Patent Protection, Technological Change and Wage Inequality*

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Abstract

We develop a directed-technological-change model to address the issue of the optimal patent system and investigate how the optimal patent system influences the direction of technological change and the inequality of wage, where patents are categorized as skill- and labor-complementary. The major results are: (i) Finite patent breadth maximizes the social welfare level; (ii) Optimal patent breadth increases with the amount of skilled (unskilled) workers; (iii) Optimal patent protection is skill-biased, because an increase in the amount of skilled workers increases the dynamic benefits of the protection for skill-complementary patents via the economy of scale of skill-complementary technology; (iv) Skill-biased patent protection skews inventions towards skills, thus increasing wage inequality.

Keywords: Patent Breadth; Skill-Biased Patent Protection; Skill-Biased Technological Change; Wage Inequality; Economic Growth

JEL Classification: O31; O34; J31

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1 Introduction

One important policy issue is how to design optimal patent protection to promote innovation. Numerous papers examines optimal patent protection in the framework of endogenous growth theory. This literature, however, does not consider the direction of technological change. Indeed, several models have been developed to investigate directed technological change, in which technology is complementary to skilled or unskilled workers (Acemoglu, 1998, 2002a, 2003, 2007, Thoenig and Verdier, 2003). To the best of our knowledge, the literature on directed technological change does not address the issue how to protect patents.

We would like to combine the literature on the optimal patent system and the one on directed technological change in this paper. For this purpose, an endogenous growth model is constructed to study the optimal patent system in the directed-technological-change economy and how the optimal patent system impacts the direction of technological change and wage inequality, in which patents are classified as skill- and labor-complementary. The first main results are: (i) Optimal patent breadth is finite, and increases with the quantity of skilled and unskilled workers; (ii) Optimal patent protection is skill-biased. The basic idea is that the more skilled workers, the greater the economy of scale of skill-complementary technology, and thus the more the dynamic benefits of protection for skill-complementary patent; (iii) Skill-biased patent protection induces technological change towards skills, thus increasing wage inequality. Consequently, an important policy implication is that it is optimal to have different patent protection for different type of innovation. For instance, it will be beneficial to strengthen protection for labor-complementary patent in developing countries, whereas it will be better off to increase protection for skill-complementary patent in developed countries. Consequently, it may be time to consider whether or not patent rules that are neutral to technologies in the real world are the best.

Ever since Schumpeter (1942), we have known that it is necessary to provide inventors with some form of market power to give them incentives to invent in the first place. Indeed, two branches of literature have been devoted to the topic of optimal patent protection, namely, that of the patent system that best solves the trade-off between providing enough incentives to invent ex ante and minimizing dead-weight losses ex post. One branch of literature has studied optimal patent protection in a partial equilibrium model (for example, Nordhaus, 1969, Gilbert and Shapiro, 1990, Klemperer, 1990, Gallini, 1992).[1] Another branch of literature has looked at the

[1] Scotchmer (2004) provides a comprehensive review on this literature.

optimal patent system within a framework of general equilibrium (e.g., Judd, 1985, Goh and Oliver, 2002, Kwan and Lai, 2003, Iwaisako and Futagami, 2003, Horii and Iwaisako, 2007, Futagami and Iwaisako, 2007, Chu, 2010, Chu and Pan, 2012). This paper follows and complements the second branch of literature by taking account of the direction of technological change.

This paper relates to Acemoglu and Akcigit (2009), Mosel (2009), Chu (2011). These models all conclude that one-size-fit-all patent policy is unlikely to provide the appropriate incentives for innovation in every industry.[2] Our model adds this literature by analyzing optimal patent protection across industries in a framework of directed technological change. The related literature also includes Cozzi and Galli (2009) and Adams (2008). Cozzi and Galli (2009) emphasize that a strengthening of intellectual property rights will lead to an increase in wage inequality. Adams (2008) reports that strengthening intellectual property rights and openness are positively correlated with income inequality in developing countries. The main difference between our paper and Cozzi and Galli's is that the direction of technological change is considered, while the one between our paper and Adam's is that the latter focus on income inequality instead of wage inequality.

The paper proceeds as follows. In the next section, building on Acemoglu (1998, 2007, 2009) and the literature on an optimal patent system, we introduce the model. In Section 3, we address the issue of optimal patent protection and explore the effect of patent protection on the direction of technological change and wage inequality. Conclusions are found in Section 4.

2 The Model

Consider an economy populated with H skilled workers and L unskilled workers, who supply one unit labor inelastically. Representative consumers are with constant relative risk aversion (CRRA) preference. These consumers maximize intertemproral utility[3]

$$\int_0^\infty \frac{C(t)^{1-\theta}-1}{1-\theta}e^{-\rho t}dt, \tag{1}$$

where $C(t)$ is consumption at time t, θ is the coefficient of relative risk aversion (or intertemporal elasticity of substitution) and ρ is the subjective discount rate. We drop the time index as long as this causes no confusion.

[2] Burk and Lemley (2009) provide overwhelming evidence that innovation works differently in different industries, and state that patent protection should be different across sectors.

[3] In this paper, we assume $0 < \theta < 1$, because only in the this circumstance, an increase protection for patent has the dynamic benefit and static cost as pointed out in Section 2.

The budget constraint of the consumer is:

$$C + I + R \leq Y = \left[(Y_l)^{\frac{\epsilon-1}{\epsilon}} + (Y_h)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{2}$$

where I is investment, and R is total R&D expenditure. The production function in (2) implies that output aggregate is defined over a constant elasticity of substitution (CES) aggregate of a labor-intensive good, Y_l, and a skill-intensive good, Y_h. Parameter $\epsilon \in [0, \infty)$ is the elasticity of substitution between the two goods. When $\epsilon = \infty$, the two goods are perfect substitutes, and the function is linear. When $\epsilon = 1$, the function is Cobb-Douglas. And when $\epsilon = 0$, there is no substitution between the two goods, and the production function is Leontieff.

