Fiscal Federalism, Public Capital Formation, and Endogenous Growth*

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This paper extends the Barro (1990) growth model with one aggregate government spending and one flat income tax to include federal and local public consumption, federal and local public capital formation, federal and local taxes, and federal transfers to locality. It derives the rate of endogenous growth and examines how the growth rate and welfare respond to changes in federal taxes, local taxes, and federal transfers.

Key Words: Fiscal federalism; Public expenditures; Public capital; Taxes; Federal transfers; Endogenous growth.
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1. INTRODUCTION

In an endogenous growth model, Barro (1990) has examined the effects on economic growth of aggregate government spending including both aggregate public consumption and aggregate public investment. The Barro model does not consider the effects of public expenditures by different levels of government. Subsequent work has extended Barro’s analysis by looking into the composition of government expenditures and economic growth.

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For example, Easterly and Rebelo (1993); and Devarajan, Swaroop, and Zou (1996) have studied the growth effects of public spending on education, transportation, defense, and social welfare. In addition, Davoodi and Zou (1997); and Zhang and Zou (1997) have examined the growth effects of various public expenditures by different levels of government. But in all these studies, public capital formation is not explicitly considered.

Another strand of literature on endogenous growth has considered public capital accumulation by extending the early work of Arrow and Kurz (1970), but it does not model capital accumulation by multiple levels of government. For example, Glomm and Ravikumar (1994); Hulten (1994); Devarajan, Xie, and Zou (1998); among many others, have paid particular attention to the association between infrastructure and output growth.\(^1\)

At the same time, the structure of public expenditures and taxes among different levels of government has a fundamental impact on economic growth in light of the arguments in fiscal federalism; see Oates (1972, 1993). In fact, the proper assignments of expenditures and taxes among multiple levels of government and the proper design of intergovernmental transfers are prerequisite for efficient and equitable public service provision at both the national and local levels. One of the most important goals of establishing a sound intergovernmental fiscal relationship is to promote local as well as national economic growth (see Rivlin, 1992; Bird, 1993; Gramlich, 1993; and Oates, 1993).

In view of the important link from the design of intergovernmental fiscal relationship to economic growth, it is natural for us to extend the Barro model and provide an analytical framework for both theoretical and empirical research on the growth effects of public consumption, public capital formation, taxes, and federal transfers in a federation or in the context of multiple levels of government. This is the main task of our paper.

Our model extends the Arrow-Kurz-Barro approach in the following aspects. (1) We allow public consumption and public capital accumulation at both the federal and local level, corresponding to expenditure assignments among different levels of government in fiscal federalism; (2) on the revenue side, our model specifies federal taxes and local taxes in light of tax assignment among different levels of government in a federation; (3) our model takes care of federal transfers to locality in the forms of matching grants for both local public capital formation and local public consumption; (4) with specific production function and utility function, we derive analytical solution to the rate of balanced growth; (5) with simulations, we derive the responses of the growth rate with respect to federal income tax, federal

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\(^1\)Of course, the empirical analyses in many of these studies have followed the much-cited work by Aschauer (1989).
consumption tax, local income tax, local property tax, local consumption tax, and federal grants for local public investment and consumption.

The paper is organized as follows. Section 2 sets up a much extended Arrow-Kurz-Barro model. Section 3 derives the rate of endogenous growth. Section 4 studies how the rate of endogenous growth changes with respect to federal taxes, local taxes, and federal transfers. Section 5 presents the welfare analysis. Section 6 concludes.

2. THE MODEL

In this paper, there are two levels of government: the federal government and local governments. Their consumption expenditures are $f$ and $s$, and their capital stocks are $k_f$ and $k_s$, respectively. In Arrow and Kurz (1970) and Barro (1990), among many others, government spending and public capital accumulation have been introduced into the utility function in aggregate terms, i.e., total government spending (Barro, 1990), or total public capital (Arrow, and Kurz, 1970), or total public consumption and investment (Barro, 1990). Along this line, we introduce public consumption and capital stocks at both the federal and local levels into the representative agent’s utility function as

$$u(c, f, k_f, s, k_s)$$

where $c$ is private consumption.

If the utility function $u(c, f, k_f, s, k_s)$ is twice differentiable, we further assume that

$$u_c > 0, \, u_f > 0, \, u_s > 0, \, u_{cc} < 0, \, u_{ff} < 0, \, u_{ss} < 0$$

$$u_{k_f} > 0, \, u_{k_s} > 0, \, u_{k_fk_f} < 0, \, u_{k_s} < 0, \, u_{k_s} < 0.$$ (1)

The cross effects $u_{cf}, u_{cs}$, and $u_{fs}$ are in general assumed to be positive.

The representative agent’s discounted utility and welfare are given by

$$U = \int_0^\infty u(c, f, k_f, s, k_s)e^{-\rho t}dt$$ (2)

where $\rho$ is the constant rate of time preference.

