz and Guido Fischer have also emphasized the quality of command and the skill of organization as the elements of national power (see Aron, 1966, p.52).

Since history has provided too many examples of this exceptional talent, I signal out only the well-known few: Alexander the Great, Julius Caesar and Napoleon. For military talent, the best example is the Carthagianian general Hannibal. Theodor Mommsen, in \3The History of Rome\1, has the following comments on him:

"He was peculiarly marked by that inventive craftiness, which forms one of the leading traits of the Phoenician character; he was fond of taking singular and unexpected routes, ambushes and stratagems of all sorts were familiar to him ... Every page of the history of that period attests his genius as a general; and his gifts as a statesman." (vol. II, p.88).

While the exceptional ability in military field is a great factor in the rising of almost all great empires in the history, the leadership talent of statesmen and legislators such as Lycurgus of Sparta, Solon of Athens, and the founding fathers of American constitution has also contributed greatly to the rise of their nations

F: Schumpeterian Innovations

Productivity parameter, A, has been the focus of growth theory. In our model, its positive impact on growth is easy to show from (10): In this paper, the definition of this productivity parameter is much broader than the usual definition. For example, it represents the improvement of productivity, the new advances in technology as the barbarous people adopted and utilized more advanced technology when they conquered and defeated

a higher civilization (Germans versus Romans, Mongolians versus the Chinese), the discovery and colonization of new land, new raw material and new sea routes. Thus parameter A is exactly what Schumpeter (1934) has included in the concept of innovation: "(1) The introduction of a new good — that is one with which consumers are not yet familiar — or of a new quality of a good. (2) The introduction of a new method of production, that is one not yet tested by experience in the branch of manufacture concerned, which need by no means be founded upon a discovery scientifically new, and can also exist in a new way of handling a commodity commercially. (3) The opening of a new market,that is a market into which the particular branch of manufacture of the country in question has not previously entered, whether or not this market has existed before. (4) The conquest of a new source of supply of raw materials or half-manufactured goods, again irrespective of whether this source already exists or whether it has first to be created. (5) The carrying out of the new organization of any industry, like the creation of a monopoly position (for example through trustification) or the breaking up of a monopoly position."(p.66)

Many conquests and expansion in the history led to Schumpeterian innovations: the opening of new market after the British won the Opium War in the 1840's; the supply of more raw materials after the Spanish discovery of American silver mine in the 17th century and the British colonization of India which supplied British textile industry with cheap cotton in the 19th century; the access to new methods in production after Germans conquered the Roman empire in the fifth century and the Mongols defeated China in the 11th century; the adoption of new economic and even social organization after the Manchurian conquered China in the 17th century. How a country can grow from a weak status to a powerful status through these, sometimes quite accidental, innovations (the discovery of new sources of supply and the opening of new market) have been stated well by H. G. Wells (1961) in the case of the rise of Spain from the 11th to the 17th century:

"So it was that Spain rose to a temporary power and prominence in the world's affairs. It was a very sudden and a very memorable rise. From the eleventh century this infertile and corrugated peninsula had been divided against itself, its Christian population had sustained a perpetual conflict with the Moors; then by what seems like an accident it achieved unity just in time to reap the first harvest from the discovery of America. Before that time Spain had always been a poor country... For a century, however, through its monopoly of the gold and silver of America, it dominated the World."

(p.624.)

In modern European history, this is how in the wake of Portugal, Spain and Holland came France and England, each in its turn taking up the role of expansion and empire overseas.

The most important still is the discovery of new technology. The industrial revolution beginning with James Watt's steam-engine in the 18th century in Great Britain ushered the beginning of a century's supreme dominance of Great Britain in the world economy, politics and military expansion. The first six decades in the 20th century witnessed the enormous technology progress in the United States and its dominance as a super economic and military power in the world. The economic miracles of Germany and Japan after World War II again illustrated the fundamental role of economic strength in the rise of nations.

While economic power is the foundation for a rising nation, military expansion has often done for both dominance and economic interest. As Fuller (1954) puts it: "it was the shortage of good agricultural land more than the desire to conquer and plunder that made of the Greeks the greatest colonizing people in classical history" (p.12). In modern history, the successive rising of Spain, Holland, France and England from the 16th century to the 19th also illustrated how economic power and military power go hand in hand in the rise and decline of nations. All this is just a factual verification of Clausewitz's (1984) famous statement: "war is nothing but the continuation of policy with other means." (p.69)

G: Adverse Shocks

In the set-up of the model, I have used the term adverse shocks and the notation \7d \1to denote the rate of "depreciation" of a nation's wealth due to physical depreciation of capital and arms, the natural rate of dissolution of the empire such as the independence of colonies, the internal conflict and civil war of a nation, etc. In this way, the narrow meaning of the depreciation rate in economics has been extended significantly. Of course,

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Thus, \7d \1can be interpreted as a shock or a defeat in a war. In this

context, is the fraction of wealth lost as a result of the defeat or \7d\1w is destroyed in the war. Examples are the British-Spanish war, and the British-French war. In these wars, Spain and France suffered major losses in their wealth, while Britain claimed major windfall in terms of colonies, sea routes and raw materials, etc..