Following Acemoglu (1998, 2002, 2009), the labor-intensive good is produced from unskilled workers and different types of labor-complementary machines or intermediates, while the skill-intensive good is produced from skilled workers and a set of differentiated skill-complementary machines. The key assumption is that none of these machines are used by both types of workers. Specifically, the production functions of the skill-intensive and the labor-intensive good are as follows[4]:

$$Y_h = \frac{1}{1-\alpha} \int_0^{A_h} k_h(i)^{1-\alpha} \, di \cdot (ZH)^{\alpha}, \tag{3}$$

and

$$Y_l = \frac{1}{1-\alpha} \int_0^{A_l} k_l(i)^{1-\alpha} \, di \cdot L^{\alpha}, \tag{4}$$

where $\alpha \in (0, 1)$, A_z is the number of machines complementary to z, $k_z(i)$ is the quantity of machines of variety i together with workers of skill level z, $z = h$ or l. Indexes h and l denote skilled and unskilled workers, respectively. $Z > 1$ measures the relative productivity of skilled workers. Consequently, ZH represents the effective amount of skilled workers. The production functions in (3) and (4) exhibit constant returns to scale in input factors: the double of labor and the quantity of all intermediate goods doubles output. However, the production possibilities set of the economy will exhibit increasing returns to scale because technological knowledge, A_z, are endogenized.

Technological progress takes the form of the increase in A_z over time. Using μ units of the final good, a firm can develop a new variety of either type of machine. Therefore, the accumulation equation of technological knowledge is given by[5]

[4] The main results are not altered when we assume that $Y_h = \frac{\Phi}{1-\alpha} \int_0^{A_h} k_h(i)^{1-\alpha} \, di \cdot (ZH)^{\alpha}$ and $Y_l = \frac{\Psi}{1-\alpha} \int_0^{A_l} k_l(i)^{1-\alpha} \, di \cdot L^{\alpha}$, where $\Phi \neq \Phi$.

[5] Assumption $\dot{A}_z = \frac{X_z}{\mu_z}$ where $\mu_h \neq \mu_l$ will not change our main results.

$$\dot{A}_z = \frac{X_z}{\mu}, \tag{5}$$

where X_z denotes total output devoted to improving the technology complementary to $z = h$ or l. Equation (5) implies that with a total expenditure of X, there will be X/μ new varieties invented. For convenience, we assume the marginal cost for the production of any machine is constant and equal to one unit of the final good.

A firm that invents a machine obtains the protection granted by patent. As do Goh and Oliver (2002), we assume that the life of a patent is infinite and we focus on the issue of patent breadth. Following Gilbert and Shapiro (1990), Diwan and Rodrik (1991), Iwaisako and Futagami (2003), we define patent breadth as the ability of the patentee to raise the prices for the patented goods over the lifetime of patent. Strong protection granted by the patent increases the number of substitute products that infringe on the patent or raises the costs of imitation, thus allowing the patentee to raise prices. In particular, the prices charged by firms producing the goods that embody inventions are given by

$$\chi_z(i) = \chi_z = 1 + \beta_z, \;\; z = h \text{ or } l, \tag{6}$$

where β_z is the measure of patent breadth. The bigger the β_z, the greater the patent breadth. Using some algebra, we know that the monopoly price maximizing the profits of patent products is $\frac{1}{1-\alpha}$. Therefore, $\beta_z \leq \frac{\alpha}{1-\alpha}$, and patent breadth is infinite as equality holds.

Taking advantage of (3), (4) and (6), we obtain the quantity of machines complementary to skilled and unskilled workers:

$$k_h(i) = \left[p_h / \left(1 + \beta_h\right)\right]^{1/\alpha} \cdot ZH, \tag{7}$$

and

$$k_l(i) = \left[p_l / \left(1 + \beta_l\right)\right]^{1/\alpha} \cdot L, \tag{8}$$

where p_h and p_l are the prices of the skill-intensive and the labor-intensive good, respectively. We normalize the price of consumption aggregate as one. Therefore, the monopoly profits of any intermediate good used by skilled and unskilled workers at time τ are:

$$\pi_h(\tau) = \beta_h \left[p_h / \left(1 + \beta_h\right)\right]^{1/\alpha} \cdot ZH, \tag{9}$$

and

$$\pi_l(\tau) = \beta_l \left[p_l / (1+\beta_l)\right]^{1/\alpha} \cdot L. \tag{10}$$

It is simple to show that when $\beta_z < \frac{\alpha}{1-\alpha}$, $\frac{\partial \pi_z(\tau)}{\partial \beta_z} > 0$ and when $\beta_z = \frac{\alpha}{1-\alpha}$, $\frac{\partial \pi_z(\tau)}{\partial \beta_z} = 0$. This implies that the monopoly profits of a new variety of machines increase with patent breadth, and maximum extent of patent breadth makes the profits highest.

Plugging (7) and (8) into (3) and (4) respectively, we obtain

$$Y_h = \frac{1}{1-\alpha}\left[p_h / (1+\beta_h)\right]^{(1-\alpha)/\alpha} \cdot A_h Z H \text{ and } Y_l = \frac{1}{1-\alpha}\left[p_l / (1+\beta_l)\right]^{(1-\alpha)/\alpha} \cdot A_l L. \tag{11}$$

The market for the skill-intensive and the labor-intensive good is competitive, thus using (2) and (11), we find that the relative price of the two goods is

$$p = \frac{p_h}{p_l} = \left(\frac{1+\beta_h}{1+\beta_l}\right)^{\frac{1-\alpha}{1+\alpha(\epsilon-1)}} \left(\frac{A_h}{A_l} \cdot \frac{ZH}{L}\right)^{-\frac{\alpha}{1+\alpha(\epsilon-1)}}. \tag{12}$$

This shows that when either the technology is highly skill-biased (high A_h/A_l) or the relative supply of skilled workers is great (high H/L), the relative supply of the skill-intensive good is large and the relative price is low. The relatively strong protection for the skill-complementary patent (high β_h/β_l) leads to a decrease in the relative demand for machines complementing skills, thus declining the relative supply of the skill-intensive good and increasing the relative price.