Again, by broadening the frameworks in Arrow and Kurz (1970), and Barro (1991), we assume that output $y$ is produced by a constant-return-to-scale production function with three inputs: private capital stock, $k_p$, federal government capital stock, $k_f$, and local government capital stock, $k_s$, namely,

$$y = y(k_p, k_f, k_s).$$ (3)
As in Arrow and Kurz (1970), the marginal productivities of private capital stock, federal government capital stock, and local government capital stock are positive and decreasing. Suppose the production function is twice differentiable, then,

\[ y_{k_p} > 0, \quad y_{k_f} > 0, \quad y_{k_s} > 0, \quad y_{k_p,k_p} < 0, \quad y_{k_f,k_f} < 0, \quad y_{k_s,k_s} < 0 \quad (4) \]

In this paper, to derive the analytical solution to the relationship between the growth rate on the one hand and tax rates and federal transfers on the other, we choose the utility function to be logarithmic

\[ u(c, f, k_f, s, k_s) = \theta_0 \ln c + \theta_1 \ln f + \theta_2 \ln k_f + \theta_3 \ln s + \theta_4 \ln k_s, \quad (5) \]

where \( \theta_0, \theta_1, \theta_2, \theta_3, \) and \( \theta_4 \) are positive constants satisfy \( \theta_0 + \theta_1 + \theta_2 + \theta_3 + \theta_4 = 1. \) And we take the production function to be Cobb-Douglas

\[ y(k_p, k_s, k_f) = k_p^{\omega_1} k_s^{\omega_2} k_f^{\omega_3} \quad (6) \]

where \( \omega_1, \omega_2, \) and \( \omega_3 \) are positive constants satisfying \( \omega_1 + \omega_2 + \omega_3 = 1. \)

### 2.1. The agent’s optimization problem

The representative agent’s budget constraint is given by the condition that the after-tax income is to equal to the spending on his consumption, \((1 + \tau_{cf} + \tau_{cs})c,\) and his gross investment, \( \frac{dk_p}{dt}, \) namely

\[ \frac{dk_p}{dt} = (1 - \tau_f - \tau_s)y(k_p, k_s, k_f) - (1 + \tau_{cf} + \tau_{cs})c - \tau_k k_p - \delta k_p \quad (7) \]

where \( \tau_f \) and \( \tau_s \) are the income tax rates of the federal government and local government, respectively; \( \tau_{cf} \) and \( \tau_{cs} \) are the consumption tax rates of the federal government and local government, respectively; and \( \tau_k \) is the property tax rate. To provide some illustrations from the reality, please note that \( \tau_s \) can be regarded as the state income tax in the United States, \( \tau_{cf} \) can be regarded as sales tax and consumption-based value-added tax collected by many central governments in Europe, \( \tau_{cs} \) is the standard sales tax in the United States, and \( \tau_k \) is the property tax collected by local governments in most countries in the world.

Given the taxes, public consumption, and public capital formation at both the federal and local levels, the agent chooses his consumption path, \( c(t), \) and capital accumulation path, \( k_p(t), \) to maximize his discounted utility, i.e.

\[ \max_{c(t), k_p(t)} \int_0^\infty u(c, s, k_s, f, k_f)e^{-\rho t}dt \quad (8) \]
subject to budget constraint (7) and the initial capital stock \( k_p(0) \)

### 2.2. Federal government’s optimization problem

The federal government collects capital income tax, \( \tau_f \), and consumption tax, \( \tau_{cf} \), as its revenue. On the spending side, first, it makes two kinds of transfers to the local government in the forms of matching grants for investment at the rate \( \alpha \) and for local public consumption at the rate \( \beta \), i.e., \( \alpha \frac{dk_s}{dt} + \beta s \); second, the federal government undertakes its own public consumption, \( f \), and its own gross investment, \( \frac{dk_f}{dt} + \delta k_f \), namely,

\[
\frac{dk_f}{dt} = \tau_f y + \tau_{cf} c - \alpha \frac{dk_s}{dt} - \beta s - f - \delta k_f. \tag{9}
\]

Now, taking local government and private behaviors as given, the federal government chooses its expenditure path, \( f(t) \), and its capital accumulation path, \( k_f(t) \), to maximize the agent’s welfare, i.e.,

\[
\max_{f(t), k_f(t)} \int_0^\infty u(c, s, f, k_s, k_f) e^{-\rho t} dt \tag{10}
\]

subject to its budget constraint (9). The initial federal capital stock, \( k_f(0) \), is given. Again, for the federal government, private consumption, \( c \), private capital stock, \( k_p \), local public consumption, \( s \), and local public capital stock, \( k_s \), are all given.

In our specification of federal government’s optimization problem, the rates of federal taxes and federal transfers are exogenously given, whereas federal consumption and federal capital accumulation are endogenous. The same approach will be applied to local government’s optimization problem in the next subsection. We take this approach for the following reasons. Our focus of our paper is to see how federal taxes, local taxes, and federal transfers affect capital accumulation and consumption by the private sector, the federal government, and local governments. Once the optimal responses of accumulation and consumption with respect to taxes and transfers have determined, the optimal choices of taxes and transfers can be derived from welfare maximization or growth maximization. This will be our main tasks in our simulation analysis in sections 4 and 5. In this sense, our approach can be viewed as a two-stage optimization.