Trade union and struggle between the capitalist and the working class can also be regarded as a shock element and \7d \1can measure the degree of internal conflict and the frequency of strikes. The adverse effects of these shocks on a nation have received extensive treatment in economic and political studies. For example, many authors have taken ethnic homogeneity, degree of social integration, and political stability as the elements of power a nation possesses; see Aron (1966, p.52-53) for a survey.

H: Effective Desire of Accumulation and the Time Preference

a higher time preference or a lower effective desire of accumulation reduces a nation's long-run wealth accumulation; and vice versa. This proposition is a fundamental one since Adam Smith, John Rae, J. S. Mill, E. Bohm-Bawerk, and I. Fisher, among many others. I quote Rae (1834) here:

"The determination to sacrifice a certain amount of present good, to obtain another greater amount of good, at some future period, may be termed the effective desire of accumulation." (p119) In Rae's view, this effective desire of accumulation is very different among nations and peoples. Concurring with Edward Gibbon (1940), Rae regards vanity and conspicuous consumption (p.267) as one of the basic factors causing the decline and the fall of the Roman empire: ancient "Romans are still more conspicuous instance of the extravagance into which this passion betrays nations." (p.277) Veblen (1915) regards the conspicuous consumption as the reason for British decline in the19th and the 20th centuries. Since the role of time preference has been extensively treated in the theory of economic growth, I will not make any further elaboration here.

The Possibility of Convergence among Nations

The model presented so far illustrates a very important point about the possibility of convergence across nations. The first case for convergence is the most obvious one: the will to power gradually disappear, equation (1), and the model returns to the standard Cass (1965) model where there exists a unique equilibrium instead of multiple equilibria. This is exactly a mathematical expression for what J. Schumpeter (1974) has said about the the wear-down of the warlike disposition and power hunger. According to Schumpeter, The gradual disappearance of the power hunger happened to

many nations in the history

"The imperialism of a warrior nation, a people's imperialism, appears in history when a people has acquired a warlike disposition and a corresponding social organization \3before \1it has had an opportunity to be absorbed in the peaceful exploitation of its definite area of settlement. Peoples who were so absorbed, such as ancient Egyptians, the Chinese, or the Slavs, never of themselves develop imperialist tendencies, though they may be induced to do so by mercenary and generally alien armies. Peoples who were not preoccupied in this fashion ... remain natural-born imperialists until centuries of peaceful work wear down that warlike disposition and undermine the corresponding social organization" (p.27).

Schumpeter takes the Franks as another example: "In the case of the Franks, the 'habit of conquest' did not go back far enough to become enduringly fixed... Even while they were engaged in conquest, the Franks remained predominantly tillers of the soil... Thus the popular will to conquest as such soon vanished, once large numbers of Franks had ensconced themselves comfortably in new areas of settlement... Once again they were swallowed up by the private sphere of agriculture, hunting, local guerrilla warfare — the life of village, estate, and province. The people soon very soon lost all interest in imperial politics, all contact with the central power" (p.45).

If the development of legal and bureaucratic organizations leads to a routinization of charisma and leadership (Weber, 1968, pp.48-65), will be the same across nations. Furthermore, if Schumpeter's prediction is right, the innovation parameter A will become the same across nations because in a future world, with the development of capitalism, the innovation function of entrepreneurs becomes obsolete (Schumpeter, 1975, p.131-134). If Veblen's prediction is right, the time preference parameter \7r \1will also become the same, and all nations will be resided by conspicuous consumers

Therefore, with the waning of the desire to power, with the routinization of personal charisma and innovation, with the coming age of conspicuous consumers, there would be convergence in wealth accumulation and power across nations indeed! But how realistic these assertions are, I leave the judgment to the reader.

In this paper, I have made a preliminary attempt to model the rise and decline of nations and great powers by focusing on highly stylized factors which have been recognized by economists, political scientists, historians, sociologists and others. Simple as the model is, it is highly illuminating to see that once the will to power or the desire for expansion comes to play on

the stage of world history, different nations with different initial conditions tend to end up with different equilibria with some being powerful and some weak. The role of military campaign and the chance of war can be seen easily in this picture of multiple equilibria. The role of personal charisma and leadership, the role of technology progress and innovations, the role of social cohesion and national moral, national spirit, the time preference and the effective desire of wealth accumulation in determining the rise and decline of nations can be grouped in an integrated picture. The crucial role of the will to power in generating multiple equilibria should be emphasized here because without its existence in the utility function, the world ends up with a unique equilibrium as in the Ramsey-Cass-Koopmans model in which the rise and decline of nations and the role of war and chance can hardly be seen. In this way, the simple model also highlights the conditions for convergence.