3 Equilibrium

We now characterize the equilibrium develop a simple model to explore why the optimal patent protection should be skill-biased and how the skill-biased patent protection affects the degree of skill bias of technology and wage inequality.

3.1 Optimal Patent System

Free-entry in the R&D business implies that the cost of invention, μ, should be equal to the present value of profits of any intermediate good, V_z. That is,

$$\mu = \int_t^{\infty} e^{-\int_t^{\tau} r(s)ds} \pi_z(\tau) d\tau = V_z, \tag{13}$$

where r is the rental price of capital. It suggests that in the equilibrium the flow profits from selling labor- and skill-complementary machines should be equal, i.e., $\pi_h = \pi_l$. Therefore, inspection of (9) and (10) results in

$$\frac{p_h}{p_l} = \frac{1+\beta_h}{1+\beta_l}\left(\frac{\beta_h}{\beta_l}\cdot\frac{ZH}{L}\right)^{-\alpha}. \tag{14}$$

Intuitively, the bigger the amount of skilled workers, the larger the market for skill-complementary machines, thus the lower the relative price of the skill-intensive good to ensure $\pi_h = \pi_l$. Moreover, it is clear that $\frac{\partial\left[(1+\beta_z)\beta_z^{-\alpha}\right]}{\partial\beta_z} \leq 0$. This implies that the stronger the protection for the skill-complementary patent, the larger the profits of new invention complementing skills, thus the relative price of the skill-intensive good has to be lower to make the flow profits of labor- and skill- complementary machines equal.

Combining (12) and (14), we obtain

$$\frac{A_h}{A_l} = \left(\frac{1+\beta_h}{1+\beta_l}\right)^{-\epsilon}\left(\frac{\beta_h}{\beta_l}\right)^{1+\alpha(\epsilon-1)}\left(\frac{ZH}{L}\right)^{\alpha(\epsilon-1)}. \tag{15}$$

By (2), (11) and (12), we know that the elasticity of substitution between skilled and unskilled workers is $1+\alpha(\epsilon-1)$. Thus, (15) shows that the relative degree of skill bias of technology, A_h/A_l, is determined by the relative factor supply and the elasticity of substitution between the two factors, skilled and unskilled workers. When $\epsilon > 1$, i.e., when the two factors are gross substitutes, technological change is towards skilled workers; while $\epsilon < 1$, i.e., when the two factors are gross complements, technological change is unskilled-biased (skill-replacing).[6] Almost all estimates show an elasticity of substitution between skilled and unskilled workers greater than 1, most likely greater than 1.4, and perhaps as large as 2, that is, $1 \leq 1+\alpha(\epsilon-1) \leq 2$ (see, for example, Freeman, 1986, Acemoglu, 2002b). Hence, we take ϵ to be greater than 1 in the rest of the paper. In addition, using some algebra, we find that $\frac{\partial(1+\beta_z)^{-\epsilon}(\beta_z)^{1+\alpha(\epsilon-1)}}{\partial\beta_z} > 0$ as $\beta_z \leq \frac{\alpha}{1-\alpha}$. Therefore, the degree of skill bias of technology, A_h/A_l, rises with the breadth of the skill-complementary patent. Strong protection for the skill-complementary patent results in an increase in the profits selling skill-complementary machines, thus inducing skill-biased technological change.

Inspection of the production function in (2) yields

$$\left(p_l^{1-\epsilon} + p_h^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} = 1 \tag{16}$$

[6] When the elasticity of substitution is equal to 1, technological change is never biased towards skilled or unskilled workers.

Using (14) and (16), we find the price indices to be

$$p_h = \left[1 + \left(\frac{1+\beta_h}{1+\beta_l}\right)^{\epsilon-1} \left(\frac{\beta_h ZH}{\beta_l L}\right)^{\alpha(1-\epsilon)}\right]^{-\frac{1}{1-\epsilon}} \text{ and } p_l = \left[1 + \left(\frac{1+\beta_h}{1+\beta_l}\right)^{1-\epsilon} \left(\frac{\beta_h ZH}{\beta_l L}\right)^{-\alpha(1-\epsilon)}\right]^{-\frac{1}{1-\epsilon}}. \tag{17}$$

Therefore, the price of the skill-intensive good is lower (and the price of the labor-intensive good is higher) when the relative supply of skilled workers is larger or the relative protection for the skill-complementary patent is stronger.

Maximization of utility function in (1), subject to a standard budget constraint, yields the usual formula for the growth rate of consumption:

$$\frac{\dot{C}}{C} = \frac{1}{\theta}(r - \rho), \tag{18}$$

It says that either the intertemporal elasticity of substitution is bigger or the discount rate is smaller, the growth rate of consumption is higher.

Combining (9), (10), (13), (17) and (18), we obtain the equilibrium economic growth rate:[7]

$$g = \frac{1}{\theta}\left\{\frac{\left[(1+\beta_h)^{1-\epsilon}(\beta_h ZH)^{\alpha(\epsilon-1)} + (1+\beta_l)^{1-\epsilon}(\beta_l L)^{\alpha(\epsilon-1)}\right]^{\frac{1}{\alpha(\epsilon-1)}}}{\mu} - \rho\right\}. \tag{19}$$

Obviously, $\frac{\partial g}{\partial \beta_z} \geq 0$, that is, strong patent protection increases the growth rate. This denotes the dynamic benefits of the patent system. Intuitively, great patent breadth raises the profits of invention, thus inducing resources devoted to inventing technology and a high growth rate. When there is a bigger amount of skilled (unskilled) workers, the market for machines complementing skilled (unskilled) workers is larger, hence the profits of invention are higher. Therefore, the growth rate increases with the quantity of skilled and unskilled workers, i.e., $\frac{\partial g}{\partial H} > 0$ and $\frac{\partial g}{\partial L} > 0$.