### 2.3. Local government’s optimization problem

The local government collects its income tax, \( \tau_s y \), its consumption tax, \( \tau_{cs} c \), and its property tax, \( \tau_k k_p \) at each time period. In additional, the local government receives federal transfer for its investment and consumption as the rates of \( \alpha \) and \( \beta \), respectively: \( \alpha \frac{dk_s}{dt} + \beta s \). Hence, the budget constraint

\[
\frac{dk_f}{dt} = \tau_s y + \tau_{cs} c - \alpha \frac{dk_s}{dt} - \beta s - f - \delta k_f. \tag{9}
\]
for the local government can be written as

\[
\frac{dk_s}{dt} = \alpha \frac{dk_s}{dt} + \beta s - s + \tau_k k_p + \tau_c s - \delta k_s
\]

(11)

Given the choices of the federal government and private agent, the local government chooses its consumption path, \(s(t)\), and its investment path, \(k_s(t)\), to maximize the agent’s welfare, i.e.,

\[
\max \int_0^\infty u(c, s, f, k_f) e^{-\rho t} dt
\]

subject to budget constraint (11) with the initial capital stock \(k_s(0)\) given.

3. THE BALANCED GROWTH RATE

To derive the balanced growth rate, we solve the optimization problems for the private agent, the federal government, and the local government, respectively.

First, from private agent’s optimization, we get

\[
\frac{dc}{dt} = (1 - \tau_f - \tau_s) \frac{\partial y(k_p, k_s, k_f)}{\partial k_p} - \rho - \tau_k - \delta
\]

(12)

\[
\frac{dk_p}{dt} = (1 - \tau_f - \tau_s) \frac{y(k_p, k_s)}{k_p} - \tau_k - (1 + \tau_c + \tau_s) \frac{c}{k_p} - \delta
\]

(13)

and the transversality condition

\[
\lim_{t \to \infty} u(c, k_p(t)) e^{-\rho t} = 0.
\]

Next, from the federal government’s optimization, we have

\[
\frac{df}{dt} = \tau_f \frac{\partial y(k_p, k_s, k_f)}{\partial k_f} - \rho - \delta + \frac{\theta_2}{\theta_1} k_f
\]

(15)
\[
\frac{dk_f}{dt} = \tau_f \frac{y(k_p, k_s, k_f)}{k_f} + \tau_{cf} \frac{c}{k_f} - \frac{f}{k_f} - \alpha \frac{dt}{k_f} - \beta s - \delta
\]
\[
= \tau_f \frac{k_p}{k_f} (k_s k_p)_{\omega_2} (k_f)_{\omega_3} + \tau_{cf} \frac{c}{k_f} \frac{k_p}{k_f} - \frac{f}{k_f} + \omega_2 \left( \frac{k_f}{k_p} \right)_{\omega_2} \omega_3 + \frac{k_p}{k_f} \omega_3 - \rho - \delta
\]
and the transversality condition
\[
\lim_{t \to \infty} u_f k_f(t) e^{-\rho t} = 0. \quad (17)
\]
Finally, from the local government’s optimization problem, we obtain
\[
\frac{ds}{dt} = \frac{\tau_s}{1 - \alpha} \frac{\partial y(k_p, k_s, k_f)}{\partial k_s} - \rho + \frac{1 - \beta \theta_4 s}{1 - \alpha \theta_3 k_s} - \frac{\delta}{1 - \alpha - \beta_1 - \alpha \theta_4 s}
\]
\[
= \frac{\omega_2 \tau_s}{1 - \alpha} \frac{k_p}{k_s} (k_s k_p)_{\omega_2} (k_f)_{\omega_3} - \rho - \frac{\delta}{1 - \alpha} + \frac{1 - \beta \theta_4 s}{1 - \alpha} \frac{1 - \alpha}{1 - \alpha \theta_3 k_s}, \quad (18)
\]
\[
\frac{dk_s}{dt} = \frac{\tau_s}{1 - \alpha} \frac{y(k_p, k_s, k_f)}{k_s}
\]
\[
+ \frac{\tau_k}{1 - \alpha} \frac{k_p}{k_s} + \frac{\tau_{cs}}{1 - \alpha} \frac{c}{k_s} - \frac{1 - \beta s}{1 - \alpha} - \frac{\delta}{1 - \alpha - \beta_1 - \alpha \theta_4 s}
\]
\[
= \frac{\tau_s}{1 - \alpha} \frac{k_p}{k_s} (k_s k_p)_{\omega_2} (k_f)_{\omega_3} + \frac{\tau_{cs}}{1 - \alpha} \frac{c}{k_s} \frac{k_p}{k_f} + \frac{\tau_k}{1 - \alpha} \frac{k_p}{k_s} - \frac{1 - \beta s}{1 - \alpha} - \frac{\delta}{1 - \alpha}, \quad (19)
\]
and the transversality condition
\[
\lim_{t \to \infty} u_s k_s(t) e^{-\rho t} = 0. \quad (20)
\]
It can be easily shown that, so long as all seven endogenous variables in the model grow at constant rates, these growth rates will be the same:
\[
\begin{align*}
\frac{dk_s}{dt} \quad & = \quad \frac{dk_p}{dt} \quad = \quad \frac{ds}{k_s} \quad = \quad \frac{dc}{k_p} \quad = \quad \frac{dy}{k_f} \quad = \quad \frac{df}{k_f} = \phi \quad (21)
\end{align*}
\]

where we denote the common growth rate as \( \phi \).