The preferences of home country is defined on the consumption, c, the home country's total wealth, $w(t)$, which equals the weapon stock , $m(t)$, plus the capital stock, $k(t)$, and the foreign total wealth, $w^*(t)$, which is defined as the sum of weapon stock, m^*,and the foreign capital stock, $k^*(t)$, mathematically, $u(c, w, w^*)$. Suppose the utility function $u(c, w, w^*)$ is twice differentiable and concave. Also, we assume that

$$\begin{aligned} u_1 &> 0, u_2 > 0, u_3 < 0, u_{11} < 0, u_{22} < 0, \\ u_{12} &= u_{21} > 0, u_{13} = u_{31} < 0, u_{23} = u_{32} > 0 \end{aligned} \tag{1}$$

which asserts that the marginal utility from consumption and home country total wealth is positive and diminishing. But the marginal utility from the foreign country total wealth is negative. The assumption $u_{23} = u_{32} > 0$ implies that an increase in foreign weapon stock will increase the marginal utility of home country stock and defense.

Follow Turnovsky, output is produced by a stochastic technology,

$$dY = F(k)dt + H(k)dy,\ F^{'}(k) > 0, F^{''}(k) < 0. \tag{2}$$

which asserts that the flow of output over the period $(t, t+dt)$, consists of two components. First, there is the deterministic component, described by the first term on the right side, with $F(k)$ representing the mean rate of output per unit time. In addition, there is a stochastic component, reflecting the various random influences that impact on production. The stochastic term dy can be explained as stochastic shock and assumed to be a temporally

independent , normally distributed with mean zero and variance variance $\sigma_y^2 dt$.

Suppose the foreign total wealth w^* follows a stochastic process,

$$dw^* = \mu_{w^*} w^* dt + \sigma_{w^*} w^* dz^* \tag{3}$$

where the same assumptions on the stochastic term dz^*.The stochastic term dz^* is assumed to be a temporally independent , normally distributed with mean zero and variance variance dt.

Now, the budget constraint of the representative agent in home country can be described as

$$dw = dY - cdt \tag{4}$$

where

$$w = k + m$$

The representative agent chooses his capital stock, weapon stock and consumption to maximize his discounted utility

$$E_0 \int_0^\infty u(c, w, w^*) e^{-\rho t} dt$$

subject to the budget constraint (4) and the initial stocks $k(0)$ and $m(0)$ are given.

0.1 Optimalities

To solve the problem, we introduce the value function $V(w, w^*, t)$ and the discounted value function $X(w, w^*)$ as the relation

$$X(w, w^*) e^{-\rho t} = V(w, w^*, t)$$

Denote the holding share of military stock as

$$n = \frac{m}{k + m}$$

The representative agent chooses his asset holding, n, and his consumption path, $c(t)$, to maximize the following expression

$$u(c, w, w^*) - \rho X + X_w(F((1-n)w) - c) + X_{w^*}\mu_{w^*} w^* + \frac{1}{2} X_{ww^*} \sigma_{yz^*} H((1-n)w) w^*$$
$$+ \frac{1}{2} X_{ww} H((1-n)w)^2 \sigma_y^2 + \frac{1}{2} X_{w^*w^*} \sigma_{w^*}^2 w^{*2}$$

Thus, we get the optimal conditions for the optimization problem

$$\frac{\partial u(c,w,w^*)}{\partial c} = X_w \tag{5}$$

$$\begin{aligned} &-X_w F^{'}((1-n)w) - \frac{1}{2}X_{ww^*}\sigma_{yz^*}H^{'}((1-n)w)w^* \\ &-X_{ww}H((1-n)w)H^{'}((1-n)w)\sigma_y^2 = 0 \end{aligned} \tag{6}$$

From the above two equations, we can derive the optimal choices for the asset holding share and the consumption path as functions of X_w, X_{ww^*}, and X_{ww}. With the substitution of the optimal values of the asset holding and consumption, the value function must satisfy the following Bellman Equation

$$\begin{aligned} &u(c,w,w^*) - \rho X + X_w(F((1-n)w) - c) + X_{w^*}\mu_{w^*}w^* + \frac{1}{2}X_{ww^*}\sigma_{yz^*}H((1-n)w) \\ &+\frac{1}{2}X_{ww}H((1-n)w)^2\sigma_y^2 + \frac{1}{2}X_{w^*w^*}\sigma_{w^*}^2 w^{*2} = 0 \end{aligned} \tag{7}$$

0.2 Explicit solutions

In order to find the explicit solutions for the asset holding and consumption, we specified the utility function and the technology as

$$u(c,w,w^*) = \frac{c^{1-\xi}}{1-\xi}(\frac{w}{w^*})^{-\eta} \tag{8}$$

$$F(k) = Ak, H(k) = Ak$$

where ξ and η satisfy $0 < \xi < 1, \eta < 0$ and $\xi > 1, \eta > 0$, A is a positive constant.

Given the specified utility, we conjecture the value function as the form

$$X(w,w^*) = \chi w^{1-\xi-\eta}(w^*)^{\eta} \tag{9}$$

where the coefficient χ is to be determined.