Now let us briefly investigate the stability of the equilibrium. For given patent breadth, according to Acemoglu and Zilibotti (2001), we know that off the balanced growth path, there will only be one type of innovation. That is, if $V_h/V_l > 1$, only skill-complementary innovation is taken place, and if $V_h/V_l < 1$, innovators only undertake labor-complementary R&D. Clearly, when A_h/A_l is lower than in (15), $V_h/V_l > 1$, and

[7] The growth rate expression suggests that the parameters must be assumed to be such that $g \geq 0$. Otherwise, the constraint that A_z cannot be decreasing would be violated, and the free-entry condition for R&D would not hold with equality. Therefore, the measure of patent breadth, β_h and β_l, will not be zero.

vice versa when A_h/A_l is too high. As a consequence, the transitional dynamics of the economy are stable.[8]

Since the utility function is continuous on $\beta_z \in \left[0, \frac{\alpha}{1-\alpha}\right]$, there exists β_z^* maximizing social welfare.[9] Output growth maximization, however, may not be equivalent to welfare maximization. Therefore, infinite patent breadth, which maximizes the output growth rate, is not optimal. Intuitively, when patent breadth is infinite, the benefit of increasing patent protection is zero (i.e., $\left.\frac{\partial g}{\partial \beta_z}\right|_{\beta_z=\frac{\alpha}{1-\alpha}} = 0$), but the cost is bigger than zero because of monopoly pricing. Hence, we state the following proposition

Proposition 1 *The patent breadth that maximizes social welfare is finite, i.e.,* $\beta_z^* < \frac{\alpha}{1-\alpha}$.

Proof. See the Appendix. ■

Propositions 1 shows that finite patent breadth maximizes the social welfare level. This result is similar to the results of many existing studies. For instance, Gilbert and Shapiro (1990) have argued that narrow patent is optimal because broad patent is costly for society in that it gives excessive monopoly power to the patent holder.

In recent decades, the relative quantity of skilled workers has increased sharply in most developed countries. To the best of our knowledge, there is no previous paper investigating the impact of an increased supply of skills on the optimal patent system.

Proposition 2 *The more the quantity of (unskilled) skilled workers, the greater the breadth of patent, that is,* $\frac{\partial (\beta_z)^*}{\partial ZH} > 0$ *and* $\frac{\partial (\beta_z)^*}{\partial L} > 0$. *Moreover, if the effective amount of skilled workers is bigger (smaller) than the amount of unskilled workers, the optimal breadth of the skill-complementary patent is broader (narrower) than that of the labor-complementary patent, and if the effective amount of skilled workers is equal to the amount of unskilled workers, the optimal breadth of the two types of patents is same. That is, when* $ZH > L$, $\beta_h^* > \beta_l^*$; *while* $ZH = L$, $\beta_h^* = \beta_l^*$; *when* $ZH < L$, $\beta_h^* < \beta_l^*$.[10]

Proof. See the Appendix. ■

[8] See Acemoglu and Zilibotti (2001) for a formal and detail proof.

[9] Maybe β_z^* is unique. However, the analysis is substantially complicated. Fortunately, the following results are independent of whether β_z^* is singleton or not. Indeed, when β_z^* is not unique, it is reasonable to choose β_z^* that maximizes the growth rate as the optimal patent breadth. Moreover, it is easy to know that $\frac{\partial U(\beta_h, \beta_l)}{\partial \beta_z} > 0$ at β_z which satisfies $g\left(\beta_h, \beta_l\right) = 0$. Therefore, optimal β_z is interior solution, i.e., $\frac{\partial U(\beta_h, \beta_l)}{\partial \beta_z} = 0$.

[10] When we take into account of the other differences in the production of the skill-intensive and the labor-intensive good, even though $ZH < L$, $\beta_h^* > \beta_l^*$ may hold. For example, if the production technology of the skill-intensive good is assumed to be $Y_h = \frac{\phi}{1-\alpha} \int_0^{A_h} k_h(i)^{1-\alpha} di \cdot (ZH)^\alpha$ where $\phi > 1$, then $\phi^{1/\alpha} ZH > L$, $\beta_h^* > \beta_l^*$.

The intuition is straightforward. The more the supply of skilled workers, the greater the economy of scale of knowledge complementing skilled workers, thus the broader the breadth of the skill-complementary patent. That is, $\frac{\partial(\beta_h)^*}{\partial H} > 0$. Moreover, the relative price of the labor-intensive good rises with the amount of skilled workers, thus the economy of scale of knowledge complementing unskilled workers becomes large. As a consequence, the protection for labor-complementary patent strengthens, namely, $\frac{\partial(\beta_l)^*}{\partial H} > 0$. By symmetry, $\frac{\partial(\beta_h)^*}{\partial L} > 0$.

As an important implication of this proposition, the patent policy should be skill-biased, that is, $\frac{\partial(\beta_h^*/\beta_l^*)^*}{\partial(H/L)} > 0$. Precisely, it is easy to see that, in any line segment of $ZH = L$, $\frac{\partial(\beta_h)^*}{\partial ZH} > \frac{\partial(\beta_l)^*}{\partial ZH} > 0$ and $\frac{\partial(\beta_l)^*}{\partial L} > \frac{\partial(\beta_h)^*}{\partial L} > 0$, respectively. This implies, at least in a neighborhood of the segment, there is a skill-biased patent protection, that is, for $ZH > L$, $ZH' > L'$ and $\frac{ZH'}{L'} > \frac{ZH}{L}$, $\frac{(\beta'_h)^*}{(\beta'_l)^*} > \frac{\beta_h^*}{\beta_l^*}$, and for $ZH < L$, $ZH' < L'$ and $\frac{ZH'}{L'} < \frac{ZH}{L}$, $\frac{(\beta'_h)^*}{(\beta'_l)^*} < \frac{\beta_h^*}{\beta_l^*}$. Hence, we would like to claim that a relative increasing of skill workers will lead to a relatively stronger protection towards skill-complementary patent. That is,

Corollary 1 *Optimal patent protection is skill-biased, i.e.,* $\frac{\partial(\beta_h^*/\beta_l^*)^*}{\partial(H/L)} > 0$*, when* $ZH \approx L$.