From equations (12) and (21), we have

\[
\frac{\phi + \rho + \tau_k + \delta}{\omega_1(1 - \tau_f - \tau_s)} = \left( \frac{k_s}{k_p} \right) \omega_2 \left( \frac{k_f}{k_p} \right) \omega_3. \quad (22)
\]

Using equations (13) and (21), we get

\[
\frac{c}{k_p} = \frac{(1 - \omega_1)\phi + (1 - \omega_1)(\tau_k + \delta) + \rho}{(1 + \tau_{cf} + \tau_{cs})\omega_1}. \quad (23)
\]

Substituting equation (22) into equations (18) and (19) and using equation (21), we obtain

\[
\frac{k_s}{k_p} = \left( 1 + \frac{\theta_3}{\theta_4} \omega_2 \right) \frac{1 - \alpha}{1 - \alpha} \frac{\phi + \rho + \tau_k + \delta}{1 - \alpha} \omega_1(1 - \tau_f - \tau_s) + \tau_{cs} \frac{(1 - \omega_1)\phi + (1 - \omega_1)(\tau_k + \delta) + \rho}{(1 + \tau_{cf} + \tau_{cs})\omega_1} + \frac{\theta_3}{\theta_4} \omega_2 \left( \frac{\phi + \rho + \tau_k + \delta}{1 - \alpha} + \frac{\delta}{1 - \alpha} \right) \quad (24)
\]

and

\[
\frac{s}{k_s} = \frac{\theta_3}{\theta_4} \frac{1 - \alpha}{1 - \beta} \left\{ \phi + \rho + \frac{\delta}{1 - \alpha} - \omega_2 \frac{\tau_s}{1 - \alpha} \frac{\phi + \rho + \tau_k + \delta}{1 - \alpha} \omega_1(1 - \tau_f - \tau_s) \right. \\
\left. \frac{\theta_3}{\theta_4} \omega_2 \left( \frac{\phi + \rho + \tau_k + \delta}{1 - \alpha} + \frac{\delta}{1 - \alpha} \right) + \phi + \frac{\delta}{1 - \alpha} \right\} \quad (25)
\]
Substituting equations (22) and (24) into the federal government’s first-order conditions (15) and (16), we get

\[
\frac{\theta_1}{\theta_2} (\phi + \rho + \delta) + \phi + \delta
\]

\[
= \left\{ \left( 1 + \frac{\theta_1}{\theta_2} \omega_3 \right) \tau_f \frac{\phi + \rho + \tau_k + \delta}{\omega_1 (1 - \tau_f - \tau_s)} + \tau_{cf} \frac{(1 - \omega_1) \phi + (1 - \omega_1) (\tau_k + \delta) + \rho}{(1 + \tau_{cf} + \tau_{cs}) \omega_1} \right\}
\]

\[
- \alpha \frac{\theta_4}{\theta_2} \phi + \rho + \delta + \phi + \frac{\delta}{1 - \alpha} + \phi + \frac{\delta}{1 - \alpha}
\]

\[
- \beta \left[ \left( \phi + \rho + \frac{\delta}{1 - \alpha} \right) \frac{\tau_s (1 - \omega_1) \phi + (1 - \omega_1) (\tau_k + \delta) + \rho}{(1 + \tau_{cf} + \tau_{cs}) \omega_1} + \tau_k \right]
\]

\[
+ \frac{\theta_3}{\theta_2} \omega_2 \left( \frac{\tau_s (1 - \omega_1) \phi + (1 - \omega_1) (\tau_k + \delta) + \rho}{(1 + \tau_{cf} + \tau_{cs}) \omega_1} + \tau_k \right)
\]

\[
- \omega_2 \left[ \frac{\theta_3}{\theta_2} \omega_2 \left( \frac{(1 - \omega_1) \phi + (1 - \omega_1) (\tau_k + \delta) + \rho}{(1 + \tau_{cf} + \tau_{cs}) \omega_1} + \tau_k \right) \right]
\]

or

\[
k_f = \frac{\theta_2}{\theta_4} \phi + \rho + \delta + \phi + \frac{\delta}{1 - \alpha} + \phi + \frac{\delta}{1 - \alpha}
\]

\[
+ \tau_{cf} \frac{(1 - \omega_1) \phi + (1 - \omega_1) (\tau_k + \delta) + \rho}{(1 + \tau_{cf} + \tau_{cs}) \omega_1} + \tau_k
\]

\[
- \alpha \frac{\theta_4}{\theta_2} \phi + \rho + \delta + \phi + \frac{\delta}{1 - \alpha} + \phi + \frac{\delta}{1 - \alpha}
\]

\[
- \beta \left[ \left( \phi + \rho + \frac{\delta}{1 - \alpha} \right) \frac{\tau_s (1 - \omega_1) \phi + (1 - \omega_1) (\tau_k + \delta) + \rho}{(1 + \tau_{cf} + \tau_{cs}) \omega_1} + \tau_k \right]
\]

\[
+ \frac{\theta_3}{\theta_2} \omega_2 \left( \frac{\tau_s (1 - \omega_1) \phi + (1 - \omega_1) (\tau_k + \delta) + \rho}{(1 + \tau_{cf} + \tau_{cs}) \omega_1} + \tau_k \right)
\]

\[
- \omega_2 \left[ \frac{\theta_3}{\theta_2} \omega_2 \left( \frac{(1 - \omega_1) \phi + (1 - \omega_1) (\tau_k + \delta) + \rho}{(1 + \tau_{cf} + \tau_{cs}) \omega_1} + \tau_k \right) \right]
\]

\[
, (26)
\]