Taking partial derivations, we have

$$\begin{aligned} X_w &= \chi(1-\xi-\eta)w^{-\xi-\eta}(w^*)^{\eta}, X_{ww} = \chi(1-\xi-\eta)(-\xi-\eta)w^{-\xi-\eta-1}(w^*)^{\eta} \\ X_{w^*} &= \chi\eta w^{1-\xi-\eta}(w^*)^{\eta-1}, X_{ww^*} = \chi(1-\xi-\eta)\eta w^{-\xi-\eta}(w^*)^{\eta-1}, \end{aligned}$$

Substituting the above expressions of derivatives into equations (5) and (6), we derive

$$c^{-\xi} = \chi(1-\xi-\eta)w^{-\xi}$$

or

$$\frac{c}{w} = (\chi(1-\xi-\eta))^{-\frac{1}{\xi}} \tag{10}$$

$$\begin{aligned}
&-(1-\xi-\eta)A\chi w^{1-\xi-\eta}(w^*)^{-\eta} - \tfrac{1}{2}(1-\xi-\eta)\eta\sigma_{yz^*}\chi w^{1-\xi-\eta}(w^*)^{-\eta} \\
&+(1-\xi-\eta)(\xi+\eta)\chi w^{1-\xi-\eta}(w^*)^{-\eta}(1-n)\sigma_y^2 = 0
\end{aligned}$$

or

$$-(1-\xi-\eta)A - \tfrac{1}{2}(1-\xi-\eta)\eta\sigma_{yz^*} + (1-\xi-\eta)(\xi+\eta)(1-n)\sigma_y^2 = 0 \tag{11}$$

Substituting equations (10) and (11) into the Bellman equation (7), yields

$$\begin{aligned}
&(\chi(1-\xi-\eta))^{-\frac{1-\xi}{\xi}}\frac{w^{1-\xi}}{1-\xi}(\frac{w}{w^*})^{-\eta} - \rho\chi w^{1-\xi-\eta}(w^*)^{\eta} + \chi\eta w^{1-\xi-\eta}(w^*)^{\eta-1}\mu_{w^*} \\
&+\chi w^{1-\xi}(\frac{w}{w^*})^{-\eta}(1-\xi-\eta)(A(1-n) - (\chi(1-\xi-\eta))^{-\frac{1}{\xi}}) \\
&-\frac{1}{2}\chi(1-\xi-\eta)\eta(1-n)w^{1-\xi}(\frac{w}{w^*})^{-\eta}\sigma_{yz^*} \\
&-\frac{1}{2}(1-\xi-\eta)(\xi+\eta)\chi(1-n)^2w^{1-\xi}(\frac{w}{w^*})^{-\eta}\sigma_y^2 \\
&+\frac{1}{2}\eta(\eta+1)\sigma_{w^*}^2\chi w^{1-\xi}(\frac{w}{w^*})^{-\eta} = 0
\end{aligned} \tag{12}$$

From the above equation, we derive

$$\begin{aligned}
(\chi(1-\xi-\eta))^{-\frac{1}{\xi}} &= \frac{(1-\xi-\eta)\frac{1}{2}[\eta(1-n)\sigma_{yz^*} + (\xi+\eta)(1-n)^2\sigma_y^2 - 2A(1-n)]}{(1-\xi-\eta)\frac{\xi}{1-\xi}} \\
&\quad + \frac{\rho - \eta\mu_{w^*} - \frac{1}{2}\eta(\eta+1)\sigma_{w^*}^2}{(1-\xi-\eta)\frac{\xi}{1-\xi}}
\end{aligned} \tag{13}$$

Substituting equation (13) into equation (10), we have

$$\frac{c}{w} = \frac{\rho - \eta\mu_{w^*} + (1-\xi-\eta)\frac{1}{2}[(\xi+\eta)(1-n)^2\sigma_y^2 - \eta(1-n)\sigma_{yz^*} - 2A(1-n)] - \frac{1}{2}\eta(\eta+1)\sigma_{w^*}^2}{(1-\xi-\eta)\frac{\xi}{1-\xi}} \tag{14}$$

From equation (11), we determine n as

$$n = \frac{-A - \frac{1}{2}\eta\sigma_{yz^*}}{(\xi+\eta)\sigma_y^2} + 1 \tag{15}$$

Using equation (4) and the specified technology, we get the mean growth rate of the economy

$$\phi_1 = E(\frac{\frac{dw}{dt}}{w}) = (A(1-n) - \frac{c}{w})$$

where $\frac{c}{w}$ and n are given in equations (14) and (15).

As for the transvesality condition

$$\lim_{t\to\infty} E[\delta w^{1-\xi-\eta}(w^*)^{-\eta}e^{-\rho t}] = 0$$

we also can prove that it equivalents to the positivity of consumption-wealth ratio.

0.3 Comparative solution

Now, we focus on the effects of foreign weapon stock on the domestic economy.

$$\frac{\partial\phi_1}{\partial\sigma_{w^*}^2} = \frac{\frac{1}{2}(1-\xi)\eta(\eta+1)}{\xi(1-\xi-\eta)}, \frac{\partial\phi_1}{\partial\mu_{w^*}} = \frac{(1-\xi)\eta}{\xi(1-\xi-\eta)}$$

$$\frac{\partial\phi_1}{\partial A} = \frac{A(1+\xi)}{\xi(\xi+\eta)\sigma_y^2}, \frac{\partial\phi_1}{\partial\sigma_y^2} = -\frac{1}{2}\frac{A^2(1+\xi)}{\xi(\xi+\eta)(\sigma_y^2)^2}$$

$$\frac{\partial\phi_1}{\partial\rho} = -\frac{1-\xi}{\xi(1-\xi-\eta)}$$

Because $0 < \xi < 1, \eta < 0$ and $\xi > 1, \eta > 0$, we have

$$\frac{\partial\phi_1}{\partial\mu_{w^*}} > 0,$$

when $\xi > 1, \eta > 0$;

$$\frac{\partial\phi_1}{\partial\mu_{w^*}} < 0$$

when $0 < \xi < 1, \eta < 0$.