Over the decades, patent systems have been harmonized worldwide. During the 1980s and early 1990s, patent protection in some developing countries proved inadequate. For instance, biotechnology, software and business methods were widely considered unpatentable, and most importantly, procedures and resources for the enforcement of patent laws were often inadequate at protecting patent holders under even the existing weak standards of patent protection (Maskus, 2000). A major turning point in the global protection of intellectual property (patent protection) was the enforcement of an agreement on TRIPs in 1995. All WTO member countries have signed on to TRIPs and are obligated to implement the most rudimentary rules for IPRs protection based on the U.S. and EU practices. Moreover, when additional countries join the WTO, they must meet the minimum standard required by TRIPs. Therefore, the implementation of the agreement on TRIPs has significantly increased harmonization of IPRs protection worldwide.[11] It is notable that pressure from the United States and the European Union played a critical role in the international harmonization of IPRs protection (e.g., Grossman and Lai, 2004). Widely publicized American negotiations and threats in the 1980s and 1990s resulted in stronger IPRs legislation in South Korea, Argentina, Brazil, Thailand, Taiwan, and China.

[11] Since TRIPs only sets out minimum standards with which countries must comply, it does not aim at complete harmonization.

Similarly, European Union negotiations and assistance advanced IPRs protection in Egypt and Turkey (Maskus, 2000).

Since there are less skilled workers in developing countries, one implication of Proposition 2 and Corollary 1 is that the harmonization of patent systems worldwide may decrease the social welfare level of many developing countries.[12]

3.2 Wage Inequality

As vast quantities of empirical papers have documented over past decades, there has been an increase in wage inequality in most developed and developing countries.[13] The standard explanation for this pattern is that technological change has been skill-biased over this period. We argue here that skill-biased patent protection could be another cause.

Now let us first address the problem of wage inequality in developed countries. Inspection of (11) yields the ratio of wages paid for skilled workers to the ones paid for unskilled workers

$$\frac{w_h}{w_l} = \left(\frac{p_h}{p_l}\right)^{1/\alpha} \cdot \left(\frac{1+\beta_h}{1+\beta_l}\right)^{-(1-\alpha)/\alpha} \frac{A_h Z}{A_l}. \tag{20}$$

It suggests that skill premia are greater when either the relative price of the skill-intensive good is higher or the technology is more skill-biased. Equation (20) also says that when the impact of skill-biased patent protection on the direction of technological change is not considered, great protection for skill-complementary patent reduces the demand for machines used by skilled workers, due to the high price, and thus the marginal product of skilled workers (or wages) declines.

Combining (12), (15) and (20), we get the ratio of wages paid for skilled workers to the ones paid for unskilled workers:

$$\frac{w_h}{w_l} = \underbrace{Z\left(\frac{1+\beta_h}{1+\beta_l}\right)^{-(\epsilon-1)} \left(\frac{\beta_h}{\beta_l}\right)^{\alpha(\epsilon-1)}}_{\substack{\text{effect of skill-biased} \\ \text{patent protection}}} \cdot \underbrace{\left(\frac{ZH}{L}\right)^{\alpha(\epsilon-1)-1}}_{\substack{\text{effect of skill-biased} \\ \text{technological change}}}. \tag{21}$$

[12] Deardorff (1992), Grossman and Lai (2004) stress that while the welfare of developed countries certainly rises with harmonization of patent systems worldwide, that of developing countries may fall, and may well fall by more than the increase in the welfare of the developed countries. Maskus (2000) point out that in the short-term, global patent protection decreases the welfare of developing countries, but it would benefit all countries in the long-run.

[13] See Acemoglu (2002b) and Goldbegr and Pavcnik (2007) for details.

When β_z and $\frac{\beta_h}{\beta_l}$ increase, $\left(\frac{1+\beta_h}{1+\beta_l}\right)^{-(\epsilon-1)}\left(\frac{\beta_h}{\beta_l}\right)^{\alpha(\epsilon-1)}$ becomes larger. Thus, we state

Proposition 3 *When $\alpha(\epsilon-1)-1>0$, skill-biased patent protection and technological change lead to an increase in skill premia in developed countries.*

Taking advantage of (15) and Proposition 2, we know that skill-biased patent protection encourages technological change towards skills, and therefore raising wage inequality. A number of studies have documented that the relative supply of skills have increased over the past decades (see Acemoglu, 2002b, and references therein). As consequence, based on Proposition 2, we conject that patent protection is biased towards skills[14], and thereby enlarging wage inequality.

Let us assume $ZH^N > L^N$ in developed countries and $ZH^S < L^S$ in developing countries.[15] By proposition 2, we know that in the autarkic economy $\beta_h^S < \beta_l^S$. Therefore, (21) implies that skill premia in developing countries in autarky are

$$\frac{w_h^S}{w_l^S} < Z\left(\frac{ZH^S}{L^S}\right)^{\alpha(\epsilon-1)-1}. \tag{22}$$

Now if patent protection is harmonized all over the world and under pressures of the developed countries the developing countries are forced to enforce the patent protection, $\left(\beta_h^N, \beta_l^N\right)$, then by (21), we obtain[16]

$$\frac{\left(w_h^S\right)'}{\left(w_l^S\right)'} > Z\left(\frac{ZH^S}{L^S}\right)^{\alpha(\epsilon-1)-1} \tag{23}$$

Comparing (22) and (23), we obtain the following proposition

Proposition 4 *Harmonization of global patent protection increases wage inequality in developing countries.*

The United States and Europe have been dissatisfied with the situation of weak IPRs protection in many developing countries. Hence, they have tried to force

[14] Clearly, most inventions of biotechnology, software and business methods are complementing to skills, although some of them are used by unskilled workers. Therefore, to some extent, the greater breadth of these patents demonstrates that patent protection is skill-biased. A famous example of broader claims in biotechnology is when Johns Hopkins University was assigned a patent that they claimed covered not only the My-10 antibody but also other antibodies that bind to CD34, although that patent only showed effects with respect to the My-10 antibody (Bar-Shalom and Cook-Deegan, 2002). Owing to 'function claims' in the field of software and business methods, that is, patents claiming 'problems' rather than 'solutions', any invention developed for that problem would infringe the patent (Martinez and Guellec, 2003). Therefore, the breadth of patent for software and business methods is substantially broad.

