ANNALS OF ECONOMICS AND FINANCE **12-1**, 1–11 (2011)

Inflation Aversion

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With inflation aversion, an increase in the monetary growth rate decreases the steady-state value of capital stock, consumption, and real balance holding.

Key Words: Inflation aversion; Capital accumulation ; Money.
JEL Classification Numbers: D91, E63, O23.

1. INTRODUCTION

In optimal growth models, positive time preferences, i.e. people systematically discount utility derived from future consumption, is taken for granted [Olson and Bailey (1981)]. In addition, a further modeling of time preferences by Uzawa (1968), where the time preference is an increasing function of current utility, has been increasingly used in growth and asset accumulation models. See Obstfeld (1981), (1982) and (1989) among others. While most economists do not question the existence of positive

* A very preliminary version of inflation aversion in this paper was developed by Xinsheng Zeng and Heng-fu Zou in 1993. Please see Xinsheng Zeng's Ph.D. thesis at Boston University, 1993.

1529-7373/2011

time preference, they have raised some doubt about Uzawa's assumption. For example, Blanchard and Fischer(1989) state that, for Uzawa's specification, "in steady state, a higher level of consumption implies a higher rate of time preference. The assumption is difficult to defend a priori; indeed, we usually think it is the rich who are more likely to be patient. ... The Uzawa function ... is not particularly attractive as a description of preferences and is not recommended for general use" (pp.72-75).

Here in a monetary growth model, we attempt to establish a link between the time preference and inflation. We will demonstrate that, if there exists positive time preference in the real world, inflation is an important element in determining its magnitude. The argument for defining the time preference as an increasing function of inflation is much stronger than for Uzawa's specification. With this new definition of time discount rate, we will show that inflation reduces long-run capital accumulation, consumption and real balance holdings.

2. ENDOGENOUS TIME PREFERENCE AND INFLATION AVERSION

While the time preference depends on various factors in a society, inflation is an important factor leading to social and economic instability and disorder. Thus, it is convincing to define the time preference, denoted as δ, to be an increasing function of expected inflation rate, denoted as π. Namely, $\delta = \delta(\pi)$, $\delta'(\pi) \geq 0$, and $\delta''(\pi) < 0$. Obviously enough we call this definition as inflation aversion. We first present the theory of endogenous time preference in Rae (1834), Bohm-Bawerk (1959), Fisher (1930), and then argue why this definition is reasonable.

The time preference theory has its direct origin in Rae (1834) and Bohm-Bawerk (1959); for this, Irving Fisher dedicated The Theory of Interest (1930) in their memory. Rae calls the time preference as the effective desire of accumulation, which has the following definition:

"The determination to sacrifice a certain amount of present good, to obtain another greater amount of good, at some future period, may be termed the effective desire of accumulation. All men may be said to have a desire of this sort, for all men prefer a greater to a less; but to be effective it must prompt to action." (Rae, 1834, p.119)

What determines this effective desire of accumulation? Rae mainly lists the following three elements: *"1. The prevalence throughout the society, of the social and benevolent affections... 2. The extent of the intellectual powers, and the consequent prevalence of habits of reflection, and prudence, in the minds of the members of the society. 3. The stability of the condition of the affairs of the society, and the reign of law and order throughout it."*(Rae, 1834, *New Principles of Political Economy*, pp.124.) In this list, Rae does not say anything about how does inflation influence the time preference. But it is quite clear from Rae's long discussion on "the social and benevolent affection" and the desire of the social stability in the moral sense that he admits the role of "inflation aversion" in strengthening the effective desire of accumulation in modern time.When he takes "the money-making spirit" as the main element of the social affection and the instability of the society, he says "(the love of) money is the root of all evil, and infallibly leads to wickedness", "these feelings, therefore, investing the concerns of futurity with a lively interest to the individual, and giving a continuity to the existence and projects of the race, must tend to strengthen very greatly the effective desire of accumulation."