As for $\frac{\partial \phi_1}{\partial \sigma^2_{w^*}}$, we have

$$\frac{\partial \phi_1}{\partial \sigma^2_{w^*}} < 0$$

when $0 < \xi < 1, -1 < \eta < 0$;

$$\frac{\partial \phi_1}{\partial \sigma^2_{w^*}} > 0$$

when $0 < \xi < 1, \eta < -1$, and $\xi > 1, \eta > 0$.

1 Model 2

The preferences of home country is defined on the consumption, c, the home country's total wealth, $w(t)$, which equals the weapon stock , $m(t)$,plus the capital stock, $k(t)$, the foreign total wealth, $w^*(t)$, which is defined as the sum of weapon stock, m^*,and the foreign capital stock, $k^*(t)$, power of the leadership of home country, l, and power of leadership of foreign country, l^*, mathematically, $u(c, w, w^*, l, l^*)$. Suppose the utility function $u(c, w, w^*, l, l^*)$ is twice differentiable and concave. Also, we assume that

$$\begin{aligned} u_1 &> 0, u_2 > 0, u_3 < 0, u_4 > 0, u_5 < 0, u_{11} < 0, u_{22} < 0, \\ u_{44} &< 0, u_{12} = u_{21} > 0, u_{13} = u_{31} < 0, u_{23} = u_{32} > 0 \end{aligned} \tag{16}$$

which asserts that the marginal utility from consumption and home country total wealth is positive and diminishing. But the marginal utility from the foreign country total wealth is negative. The assumption $u_{23} = u_{32} > 0$ implies that an increase in foreign weapon stock will increase the marginal utility of home country stock and defense.

The same assumptions on the technology and the foreign wealth as equations (2) and (3).

We suppose that both the power of the leadership of the home country and the leadership of the foreign country are follow the stochastic processes

$$dl = \mu_l l dt + \sigma_l l dL \tag{17}$$

$$dl^* = \mu_{l^*} l^* dt + \sigma_{l^*} l^* dL^* \tag{18}$$

where the same assumptions on the stochastic terms dL and dL^*.They are assumed to be a temporally independent , normally distributed with mean zero and variance variance dt.

Similarly, the representative agent chooses his capital stock, weapon stock and consumption to maximize his discounted utility, namely,

$$\max E_0 \int_0^\infty u(c, w, w^*, l, l^*)e^{-\rho t}dt$$

subject to the budget constraint (4) and the initial stocks $k(0)$ and $w(0)$ are given.

1.1 Optimalities

Similarly, we introduce the value function $\overline{V}(w, w^*, l, l^*, t)$ and the discounted value function $\overline{X}(w, w^*, l, l^*)$ as the relation

$$\overline{X}(w, w^*, l, l^*)e^{-\rho t} = \overline{V}(w, w^*, l, l^*, t)$$

The representative agent chooses his asset holding, n, and his consumption path, $c(t)$, to maximize the following expression

$$\begin{aligned}
&u(c, w, w^*, l, l^*) - \rho\overline{X} + \overline{X}_w(F((1-n)w) - c) + \overline{X}_{w^*}\mu_{w^*}w^* + \overline{X}_l\mu_l l + \overline{X}_{l^*}\mu_{l^*}l^* \\
&+\frac{1}{2}\overline{X}_{ww^*}\sigma_{yz^*}H((1-n)w)w^* + \frac{1}{2}\overline{X}_{ll^*}ll^*\sigma_{ll^*} + \frac{1}{2}\overline{X}_{ll}l^2\sigma_l^2 + \frac{1}{2}\overline{X}_{l^*l^*}l^{*2}\sigma_{l^*}^2 \\
&+\frac{1}{2}\overline{X}_{ww}H((1-n)w)^2\sigma_y^2 + \frac{1}{2}\overline{X}_{w^*w^*}\sigma_{w^*}^2 w^{*2}
\end{aligned}$$

Thus, we get the optimal conditions for the optimization problem

$$\frac{\partial u(c, w, w^*, l, l^*)}{\partial c} = \overline{X}_w \tag{19}$$

$$\begin{aligned}
&-\overline{X}_w F^{'}((1-n)w) - \frac{1}{2}\overline{X}_{ww^*}\sigma_{yz^*}H^{'}((1-n)w)w^* \\
&-\overline{X}_{ww}H((1-n)w)H^{'}((1-n)w)\sigma_y^2 = 0.
\end{aligned} \tag{20}$$