Thus, we have

\[
f = \frac{\theta_3}{\theta_2} \omega_2 \frac{\phi + \rho + \tau_k + \delta}{(1 - \tau_f - \tau_s) \left( \frac{k_p}{k_f} \frac{\theta_1}{\theta_2} \right)}.
\]

(27)
Now, substituting equations (24) and (26) into equation (18), we get

\[
\frac{\phi + \rho + \tau_k + \delta}{\omega_1(1 - \tau_f - \tau_s)} = \left( \frac{1}{\theta_4} \right) \frac{(1 + \theta_1 \omega_1(1 - \tau_f - \tau_s))}{\theta_1 \omega_1(1 - \tau_f - \tau_s) + \tau_c s (1 - \omega_1)(\phi + (1 - \omega_1)(\tau_k + \delta) + \rho) + \tau_k}\right)^{\omega_2} \tag{28}
\]

which is a highly nonlinear equation defining the balanced growth rate, \(\phi\), as a function of taxes \((\tau_f, \tau_s, \tau_c s, \tau_c f, \text{ and } \tau_k)\), federal transfers \((\alpha \text{ and } \beta)\), technology parameters \((\omega_1, \omega_2, \omega_3, \text{ and } \delta)\), and preference parameters \((\theta_0, \theta_1, \theta_2, \theta_3, \text{ and } \rho)\). We admit that we cannot even obtain a nonlinear equation defining the balanced growth rate when the preferences and technology are assumed to be CES instead of Cobb-Douglas. But without public capital accumulation by the federal and local governments, nonlinear or even explicit solutions to the growth rate are possible with more general preferences and production technology; see Gong and Zou (1997).

\[4. \text{ EFFECTS OF TAXES AND TRANSFERS ON THE GROWTH RATE}\]

Although the growth rate defined in equation (28) is highly nonlinear in taxes and federal transfers, it is rather simple to find out the effects of various taxes and federal transfers on economic growth with simulations. In this section, we report some results on the simulation exercise based on equation (28).
Figure 1 shows the relationship between the rate of endogenous growth, $\phi$, and federal government’s income tax rate, $\tau_f$, on the basis of the following base values for the structure of other taxes and transfers: a federal consumption tax at zero percent, $\tau_{cf} = 0$, a local income tax at ten percent: $\tau_s = .10$, a local consumption tax at five percent, $\tau_{cs} = .05$, a capital tax or local property tax at two percent: $\tau_k = .02$, a federal matching grant for local investment at thirty percent: $\alpha = .3$, and a federal matching grant for local consumption also at thirty percent, $\beta = .3$. The preference and technology parameters are all the same for all simulations, and they are given in the legend of each figure. Figure 1 presents a typical Laffer curve relating the growth rate to federal income tax. Given local taxes, federal transfers, and all other parameters in our model, a rise in federal income tax will increases the growth rate before the tax rate hits around thirty percent. In fact, when the federal income tax is zero, coupled with a zero rate of federal consumption tax, the growth rate is negative. With the rise of federal income tax rate from zero to ten percent, the growth rate rises from a negative seven percent to a positive three percent. After the federal income tax reaches thirty percent, further increases in the federal income taxation lower the growth rate. The growth rate is around zero when federal income tax is seventy percent.

The explanation for this Laffer curve is now becoming standard; see Barro (1990). A change in federal income tax has three effects. First, a higher federal income tax reduces the return on private capital and the
growth rate directly. But second, a larger tax revenue implies a higher federal consumption and capital investment that are assumed to increase both private utility and private productivity, which raises the growth rate. Third, at the same time, a larger tax revenue can lead to more federal transfers to the local government whose public consumption and public investment are also utility- and productivity-enhancing. When the federal income tax rate is initially very small, the second and the third forces dominate the first force. When the federal income tax is already high, the first force will dominate the second and the third forces.

\[ y = k^{\omega_1}f^{\omega_2}k_s^{\omega_3} \] and
\[ u(c, f, s, k_f, k_s) = \theta_0 \ln c + \theta_1 \ln f + \theta_2 \ln k_f + \theta_3 \ln s + \theta_4 \ln k_s, \] respectively. Parameters are selected as: \( \theta_0 = 0.5, \theta_1 = 0.1, \theta_2 = 0.1, \theta_3 = 0.1, \theta_4 = 0.1, \omega_1 = 0.5, \omega_2 = 0.25, \omega_3 = 0.25, \rho = 0.08, \delta = 0.08, \tau_f = 0.20, \alpha = 0.3, \tau_k = 0.01, \tau_{cf} = 0, \) and \( \tau_{cs} = 0.05. \)