[15] As a matter of fact, the main results remain unchanged if $\frac{\beta_h^N}{\beta_l^N} > \frac{\beta_h^S}{\beta_l^S}$.

[16] The main results still apply to the case where the patent protection in the developing country is $\left(\phi\beta_h^N, \psi\beta_l^N\right)$, where $\phi < 1$, $\psi < 1$, $\phi\beta_h^N > \beta_h^S$, $\psi\beta_l^N > \beta_l^S$ and $\phi\beta_h^N > \psi\beta_l^N$.

developing countries to increase the protection for patents, in particular for skill-complementary patents. Intuitively, stronger protection of skill-complementary patents forced by the U.S. and European countries induces skill-biased technological change, thus increasing skill premia in developing countries.

4 Conclusion

In this paper, a simple model has been constructed to explore the optimal patent system in the directed-technological-change economy and the impact of the optimal patent system on the direction of technological change and wage inequality, in which patent is categorized as skill- and labor-complementary. We show that: (i) Optimal patent breadth is finite, and rises with the quantity of skilled workers; (ii) An increase in the amount of skilled workers increases the protection for the skill-complementary patent, i.e., optimal patent protection is skill-biased; (iii) Skill-biased patent protection increases wage inequality by encouraging skill-biased technological change.

It must be stressed that we relied on the strong assumption of infinite patent life to obtain unambiguous results. The obvious next issue on the research agenda is to check the robustness of our results to departures from the assumption.

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Appendix: Proof of Propositions

Proof of Proposition 1: As in most of literature, we assume the economy is in its BGP at time 0. Obviously, (15) suggests that

$$A_h(0) = \frac{\beta_h(1+\beta_l)\Omega_l}{\beta_l(1+\beta_h)\Omega_h} A_l(0), \tag{A1}$$

where $\Omega_h = (1+\beta_h)^{\epsilon-1}(\beta_h ZH)^{\alpha(1-\epsilon)}$ and $\Omega_l = (1+\beta_l)^{\epsilon-1}(\beta_l L)^{\alpha(1-\epsilon)}$. In order to investigate whether the optimal patent protection β_z^* is finite or not, we have to compute the utility of a representative agent in the equilibrium. From (1), the discounted sum of utility could be written as, for $0 < \theta < 1$,

$$U(\beta_h, \beta_l) = \frac{C(0)^{1-\theta}}{(1-\theta)[\rho-(1-\theta)g]} - \frac{1}{\rho(1-\theta)}, \tag{A2}$$

where

$$C(0) = p_h Y_h(0) + p_l Y_l(0) - A_h(0)k_h(0) - A_l(0)k_l(0) - \mu g A_h(0) - \mu g A_l(0) \tag{A3}$$

denotes the agent's consumption. Since the amount of initial total capital, $A_h(0)k_h(0) + A_l(0)k_l(0)$, should be taken as given, thus

$$\frac{\partial\left(A_h(0)k_h(0) + A_l(0)k_l(0)\right)}{\partial\,\beta_z} = 0. \tag{A4}$$

From (19), the infinite patent breadth $\beta_z = \frac{\alpha}{1-\alpha}$ maximizes the growth rate, explicitly,

$$\left.\frac{\partial g}{\partial \beta_z}\right|_{\beta_z^* = \frac{\alpha}{1-\alpha}} = 0. \tag{A5}$$

In fact,

$$\left.\frac{\partial \Omega_z}{\partial \beta_z}\right|_{\beta_z^* = \frac{\alpha}{1-\alpha}} = 0, \tag{A6}$$

and from (17),

$$\left.\frac{\partial\, p_z}{\partial \beta_z}\right|_{\beta_z^* = \frac{\alpha}{1-\alpha}} = 0. \tag{A7}$$

By (11) and (A1), direct computation on (A3) gives

$$\begin{aligned} C(0) &= \frac{\beta_h(1+\beta_h)^{-\frac{1}{\alpha}}(1+\beta_l)}{(1-\alpha)\beta_l X_h} p_h^{\frac{1}{\alpha}}\Omega_l ZHA_l(0) + \frac{(1+\beta_l)^{1-\frac{1}{\alpha}}}{1-\alpha} p_l^{\frac{1}{\alpha}} LA_l(0) \\ & \quad -[A_h(0)k_h(0) + A_l(0)k_l(0)] - \mu g\left(\frac{\beta_h(1+\beta_l)\Omega_l}{\beta_l(1+\beta_h)\Omega_h} + 1\right) A_l(0), \end{aligned}$$

where we take $A_l(0)$ be given. Then, we deduce from (A4)—(A7) and the fact $\frac{\partial\,[\beta_h(1+\beta_h)^{-\frac{1}{\alpha}}]}{\partial\,\beta_h}|_{\beta_z=\frac{\alpha}{1-\alpha}} = 0$ that

$$\left.\frac{\partial\, C(0)}{\partial \beta_z}\right|_{\beta_z^*=\frac{\alpha}{1-\alpha}} = -\mu g \frac{(1+\beta_l)}{(1+\beta_h)^2\beta_l\Phi_h}\Omega_l A_l(0)\Big|_{\beta_z^*=\frac{\alpha}{1-\alpha}} < 0.$$

As the partial derivative of $U(\beta_h, \beta_l)$ is

$$\frac{\partial\, U}{\partial\, \beta_z} = \frac{C(0)^{-\theta}}{\rho-(1-\theta)g}\left(\frac{\partial\, C(0)}{\partial\, \beta_z} + \frac{C(0)}{\rho-(1-\theta)g}\frac{\partial\, g}{\partial\, \beta_z}\right), \tag{A8}$$

we conclude from above that $\left.\frac{\partial\, U}{\partial\, \beta_h}\right|_{\beta_z^*=\frac{\alpha}{1-\alpha}} < 0$. Similarly, $\left.\frac{\partial\, U}{\partial\, \beta_L}\right|_{\beta_z^*=\frac{\alpha}{1-\alpha}} < 0$. Therefore, $\beta_z^* < \frac{\alpha}{1-\alpha}$ holds, that is, the patent breadth maximizing social welfare is finite.