From quations (19) and (20), we can derive the optimal choices for the asset holding share and the consumption path as functions of $\overline{X}_w$, $\overline{X}_{ww^*}$, and $\overline{X}_{ww}$. With the sustitution of the optimal values for the asset holding and consumption, the value function must satisfy the following Bellman Equation

$$u(c,w,w^*,l,l^*) - \rho\overline{X} + \overline{X}_w(F((1-n)w) - c) + \overline{X}_{w^*}\mu_{w^*}w^* + \overline{X}_l\mu_l l + \overline{X}_{l^*}\mu_{l^*}l^*$$
$$+\frac{1}{2}\overline{X}_{ww^*}\sigma_{yz^*}H((1-n)w)w^* + \frac{1}{2}\overline{X}_{ll^*}ll^*\sigma_{ll^*} + \frac{1}{2}\overline{X}_{ll}l^2\sigma_l^2 + \frac{1}{2}\overline{X}_{l^*l^*}l^{*2}\sigma_{l^*}^2$$
$$+\frac{1}{2}\overline{X}_{ww}H((1-n)w)^2\sigma_y^2 + \frac{1}{2}\overline{X}_{w^*w^*}\sigma_{w^*}^2w^{*2} = 0 \tag{21}$$

1.2 Explicit solutions

To derive the explicit solutions for the optimization problem, we specified the utility function and the technology as

$$u(c,w,w^*,l,l^*) = \frac{c^{1-\gamma}}{1-\gamma}(\frac{w}{w^*})^{-\lambda}(\frac{l}{l^*})^{-\beta} \tag{22}$$

$$F(k) = Ak, H(k) = Ak$$

where γ ,β, and λ satisfy $0 < \gamma < 1, \lambda, \beta < 0$ and $\gamma > 1, \lambda, \beta > 0$, A is a positive constant.

We conjecture the value function as the form

$$\overline{X}(w,w^*,l,l^*) = \delta w^{1-\gamma-\lambda}(w^*)^{\lambda}(\frac{l}{l^*})^{-\beta} \tag{23}$$

wiht the coefficient δ is to be determined.

Taking partial derivations, we have

$$\begin{aligned}
\overline{X}_w &= \delta(1-\gamma-\lambda)w^{-\gamma-\lambda}(w^*)^{\lambda}(\frac{l}{l^*})^{-\beta},\ \overline{X}_{ww} = \delta(1-\gamma-\lambda)(-\gamma-\lambda)w^{-\gamma-\lambda-1}(w^*)^{\lambda}(\frac{l}{l^*})^{-\beta},\\
\overline{X}_{w^*} &= \delta\lambda w1^{-\gamma-\lambda}(w^*)^{\lambda-1}(\frac{l}{l^*})^{-\beta},\ \overline{X}_{w^*w^*} = \delta\lambda(\lambda+1)w^{1-\gamma-\lambda}(w^*)^{\lambda-}(\frac{l}{l^*})^{-\beta},\\
\overline{X}_l &= -\delta\beta w1^{-\gamma-\lambda}(w^*)^{\lambda}(l)^{-\beta-1}l^{*\beta},\ \overline{X}_{ll} = \delta\beta(\beta+1)w^{1-\gamma-\lambda}(w^*)^{\lambda}(l)^{-\beta-2}l^{*\beta},\\
\overline{X}_{l^*} &= \delta\beta w^{1-\gamma-\lambda}(w^*)^{\lambda}(l)^{-\beta}(l^*)^{\beta-1},\ \overline{X}_{l^*l^*} = \delta\beta(\beta-1)w^{1-\gamma-\lambda}(w^*)^{\lambda}(l)^{-\beta}(l^*)^{\beta-2},\\
\overline{X}_{ll^*} &= -\delta\beta^2w^{1-\gamma-\lambda}(w^*)^{\lambda}(l)^{-\beta-1}(l^*)^{\beta-1}
\end{aligned}$$

Substituting the above expressions of partial derivatives into equations (19) and (20), we derive

$$c^{-\gamma} = \delta(1-\gamma-\lambda)w^{-\gamma}$$

or

$$\frac{c}{w} = (\delta(1-\gamma-\lambda))^{-\frac{1}{\gamma}} \tag{24}$$

$$-(1-\gamma-\lambda)A\delta w^{1-\gamma-\lambda}(w^*)^{\lambda}(\frac{l}{l^*})^{-\beta} - \tfrac{1}{2}(1-\gamma-\lambda)\lambda\sigma_{yz^*}\delta w^{1-\gamma-\lambda}(w^*)^{\lambda}(\frac{l}{l^*})^{-\beta}$$
$$+(1-\gamma-\lambda)(\gamma+\lambda)\delta w^{1-\gamma-\lambda}(w^*)^{\lambda}(\frac{l}{l^*})^{-\beta}(1-n)\sigma_y^2 = 0$$

or

$$-(1-\gamma-\lambda)A - \tfrac{1}{2}(1-\gamma-\lambda)\lambda\sigma_{yz^*} + (1-\gamma-\lambda)(\gamma+\lambda)(1-n)\sigma_y^2 = 0. \tag{25}$$