The time preference theory was fully developed by Eugen von Bohm-Bawek. Indeed, Olson and Baily (1981) are right, "the clearest conception of positive time preference that we have been able to find was in Bohm-Bawerk's original account." According to Bohm-Bawerk,

"we feel less concerned about future sensation of joy and sorrow simply because they do lie in the future, and the lessening of our concern is in proportion to the remoteness of that future. Consequently we accord to goods which are intended to serve future ends a value which falls short of the true intensity of their future marginal utility. We systematically undervalue our future wants and also the means which serve to satisfy them. That is a fact of that there can be no doubt." (Bohm-Bawerk, *Capital and Interest*, Vol. II, p.268).

Bohm-Bawerk provides three causes for this positive time preference: (1) "the fragmentary nature of the imaginary picture that we construct of the future state of our wants" (p.269); (2) "a failure of will power and lossing control over ourselves in facing immediate enjoyment"(p.269); and (3) "consideration of the brevity and uncertainty of human life." (p.270)

The causes listed by Bohm-Bawerk in determining positive time preference manifest themselves fully in an inflationary world. First, inflation leads to economic, social and institutional uncertainty, and causes "disillusionment and discontent" (Arthur Burns) in the society and "strikes at confidence" (John Maynard Keynes) of the people. All those uncertainty and anxiety, of course, result in more impatience and larger time discount rate. We cite two excellent quotations of Keynes and Burns from Fabricant (1976).

For John Maynard Keynes,

"There is no subtler, no sure means of overturning the existing basis of society than to debauch the currency. The process engages all the hidden forces of economic law on the side of destruction, and does it in a manner which not one man in a million is able to diagnose ... [The] arbitrary arrangement of riches [caused by inflation] strikes not only at security but at confidence in the equity of the existing distribution of wealth ... All

permanent relations between debtors and creditors, which form the ultimate foundation of capitalism, become so utterly disordered as to be almost meaningless; and the process of wealth-getting degenerates into a gamble and a lottery." (Keynes, 1919, *The Economic Consequences of the Peace*, pp. 235-248.)

Chairman Arthur Burns of the Federal Reserve Board warns "the menace of inflation":

"Concerned as we all are about the economic consequences of inflation, there is even greater reason for concern about the impact on our social and political institution. We must not risk the social stress that persistent inflation breeds. Because of its capricious effects on the income and wealth of a nation's families and businesses, inflation inevitably causes disillusionment and discontent. ... Discontent bred by inflation can provoke profoundly disturbing social and political change, as the history of other nations teaches. I do not believe I exaggerate in saying that the ultimate consequence of inflation could well be a significant decline of economic and political freedom for the American people."

Secondly, while high inflation makes rational economic calculation more difficult or impossible, it makes people possess even less "adequate power to imagine and to abstract" the future world. This corresponds cause (1) of positive time preference pointed out by Bohm-Bawerk. Indeed expected high inflation often leads people to perceive the future in dark color, and people may enjoy more today at the sacrifice of future consumption. This weakness of human will in facing high inflation results in large time discount rate.

Thirdly, the obvious thing about inflation is that with high inflation, even if their real income has kept constant or increased substantially, people still feel being cheated and psychologically they regard inflation as a "bad thing" [Katona (1975)]. This psychological "irrationality" has been proved again and again in experience. According to Katona (1980), in America "there is no doubt that most people consider inflation an evil. In the late 1970s many more Americans said that inflation was the most serious problem confronting them. When asked which causes more serious hardship, inflation or unemployment, about two-thirds of the respondents in 1979 named inflation and one-fourth mentioned unemployment. This despite the fact that ... very many Americans did not feel hurt by inflation" (p.81). Recent experience in China provides alarming signal about how dangerous the high inflation would be, even accompanied by rapid income growth in the decade of economic reforms. People suffer most psychologically from inflation, and if the future is an inflation world, there is no way to stop people from discounting the future heavily.