The similar picture appears in figure 2, which shows the relationship between the growth rate, \( \phi, \) and local income tax rate, \( \tau_s \) on the basis of the following base values for the structure of federal taxes, local taxes other than local income tax, and federal transfers: a federal income tax at twenty percent, \( \tau_f = .20, \) a federal consumption tax at zero percent, \( \tau_{cf} = 0, \) a local consumption tax at five percent, \( \tau_c = .05, \) a capital tax or local property tax at two percent: \( \tau_k = .02, \) a federal matching grant for local investment at 30 percent: \( \alpha = .3, \) and a federal matching grant for local consumption also at thirty percent, \( \beta = .3. \)

Since the base federal income tax is already at a relatively high rate of twenty percent, the growth rate is rising with local income tax until \( \tau_s \) reaches about eighteen percent. When local income tax rate is set at fifty-five percent, the growth rate is zero. Because the local government receives two matching grants from the federal government at a rate of thirty percent
each, and because it also raises tax revenues from the consumption tax and property tax, the local government can still finance its productive public expenditures without resorting to income tax. This is why the growth rate is still positive even though local income tax is zero in figure 2.

\[
\begin{align*}
\phi &= k_p^{\omega_1} k_f^{\omega_2} k_s^{\omega_3} \\
u(c, f, s, k_f, k_s) &= \theta_0 \ln c + \theta_1 \ln f + \theta_2 \ln k_f + \theta_3 \ln s + \theta_4 \ln k_s,
\end{align*}
\]

Parameters are selected as: \(\theta_0 = 0.5\), \(\theta_1 = 0.1\), \(\theta_2 = 0.1\), \(\theta_3 = 0.1\), \(\theta_4 = 0.1\), \(\omega_1 = 0.5\), \(\omega_2 = 0.25\), \(\omega_3 = 0.25\), \(\rho = 0.08\), \(\delta = 0.08\), \(\tau_s = 0.10\), \(\alpha = \beta = 0.3\), \(\tau_f = 0.20\), \(\tau_{cf} = 0\), and \(\tau_{cs} = 0.05\).

Figure 3 reveals the alarming negative effect of local property tax on the growth rate. The curve is drawn by assuming a federal income tax at twenty percent, a federal consumption tax at zero percent, a local income tax at ten percent, a local consumption tax at five percent, and the two federal matching grants at thirty percent each. Given the distortions of federal and local taxes, the growth rate is around six percent when the property tax is zero; and it reaches zero when the property tax hits twenty-four percent. Compared to local income tax and local consumption tax, local property tax is the most distortionary tax in raising local public revenue.

Figure 4 illustrates the relationship between the growth rate, \(\phi\), and federal consumption tax, \(\tau_{cf}\), by assuming that a federal income tax at twenty percent: \(\tau_f = .20\), a local income tax at ten percent: \(\tau_s = .10\), a capital tax or local property tax at two percent: \(\tau_k = .02\), a local consumption tax at five percent, \(\tau_c = .05\), a federal matching grant for local investment at 30 percent: \(\alpha = .3\), and a federal matching grant for local consumption also at thirty percent, \(\beta = .3\). We find that federal consumption tax has a positive effect on the growth rate. When the consumption tax increases from zero to one hundred percent, the growth rate rises from five percent to 8.5 percent in figure 4. This is because a high consumption tax raises the cost of consumption and forces the consumer to save more (the sub-
\[ y = k^{\omega_1} k_f^{\omega_2} k_s^{\omega_3} \]
\[ u(c, f, s, k_f, k_s) = \theta_0 \ln c + \theta_1 \ln f + \theta_2 \ln k_f + \theta_3 \ln s + \theta_4 \ln k_s, \]

Parameters are selected as: \( \theta_0 = 0.5, \theta_1 = 0.1, \theta_2 = 0.1, \theta_3 = 0.1, \theta_4 = 0.1, \omega_1 = 0.5, \omega_2 = 0.25, \omega_3 = 0.25, \rho = 0.08, \delta = 0.08, \tau_s = 0.10, \alpha = \beta = 0.3, \tau_f = 0.20, \tau_k = 0.01, \) and \( \tau_{cs} = 0.05. \)

The substitution effect dominating the income effect with our specifications on the preferences and technology. The rise in savings results in more capital accumulation and higher economic growth. Please note that the effect of a rise in consumption tax on the growth rate is relatively modest in our simulation given the existing tax distortions of federal and local income taxes and local property tax.

Figure 5 illustrates the relationship between the growth rate and local consumption tax \( \tau_{cs}. \) It has essentially the same feature as the relationship between federal consumption tax and the growth rate in figure 4, namely, a rise in local consumption growth rate raises the growth rate. As shown in figure 5, a rise of local consumption tax from zero percent to 100 percent can raise the growth rate from 4.5 percent to around seven percent.