Substituting equations (24) and (25) into the Bellman equation (21) leads to

$$(\delta(1-\gamma-\lambda))^{-\frac{1-\gamma}{\gamma}}\frac{w^{1-\gamma}}{1-\gamma}(\frac{w}{w^*})^{-\lambda}(\frac{l}{l^*})^{-\beta} - \rho\delta w^{1-\gamma-\lambda}(w^*)^{\lambda}(\frac{l}{l^*})^{-\beta} + \delta\lambda w^{1-\gamma-\lambda}(w^*)^{\lambda-1}(\frac{l}{l^*})^{-\beta}\mu_{w^*}w^*$$
$$+\delta w^{1-\gamma}(\frac{w}{w^*})^{-\lambda}(1-\gamma-\lambda)[A(1-n) - (\delta(1-\gamma-\lambda))^{-\frac{1}{\gamma}}] + \delta\beta w^{1-\gamma-\lambda}(w^*)^{\lambda}l^{-\beta}l^{*\beta}\mu_l$$
$$-\mu_{l^*}\delta\beta w^{1-\gamma-\lambda}(w^*)^{\lambda}l^{-\beta}(l^*)^{\beta} + \frac{1}{2}\delta(1-\gamma-\lambda)\lambda(1-n)w^{1-\gamma}(\frac{w}{w^*})^{-\lambda}(\frac{l}{l^*})^{-\beta}\sigma_{yz^*}$$
$$-\frac{1}{2}\delta\beta^2 w^{1-\gamma-\lambda}(w^*)^{\lambda}l^{-\beta}(l^*)^{\beta}\sigma_{ll^*} + \frac{1}{2}\delta\beta(\beta+1)w^{1-\gamma-\lambda}(w^*)^{\lambda}l^{-\beta}l^{*\beta}\sigma_l^2$$
$$+\frac{1}{2}\delta\beta(\beta-1)w^{1-\gamma-\lambda}(w^*)^{\lambda}l^{-\beta}(l^*)^{\beta} + \frac{1}{2}\lambda(\lambda+1)\sigma_{w^*}^2\delta w^{1-\gamma}(\frac{w}{w^*})^{-\lambda}(\frac{l}{l^*})^{-\beta}$$
$$-\frac{1}{2}(1-\gamma-\lambda)(\gamma+\lambda)\delta(1-n)^2w^{1-\gamma}(\frac{w}{w^*})^{-\lambda}(\frac{l}{l^*})^{-\beta}\sigma_y^2 = 0$$

From the above equation, we derive

$$\begin{aligned}
&(\delta(1-\gamma-\lambda))^{-\frac{1}{\gamma}} \\
= \;& \frac{\rho - \lambda\mu_{w^*} - \mu_{l^*}\beta + \beta\mu_l - \frac{1}{2}\lambda(\lambda+1)\sigma_{w^*}^2 + \frac{1}{2}\beta^2\sigma_{ll^*} - \frac{1}{2}\beta(\beta-1)\sigma_{l^*}^2 - \frac{1}{2}\beta(\beta+1)\sigma_l^2}{(1-\gamma-\lambda)^{\frac{\gamma}{1-\gamma}}} \\
&+\frac{(1-\gamma-\lambda)^{\frac{1}{2}}[(\gamma+\lambda)(1-n)^2\sigma_y^2 - \lambda(1-n)\sigma_{yz^*} - 2A(1-n)]}{(1-\gamma-\lambda)^{\frac{\gamma}{1-\gamma}}}
\end{aligned} \tag{26}$$

Substitution equation (26) into equation (24), we have

$$\begin{aligned}
\frac{c}{w} = \;& \frac{\rho - \lambda\mu_{w^*} - \mu_{l^*}\beta + \beta\mu_l - \frac{1}{2}\lambda(\lambda+1)\sigma_{w^*}^2 + \frac{1}{2}\beta^2\sigma_{ll^*} - \frac{1}{2}\beta(\beta-1)\sigma_{l^*}^2 - \frac{1}{2}\beta(\beta+1)\sigma_l^2}{(1-\gamma-\lambda)^{\frac{\gamma}{1-\gamma}}} \\
&+\frac{(1-\gamma-\lambda)^{\frac{1}{2}}[(\gamma+\lambda)(1-n)^2\sigma_y^2 - \lambda(1-n)\sigma_{yz^*} - 2A(1-n)]}{(1-\gamma-\lambda)^{\frac{\gamma}{1-\gamma}}}
\end{aligned}$$

(27)

where n is determined by equation (25)

$$n = \frac{-A - \frac{1}{2}\lambda\sigma_{yz^*}}{(\gamma+\lambda)\sigma_y^2} + 1 \tag{28}$$

Similarly, from equation (4) and the specified technology, we get the mean growth rate of the economy

$$\phi_2 = E(\frac{\frac{dw}{dt}}{w}) = (A(1-n) - \frac{c}{w})$$

where $\frac{c}{w}$ and n are given in equations (27) and (28).

As for the transvesality condition

$$\lim_{t\to\infty} E[\delta w^{1-\gamma-\lambda}(w^*)^{\lambda}(\frac{l}{l^*})^{-\beta}e^{-\rho t}] = 0$$

we also can prove that it equivalents to the positivity of consumption-wealth ratio.