To sum up, the assumption of time preference as a positive function of expected inflation is quite convincing to us. The only thing we feel strange is why this inflation aversion approach has not been widely used in monetary growth and asset accumulation models.

3. THE INFLATION AVERSION MODEL AND ITS RESULTS

A representative family, whose size grows at natural rate n, maximizes the discounted utility over an infinite horizon ,

$$W = \int_0^\infty u(c,m)e^{-\int_0^t \delta(\pi(s))ds}dt \tag{1}$$

where c is per-capita consumption, m is per-capita real balances holding, π is the expected inflation rate, and δ is the time preference generation function of inflation rate with following properties: $0 < \delta_{\min} \leq \delta(\pi) < 1$, $\delta'(\pi) \geq 0$, and $\delta''(\pi) < 0$, and there exists an inflation rate such that δ retains its minimum $\delta_{\min}$. The instantaneous utility function u is increasing, concave, and continuously differentiable in c and m, namely

$$u_c > 0, u_m > 0, u_{cc} < 0, u_{mm} < 0.$$

Output is produced by a standard neoclassical production function, $f(k)$, and k is per capita capital stock, and $f'(k) > 0$, $f''(k) < 0$.

There are two assets in the representative family's portfolios: money and capital. The dynamic budget constraint for the family is

$$\dot{a} = f(k) + x - c - nk - (\pi + n)m \tag{2}$$

$$a = k + m \tag{3}$$

where a is the total asset, and x is real transfer from the government; and a dot over a variable denotes time derivative.

Under the budget constraints (2), (3), and the given initial capital stock $k(0)$ and $m(0)$, the representative agent is to select the money holding path, captal stock accumulation path, and the consumption level to maximize the discounted utility expressed in equation (1).

Let

$$\Delta_t = \int_0^t \delta(\pi(s))ds \tag{4}$$

and so,

$$d\Delta_t = \delta(\pi(t))dt \tag{4'}$$

Upon substitution, equations (1) and (2) can be written as

$$W = \int_0^\infty \frac{u(c,m)}{\delta(\pi)} e^{-\Delta_t} dt \tag{5}$$

$$\frac{da}{d\Delta} = \frac{f(k) + x - c - nk - (\pi + n)m}{\delta(\pi)} \tag{6}$$

Thus the optimization problem can be reduecd to maximize the function in equation (5) subject to the constraints (6) and (3), with the initial capital stock $k(0)$ and real balances holding $m(0)$ are given.

Associated with the optimization problem, the Hamiltonian is defined as

$$H = \frac{u(c,m)}{\delta(\pi)} + \nu_1 \frac{f(k) + x - c - nk - (\pi + n)m}{\delta(\pi)} + \nu_2(a - k - m) \tag{7}$$

where ν_1 is the Hamilton multiplier associated with the equation (6), and ν_2 is the Lagrange multiplier associated with the wealth constraint (3).

The necessary conditions for optimization are

$$u_c = \nu_1 \tag{8}$$

$$\nu_1 \frac{f'(k) - n}{\delta(\pi)} = \nu_2 \tag{9}$$

$$\frac{u_m(c,m)}{\delta(\pi)} - \nu_1 \frac{\pi + n}{\delta(\pi)} = \nu_2 \tag{10}$$

$$\frac{d\nu_1}{d\Delta} = \nu_1 - \nu_2 \tag{11}$$

and the transversality condition $\lim_{\Delta \to \infty} e^{-\Delta_t} a\nu_1 = 0$.

Substituting euqtions (8) and (9) into equations (10) and (11), and using equation (4'), we get

$$u_m = u_c(f'(k) + \pi) \tag{12}$$

$$\frac{du_c}{dt} = u_c(\delta(\pi) + n - f'(k)) \tag{13}$$

By the definition of the per-capita real balance, we get the accumulation equations for the real balance

$$\dot{m} = m(\theta - \frac{\dot{p}}{p} - n) \tag{14}$$

where θ is the growth rate of nominal money supply, p is the price level.