Proof of Proposition 2: The economy is assumed to be in its BGP. By (A3), when $A_l(0)$ is fixed, we consider the utility $U(\beta_z)$ as a function with parametric variables H and L, and rewrite it as $U(\beta_z; H, L)$. Let $\beta_z^*(\xi)$ be the finite optimal patent breadth in the economy with ξH skilled workers and ξL unskilled workers. For the simplicity, denote $\beta_z^*(1)$ by β_z^*.

For fixed $A_z(0)$ and β_z, $C(0)$ increases as a function of ξ. Otherwise, we can find a sufficient large ξ such that $C(0;\xi) < 0$, which is impossible. Since the more the workers are, the higher the growth rate g is, we obtain, for $\xi > 1$,

$$\frac{C(0;\xi)}{\rho-(1-\theta)g(\xi)} > \frac{C(0)}{\rho-(1-\theta)g}.$$

In addition,

$$\left.\frac{\partial\, C(0;\xi)}{\partial\, \beta_z}\right|_{\beta_z^*} = \xi\left.\frac{\partial\, C(0)}{\partial\, \beta_z}\right|_{\beta_z^*}, \quad \left.\frac{\partial\, g(\xi)}{\partial\, \beta_z}\right|_{\beta_z^*} = \xi\left.\frac{\partial\, g}{\partial\, \beta_z}\right|_{\beta_z^*}.$$

Therefore, we get from (A8) that

$$\frac{\partial\, U}{\partial\, \beta_z}(\beta_z^*; \xi H, \xi L) > 0.$$

It says that $\beta_z^*(\xi) > \beta_z^*$. Hence, we can claim here that $\frac{\partial \beta_h^*}{\partial ZH} > 0$ (i.e., $\frac{\partial \beta_l^*}{\partial L} > 0$) or $\frac{\partial \beta_h^*}{\partial L} > 0$ (i.e., $\frac{\partial \beta_l^*}{\partial ZH} > 0$) hold. If the claim is false, we will have both $\frac{\partial \beta_h^*}{\partial ZH} \leq 0$ (i.e, $\frac{\partial \beta_l^*}{\partial L} < 0$) and $\frac{\partial \beta_h^*}{\partial L} \leq 0$ (i.e., $\frac{\partial \beta_l^*}{\partial ZH} < 0$), and then create a contradiction to $\beta_z^*(\xi) > \beta_z^*$.

From (A8), at β_z^*, we have

$$\frac{\partial C(0)}{\partial \beta_h} / \frac{\partial g}{\partial \beta_h} = -\frac{C(0)}{\rho - (1-\theta)g} = \frac{\partial C(0)}{\partial \beta_l} / \frac{\partial g}{\partial \beta_l}, \tag{A9}$$

and the following long identity

$$\begin{aligned} & -\frac{(1+\beta_l)(1+\alpha(\epsilon-1)-(1-\alpha)(\epsilon-1)\beta_h)}{(1+\beta_h)(1+\alpha(\epsilon-1)-(1-\alpha)(\epsilon-1)\beta_l)} \\ = \quad & \frac{(1+\beta_h)^{-\epsilon}\beta_h^{\alpha(\epsilon-1)}(1-\frac{1-\alpha}{\alpha}\beta_h)(ZH)^{\alpha(\epsilon-1)}}{(1+\beta_l)^{-\epsilon}\beta_l^{\alpha(\epsilon-1)}(1-\frac{1-\alpha}{\alpha}\beta_l)L^{\alpha(\epsilon-1)}} \\ & -\frac{(\mu\theta g+\mu\rho)^{1+\alpha(\epsilon-1)}(1-\frac{1-\alpha}{\alpha}\beta_h)}{(1-\alpha)\mu g\beta_h(1+\beta_l)^{-\epsilon}\beta_l^{\alpha(\epsilon-1)}}. \end{aligned} \tag{A10}$$

After a simple algebra, we know that when H is enlarged and β_z is given, the right hand of the identity becomes greater than the left hand of the identity. Consequently, $\frac{\partial \beta_l^*}{\partial ZH} > 0$. In addition, when $ZH = L$, (A10) implies

$$\frac{(\mu\theta g+\mu\rho)^{1+\alpha(\epsilon-1)}(1-\frac{1-\alpha}{\alpha}\beta_h)}{(1-\alpha)\mu g\beta_h(1+\beta_l)^{-\epsilon}\beta_l^{\alpha(\epsilon-1)}} = 2 \tag{A11}$$

Equation (A11) together with $\frac{\partial \beta_l^*}{\partial ZH} > 0$ suggests that $\frac{(\mu\theta g+\mu\rho)^{1+\alpha(\epsilon-1)}}{\mu g}$ is an increasing function of g. Therefore, when L increases and β_z is given, the right hand of (A10) becomes smaller than the left hand. It is followed that $\frac{\partial \beta_h^*}{\partial L} > 0$. Using symmetry, we get $\frac{\partial \beta_z^*}{\partial ZH} > 0$ and $\frac{\partial \beta_z^*}{\partial L} > 0$.