1.3 Comparative solution

Now, we focus on the effects of foreign weapon stock on the domestic economy.

$$\frac{\partial\phi_2}{\partial\sigma_{w^*}^2} = \frac{\frac{1}{2}(1-\gamma)\lambda(\lambda+1)}{\gamma(1-\gamma-\lambda)}, \frac{\partial\phi_2}{\partial\mu_{w^*}} = \frac{(1-\gamma)\lambda}{\gamma(1-\gamma-\lambda)}$$

$$\frac{\partial\phi_2}{\partial\sigma_l^2} = \frac{\frac{1}{2}(1-\gamma)\beta(\beta+1)}{\gamma(1-\gamma-\lambda)}, \frac{\partial\phi_2}{\partial\mu_l} = -\frac{(1-\gamma)\beta}{\gamma(1-\gamma-\lambda)}$$

$$\frac{\partial\phi_2}{\partial\sigma_{l^*}^2} = \frac{\frac{1}{2}(1-\gamma)\beta(\beta-1)}{\gamma(1-\gamma-\lambda)}, \frac{\partial\phi_2}{\partial\mu_{l^*}} = \frac{(1-\gamma)\beta}{\gamma(1-\gamma-\lambda)}$$

$$\frac{\partial\phi_2}{\partial A} = \frac{A(1+\gamma)}{\gamma(\gamma+\lambda)\sigma_y^2}, \frac{\partial\phi_2}{\partial\sigma_y^2} = -\frac{1}{2}\frac{A^2(1+\gamma)}{\gamma(\gamma+\lambda)(\sigma_y^2)^2}$$

$$\frac{\partial\phi_2}{\partial\rho} = -\frac{1-\gamma}{\gamma(1-\gamma-\lambda)}$$

Because $0 < \gamma < 1, \lambda < 0$ and $\gamma > 1, \lambda > 0$, we have

$$\frac{\partial \phi_2}{\partial \mu_{w^*}} > 0, \frac{\partial \phi_2}{\partial \mu_{l^*}} > 0, \frac{\partial \phi_2}{\partial \mu_l} < 0$$

when $\gamma > 1, \lambda > 0, \beta > 0$;

$$\frac{\partial \phi_2}{\partial \mu_{w^*}} < 0, \frac{\partial \phi_2}{\partial \mu_{l^*}} < 0, \frac{\partial \phi_2}{\partial \mu_l} > 0$$

when $0 < \gamma < 1, \lambda < 0, \beta < 0$.

As for $\frac{\partial \phi_2}{\partial \sigma^2_{w^*}}$, we have

$$\frac{\partial \phi_2}{\partial \sigma^2_{w^*}} < 0, \frac{\partial \phi_2}{\partial \sigma^2_{l^*}} > 0, \frac{\partial \phi_2}{\partial \sigma^2_l} < 0$$

when $0 < \gamma < 1, -1 < \lambda, \beta < 0$;

$$\frac{\partial \phi_2}{\partial \sigma^2_{w^*}} > 0, \frac{\partial \phi_2}{\partial \sigma^2_{l^*}} > 0, \frac{\partial \phi_2}{\partial \sigma^2_l} > 0$$

when $0 < \gamma < 1, \lambda, \beta < -1$, and

$$\frac{\partial \phi_2}{\partial \sigma^2_{w^*}} > 0, \frac{\partial \phi_2}{\partial \sigma^2_{l^*}} > 0, \frac{\partial \phi_2}{\partial \sigma^2_l} > 0$$

when $\gamma > 1, \lambda, \beta > 0$.

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Growth rate varus the time preference ξ
Parameters are $A = .2$, $\mu_{w^*} = 0.005$, $\sigma^2_{w^*} = 0.04$, $\sigma^2_y = 0.4$, $\rho = 0.011$, and $\eta = -.3$.

Growth rate varus the effective desire η
Parameters are $A = .2$, $\mu_{w^*} = 0.005$, $\sigma^2_{w^*} = 0.04$, $\sigma^2_y = 0.4$, $\rho = 0.011$, and $\xi = 2$

Growth rate varus the time preference γ
Parameters are $A = .2$, $\mu_{w^*} = 0.005$, $\sigma^2_{w^*} = 0.04$, $\sigma^2_y = 0.4$, $\mu_l = 0.005$, $\sigma^2_l = 0.04$, $\mu_{l^*} = 0.005$, $\sigma^2_{l^*} = 0.04$, $\rho = 0.011$, $\beta = -0.3$, and $\lambda = -.3$

Growth rate varus the effective desire λ
Parameters are $A = .2$, $\mu_{w^*} = 0.005$, $\sigma^2_{w^*} = 0.04$, $\sigma^2_y = 0.4$, $\mu_l = 0.005$, $\sigma^2_l = 0.04$, $\mu_{l^*} = 0.005$, $\sigma^2_{l^*} = 0.04$, $\rho = 0.011$, $\beta = -0.3$, and $\xi = 2$

Growth rate varus the effective desire β
Parameters are $A = .2$, $\mu_{w^*} = 0.005$, $\sigma^2_{w^*} = 0.04$, $\sigma^2_y = 0.4$, $\mu_l = 0.005$, $\sigma^2_l = 0.04$, $\mu_{l^*} = 0.005$, $\sigma^2_{l^*} = 0.04$, $\rho = 0.011$, $\lambda = -0.3$, and $\xi = 2$