On the perfect foresight path, the expected inflation rate is equal to the actual inflation rate:

$$\frac{\dot{p}}{p} = \pi \tag{15}$$

Government revenue comes from money creation and makes transfer, x, to the representative agent, so, we have

$$x = \theta m \tag{16}$$

Summarizing the discussion above, the full dynamics of the economy can be described by

$$\dot{c} = -\frac{u_{cm}}{u_{cc}}[f'(k) + \theta - n - \frac{u_m}{u_c}]m + \frac{u_c}{u_{cc}}[\delta(\frac{u_m}{u_c} - f'(k)) + n - f'(k)] \tag{17}$$

$$\dot{m} = m(\theta - \frac{u_m}{u_c} + f'(k) - n) \tag{18}$$

$$\dot{k} = f(k) - nk - c \tag{19}$$

Using equations (17), (18), and (19), we can analyze the dynamic characters of conusmption. capital stock, and the real balance, and from equation (12), we can determine inflation rate.

4. LONG-RUN EFFECTS

In this section, we analyze the long-run effects of money growth rate on the economy[1]. The steady-state value (c^*, m^*, k^*) of consumption, real balances, and the capital stock, reachs when $\dot{c} = \dot{m} = \dot{k} = 0$, hence,

$$f'(k^*) = \delta(\theta - n) + n \tag{20}$$

[1]Appendix A shows that the dynamic system of economy (17)-(19) is saddle-point stable.

$$\frac{u_m(c^*, m^*)}{u_c(c^*, m^*)} = \theta + f'(k^*) - n \tag{21}$$

$$f(k^*) - nk^* - c^* = 0 \tag{22}$$

Equation (20) says that optimal long-run capital is determined by equating the marginal productivity of capital to the sum of discount rate and population growth; equation (21) is the optimal condition for money holding: marginal rate of substitution between real balances and consumption is equal to the ratio of the cost of money holding, $\theta + f'(k^*) - n = f'(k^*) + \pi$, over the cost of consumption, which is one. Except for the dependence of time preference on inflation rate, all these steady state conditions are identical to Sidrauski's (1967).

From (20), it is very easy to see that increase in inflation reduces long-run capital stock:

$$\frac{dk}{d\theta} = \frac{\delta'(\theta - n)}{f''(k^*)} < 0. \tag{23}$$

The reason is quite simple, high inflation leads to more impatience, and the representative family discounts further the future consumption and increase its current consumption; in the end, saving and capital stock will be reduced in the new equilibrium.

The effect of inflation on long-run consumption is negative:

$$\frac{dc}{d\theta} = (f'(k^*) - n)\frac{dk}{d\theta} = \delta\delta'(\theta - n)\frac{dk}{d\theta} < 0. \tag{24}$$

Here we have used steady state condition (20) to get the second equality in (24). The reason for this result is following: in the long run, optimal capital stock is determined by the modified golden rule, and is less than the golden rule level; and there does not exist dynamic inefficiency (overaccumulation of capital) in this economy. Hence any reduction in the capital stock caused by inflation leads to reduction in consumption.

The effect of inflation on real balance holdings is also negative:

$$\frac{dm}{d\theta} = \frac{u_{cc} - u_{cm}}{u_{mm} - u_{cm}}\frac{dc}{d\theta} + \frac{u_c(\delta'(\theta - n) + 1)}{u_{mm} - u_{cm}} \tag{25}$$

Both terms on the right hand side of (25) are negative, for u_{cm} is positive and u_{cc}, u_{mm} are negative. Real balances are reduced because money is more costly to hold (substitution effect), and income is lower as a result of reduced capital (income effect).