Figure 6 relates the growth rate, \( \phi, \) to federal matching grants for local investment and consumption, \( \alpha \) and \( \beta. \) A rise in the rate of the matching grant for local investment always raises the growth rate (the curve \( \phi(\alpha) \)), whereas a rise in the rate of the matching grant for local consumption always reduces the growth rate (the curve \( \phi(\beta) \)). Obviously enough, federal matching grant for local investment stimulates local public capital formation, which in turn raises private marginal productivity of capital and increases the growth rate. For example, in figure 6, when the matching grant for local investment from zero rises to sixty percent, the growth rate increases from around four percent to seven percent. But we also need to note
FIG. 5. Growth rate versus the local consumption tax rate. Where the production function and the utility function are specified as $y = k_{p}^{\omega_{1}} k_{f}^{\omega_{2}} k_{s}^{\omega_{3}}$ and $u(c, f, s, k_f, k_s) = \theta_0 \ln c + \theta_1 \ln f + \theta_2 \ln k_f + \theta_3 \ln s + \theta_4 \ln k_s$, respectively. Parameters are selected as: $\theta_0 = 0.5, \theta_1 = 0.1, \theta_2 = 0.1, \theta_3 = 0.1, \theta_4 = 0.1, \omega_1 = 0.5, \omega_2 = 0.25, \omega_3 = 0.25, \rho = 0.08, \delta = 0.08, \tau_s = 0.10, \alpha = \beta = 0.3, \tau_f = 0.20, \tau_k = 0.01$, and $\tau_{cs} = 0$. That a federal matching grant for local consumption has both a income effect and a price (substitution) effect. While the price effect of federal grant for local consumption discourages local investment, the income effect does encourage local investment. From our simulation, the price effect of federal

FIG. 6. Growth rate versus the federal transfers. Where the production function and the utility function are specified as $y = k_{p}^{\omega_{1}} k_{f}^{\omega_{2}} k_{s}^{\omega_{3}}$ and $u(c, f, s, k_f, k_s) = \theta_0 \ln c + \theta_1 \ln f + \theta_2 \ln k_f + \theta_3 \ln s + \theta_4 \ln k_s$, respectively. Parameters are selected as: $\theta_0 = 0.5, \theta_1 = 0.1, \theta_2 = 0.1, \theta_3 = 0.1, \theta_4 = 0.1, \omega_1 = 0.5, \omega_2 = 0.25, \omega_3 = 0.25, \rho = 0.08, \delta = 0.08, \tau_s = 0.10, \alpha = \beta = 0.3, \tau_f = 0.20, \tau_k = 0.01$, and $\tau_{cs} = 0.05$. That a federal matching grant for local consumption has both a income effect and a price (substitution) effect. While the price effect of federal grant for local consumption discourages local investment, the income effect does encourage local investment. From our simulation, the price effect of federal
grant for local consumption always dominates the its income effect. In fact, when the matching rate for local public consumption increases from zero to sixty-five percent, the growth rate is down from 5.8% to around two percent.

5. WELFARE ANALYSIS

With $\phi$ implicitly defined in (28), we can now determine the time paths of various capital stocks, $k_p(t)$, $k_f(t)$, and $k_s(t)$), private consumption, $c(t)$, federal consumption, $f(t)$, and local consumption $s(t)$:

$$k_p(t) = k_p(0)e^{\phi t}, \quad k_f(t) = k_f(0)e^{\phi t}, \quad k_s(t) = k_s(0)e^{\phi t},$$
$$c(t) = c(0)e^{\phi t}, \quad f(t) = f(0)e^{\phi t}, \quad s(t) = s(0)e^{\phi t},$$

where the initial capital stocks $k_p(0), k_f(0),$ and $k_s(0)$ are given. $c(0)$ is determined by equation (23), $s(0)$ is determined from equation (25), and $f(0)$ is determined from equation (27).

The agent’s welfare is given as

$$W = \int_0^\infty u(c, s, k_s, f, k_f) e^{-\rho t} dt$$
$$= \int_0^\infty (\theta_0 \ln c + \theta_1 \ln f + \theta_2 \ln k_f + \theta_3 \ln s + \theta_4 \ln k_s) e^{-\rho t} dt$$
$$= \int_0^\infty (\theta_0 \ln c(0) + \theta_1 \ln f(0) + \theta_2 \ln k_f(0) + \theta_3 \ln s(0) + \theta_4 \ln k_s(0) + 5\phi t) e^{-\rho t} dt$$
$$= \frac{1}{\rho} (\theta_0 \ln c(0) + \theta_1 \ln f(0) + \theta_2 \ln k_f(0) + \theta_3 \ln s(0) + \theta_4 \ln k_s(0)) + \frac{5\phi}{\rho^2}$$

Unlike Barro (1990), it is not a simple exercise here to show that welfare maximization and growth maximization are consistent from equation (30) because the complicated relationship between the growth rate and the initial values of private consumption, federal consumption, and local consumption. But for reasonable values of initial capital stocks in our simulations, welfare is indeed an increasing function of the growth rate. That is to say, in equation (30), the term $\frac{5\phi}{\rho^2}$ far dominates the term $\frac{1}{\rho} (\theta_0 \ln c(0) + \theta_1 \ln f(0) + \theta_2 \ln k_f(0) + \theta_3 \ln s(0) + \theta_4 \ln k_s(0))$. This is not surprising because $\rho^2$ is rather small, and $\frac{5\phi}{\rho^2}$ can be rather big, whereas the sum of $(\theta_0 \ln c(0) + \theta_1 \ln f(0) + \theta_2 \ln k_f(0) + \theta_3 \ln s(0) + \theta_4 \ln k_s(0))$ is not very significant with the logarithmic utility function and the coefficient constrain, i.e., $\theta_0 + \theta_1 + \theta_2 + \theta_3 + \theta_4 = 1$. Therefore, in our simulations, we find