In order to prove the rest of proposition 2, we now first prove when $ZH = L$, $\frac{\partial \beta_h^*}{\partial ZH} > \frac{\partial \beta_l^*}{\partial ZH} > 0$. Due to symmetry, when $ZH = L$, we have $\beta_h^* = \beta_l^*$. Suppose that $\overline{ZH} = (1+\zeta)\, ZH$ and $\overline{L} = (1-\varsigma)\, ZH$ where $\zeta \to 0^+$, $\varsigma \to 0^+$ and $(1+\zeta)^{\alpha(\epsilon-1)} + (1-\varsigma)^{\alpha(\epsilon-1)} = 2$, and that patent protection remains unchanged. Then, the growth rate remains the same. Hence, when $A_l(0)$ is unchanged, $p_l Y_l(0) - A_l(0)\, k_l(0) - gA_l(0)\, \mu = \frac{\beta_l^*+\alpha}{1+\beta_l^*}\, p_l Y_l(0) - gA_l(0)\, \mu$ remains invariant. Some calculation yields that $\frac{p_h Y_h(0) - A_h(0)k_h(0) - gA_h(0)\mu}{p_l Y_l(0) - A_l(0)k_l(0) - gA_l(0)\mu} = \frac{A_h(0)}{A_l(0)}$. Therefore, (A3) says that when $\overline{ZH} = (1+\zeta)\, ZH$ and $\overline{L} = (1-\varsigma)\, ZH$, if $\beta_z = \beta_z^*$, then $C(0)$ goes up. This means that in order to increase utility, patent protection should be adjusted to increase the growth rate. Using (A11), we know that when $\alpha(\epsilon-1) = 1$, $\frac{\partial \beta_z^*}{\partial ZH} \neq \frac{\partial \beta_z^*}{\partial L}$. Thus, some algebra implies that for $\alpha(\epsilon-1) >$

0, $\beta_h^*\left((1+\zeta)ZH,(1-\varsigma)ZH\right) > \beta_h^*\left(ZH,ZH\right)$ and $\beta_l^*\left((1+\zeta)ZH,(1-\varsigma)ZH\right) < \beta_l^*\left(ZH,ZH\right)$ hold, or $\beta_h^*\left((1+\zeta)ZH,(1-\varsigma)ZH\right) < \beta_h^*\left(ZH,ZH\right)$ and $\beta_l^*\left((1+\zeta)ZH,(1-\varsigma)ZH\right) > \beta_l^*\left(ZH,ZH\right)$ are satisfied. Suppose the latter two inequalities hold. Then, $\beta_h^*\left((1+\zeta)ZH,(1-\varsigma)ZH\right)+$ $\beta_l^*\left((1+\zeta)ZH,(1-\varsigma)ZH\right) > \beta_h^*\left(ZH,ZH\right)+\beta_l^*\left(ZH,ZH\right)$ must be satisfied to increase the growth rate. Since $\frac{\partial\beta_h^*}{\partial ZH}=\frac{\partial\beta_l^*}{\partial L}$ and $\frac{\partial\beta_h^*}{\partial L}=\frac{\partial\beta_l^*}{\partial ZH}$ when $ZH=L$, this inequality cannot hold. Thus, inequalities $\beta_h^*\left((1+\zeta)ZH,(1-\varsigma)ZH\right) > \beta_h^*\left(ZH,ZH\right)$ and $\beta_l^*\left((1+\zeta)ZH,(1-\varsigma)ZH\right) <$ $\beta_l^*\left(ZH,ZH\right)$ should be satisfied. It is followed that when $ZH=L$, $\frac{\partial\beta_h^*}{\partial ZH} > \frac{\partial\beta_l^*}{\partial ZH} > 0$.

Suppose when $ZH > L$, $\beta_h^* < \beta_l^*$. Then, there exists $\widetilde{ZH} > L$ such that $\beta_h^* = \beta_l^*$. By symmetry, the following two equations should be satisfied

$$
\begin{aligned}
& -\frac{(1+\beta_l)(1+\alpha(\epsilon-1)-(1-\alpha)(\epsilon-1)\beta_h)}{(1+\beta_h)(1+\alpha(\epsilon-1)-(1-\alpha)(\epsilon-1)\beta_l)} \\
= \quad & \frac{(1+\beta_h)^{-\epsilon}\beta_h^{\alpha(\epsilon-1)}(1-\frac{1-\alpha}{\alpha}\beta_h)(\widetilde{ZH})^{\alpha(\epsilon-1)}}{(1+\beta_l)^{-\epsilon}\beta_l^{\alpha(\epsilon-1)}(1-\frac{1-\alpha}{\alpha}\beta_l)L^{\alpha(\epsilon-1)}} \\
& -\frac{(\mu\theta g+\mu\rho)^{1+\alpha(\epsilon-1)}(1-\frac{1-\alpha}{\alpha}\beta_h)}{(1-\alpha)\mu g\beta_h(1+\beta_l)^{-\epsilon}\beta_l^{\alpha(\epsilon-1)}}.
\end{aligned} \tag{A12}
$$

and

$$
\begin{aligned}
& -\frac{(1+\beta_l)(1+\alpha(\epsilon-1)-(1-\alpha)(\epsilon-1)\beta_h)}{(1+\beta_h)(1+\alpha(\epsilon-1)-(1-\alpha)(\epsilon-1)\beta_l)} \\
= \quad & \frac{(1+\beta_h)^{-\epsilon}\beta_h^{\alpha(\epsilon-1)}(1-\frac{1-\alpha}{\alpha}\beta_h)L^{\alpha(\epsilon-1)}}{(1+\beta_l)^{-\epsilon}\beta_l^{\alpha(\epsilon-1)}(1-\frac{1-\alpha}{\alpha}\beta_l)(\widetilde{ZH})^{\alpha(\epsilon-1)}} \\
& -\frac{(\mu\theta g+\mu\rho)^{1+\alpha(\epsilon-1)}(1-\frac{1-\alpha}{\alpha}\beta_h)}{(1-\alpha)\mu g\beta_h(1+\beta_l)^{-\epsilon}\beta_l^{\alpha(\epsilon-1)}}.
\end{aligned} \tag{A13}
$$

Clearly, when (A12) and (A13) cannot hold simultaneously. Therefore, we get a contradiction to the case $\beta_h^* = \beta_l^*$ when $\widetilde{ZH} > L$. As a result, when $ZH > L$, $\beta_h^* > \beta_l^*$; when $ZH < L$, $\beta_h^* < \beta_l^*$.