\footnote{See Zou (1994, 1996) for related discussions on the ambiguities of matching grants on local public consumption and public investment in dynamic models.}
that welfare effects of federal taxes, local taxes, and federal transfers are qualitatively the same as the growth effects. For example, figure 7 presents the Laffer curve of welfare versus federal income taxation, which is similar to the Laffer curve of growth rate versus federal income taxation in figure 1. Figure 8 relates welfare to local property tax, and it has the same shape as figure 3. Finally, figure 9 suggests that consumption tax always raises welfare. It may sound surprising. But the reason is obvious after a second thought. A higher consumption tax raises the growth rate, and income will rise faster. With fast-rising income, the agent's actual consumption after paying a high consumption tax is also rising, which leads to a higher welfare.

\[ y = k^\omega_1 k_f^\omega_2 k_s^\omega_3 \] and \[ u(c, f, k_f, k_s) = \theta_0 \ln c + \theta_1 \ln f + \theta_2 \ln k_f + \theta_3 \ln s + \theta_4 \ln k_s, \] respectively. Parameters are selected as: \( \theta_0 = 0.5, \theta_1 = 0.1, \theta_2 = 0.1, \theta_3 = 0.1, \theta_4 = 0.1, \omega_1 = 0.5, \omega_2 = 0.25, \omega_3 = 0.25, \rho = 0.08, \delta = 0.08, \tau_s = 0.10, \alpha = \beta = 0.3, \tau_k = 0.01, \tau_{cf} = 0, \text{and } \tau_{cs} = 0.05. \]

\section{6. Conclusions}

This paper has extended the Barro (1990) model with one aggregate government spending and one flat income tax to include federal and local public consumption, federal and local public capital formation, federal and local taxes, and federal transfers to locality. It has derived the rate of endogenous growth and examined how the growth rate and welfare respond to changes in federal taxes, local taxes, and federal transfers. With simulations, the paper has examined how the rate of endogenous growth changes with respect to federal income tax, federal consumption tax, local income tax, local consumption tax, local property tax, and federal transfers. Even though, analytically, the growth-maximizing choices of taxes and transfers
FIG. 8. Welfare versus the property tax rate. Where the production function and the utility function are specified as \( y = k_p^{\omega_1} k_f^{\omega_2} k_s^{\omega_3} \) and \( u(c, f, s, k_f, k_s) = \theta_0 \ln c + \theta_1 \ln f + \theta_2 \ln k_f + \theta_3 \ln s + \theta_4 \ln k_s \), respectively. Parameters are selected as: \( \theta_0 = 0.5, \theta_1 = 0.1, \theta_2 = 0.1, \theta_3 = 0.1, \theta_4 = 0.1, \omega_1 = 0.5, \omega_2 = 0.25, \omega_3 = 0.25, \rho = 0.08, \delta = 0.08, \tau_s = 0.10, \alpha = \beta = 0.3, \tau_f = 0.20, \tau_{cs} = 0.05, \) and \( \tau_{cf} = 0 \).

FIG. 9. Welfare versus the local consumption tax rate. Where the production function and the utility function are specified as \( y = k_p^{\omega_1} k_f^{\omega_2} k_s^{\omega_3} \) and \( u(c, f, s, k_f, k_s) = \theta_0 \ln c + \theta_1 \ln f + \theta_2 \ln k_f + \theta_3 \ln s + \theta_4 \ln k_s \), respectively. Parameters are selected as: \( \theta_0 = 0.5, \theta_1 = 0.1, \theta_2 = 0.1, \theta_3 = 0.1, \theta_4 = 0.1, \omega_1 = 0.5, \omega_2 = 0.25, \omega_3 = 0.25, \rho = 0.08, \delta = 0.08, \tau_s = 0.10, \alpha = \beta = 0.3, \tau_f = 0.20, \tau_{cf} = 0, \) and \( \tau_{cs} = 0.05 \).

are not the same as the welfare-maximizing choices, simulations show that these two kinds choices are consistent.

The model in this paper sets up a positive framework for evaluating how the assignments of taxes and expenditures among different levels of government and intergovernmental transfers affect economic growth. For example, in our very preliminary simulation analysis it has been shown
that local property tax has the largest negative impact on the rate of economic growth, whereas both federal and local consumption taxes are always growth-enhancing. This positive effect of consumption taxes shall be compared to Rebelo (1991). Without public spending, public capital formation, and taxation by multiple levels of government, Rebelo (1991) has found that a consumption tax has no effect on the growth rate. It is also interesting to note that higher consumption taxes at both the federal and local levels are welfare-improving through their positive effects on economic growth. Our analysis also sheds light on the role of intergovernmental transfers in economic growth. The matching grant for local public investment always promotes economic growth, whereas the matching grant for local public consumption is growth-retarding.

For future work, theoretically, we can formulate a game-theoretical growth model allowing strategic interactions between the federal government and multiple local governments in taxes, public expenditures, and intergovernmental transfers. Empirically, we can examine the effects of federal taxes, local taxes, federal transfers on private capital accumulation, federal consumption and investment, and local consumption and investment. Furthermore, the model is also useful for normative discussion on the welfare-maximizing and growth maximizing choices of taxes, transfer, and expenditures in the context of fiscal federalism.

REFERENCES