Therefore inflation is an "evil" which brings about a high time discount rate and low instantaneous utility by reducing both consumption and real balances in the long run. if the government intends to maximize the steady state welfare of the representative family, the simple rule is to choose an inflation rate which minimizes the time discount rate. In this case, Milton Friedman's (1969) rule may not be right as consumption and capital accumulation, unlike the original Sidrauski's model, are decreasing function of inflation, and

$$\frac{du}{d\theta} = u_c \frac{dc}{d\theta} + u_m \frac{dm}{d\theta} \tag{26}$$

If the minimum of time discount rate, $\delta_{\min}$, is obtained before the deflation rate reaches "$-f'(k)$ ", it is still desirable to deflate further following Friedman's prescription, $\theta = -\delta(\theta - n)$, the optimal growth rate of money supply equals the inverse of time prefence.

5. CONCLUDING REMARKS

We belive that inflation aversion will be widely used in monetary economies and international finance.

APPENDIX A

The Stability of the Model

Linearizing (17), (18) and (19) around the steady state values:

$$\begin{pmatrix} \dot{c} \\ \dot{m} \\ \dot{k} \end{pmatrix} = A \begin{pmatrix} c - c^* \\ m - m^* \\ k - k^* \end{pmatrix} \tag{A.1}$$

where

$$A = \begin{pmatrix} \kappa J_1 & \kappa J_2 & -f''(\frac{u_c}{u_{cc}}(\delta' + 1) + \frac{u_{cm}}{u_{cc}} m)) \\ -mJ_1 & -mJ_2 & mf''(k) \\ -1 & 0 & \delta \end{pmatrix}$$

and $\kappa = (\frac{u_c}{u_{cc}}\delta' + \frac{u_{cm}}{u_{cc}} m)$, $J_1 = \frac{u_{mc}u_c - u_m u_{cc}}{u_c^2}$, and $J_2 = \frac{u_{mm}u_c - u_m u_{cm}}{u_c^2}$. It is assumed that $J_1 > 0$ and $J_2 < 0$.

Denote the 3×3 matrix as A and denote the three characteristic roots as λ_1, λ_2 and λ_3. It is known that the product of the three characteristic roots of the system is given by the determinant of matrix A, and the sum of the three roots is given by the trace of A. We first calculate the determinant of A, $\det(A)$:

$$\lambda_1\lambda_2\lambda_3 = \det(A) = m\frac{u_c}{u_{cc}} f''(k) J_2 < 0 \tag{A.2}$$

So the system has either one negative root or three negative roots. The trace of A does not give us clear sign:

$$\lambda_1 + \lambda_2 + \lambda_3 = tr(A) = \kappa J_1 - mJ_2 + \delta \tag{A.3}$$

where the second and the third terms on the right hand side are positive, but the first term is negative, given $\kappa < 0$, $J_1 > 0$ and $J_2 < 0$.

We also know that the sum of the three second order principal minors

$$\begin{aligned} &\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 \\ = \ &\det\begin{pmatrix} \kappa J_1 & \kappa J_2 \\ -mJ_1 & -mJ_2 \end{pmatrix} + \det\begin{pmatrix} -mJ_2 & mf''(k) \\ 0 & \delta \end{pmatrix} \\ &+\det\begin{pmatrix} \kappa J_1 & -f''(\frac{u_c}{u_{cc}}(\delta'+1) + \frac{u_{cm}}{u_{cc}}m)) \\ -1 & \delta \end{pmatrix} \\ = \ &-\delta mJ_2 + \delta\kappa J_1 - f''(\kappa + \frac{u_c}{u_{cc}}) < 0 \end{aligned} \tag{A.4}$$

This is because that the third term on the right hand side of the second equality of (A.4) are negative, and the sum of the first term and the second term is also negative. To see the latter, the sum of these two terms is given by

$$-\delta mJ_2 + \delta\kappa J_1 = \frac{u_c}{u_{cc}}\delta'\delta J_1 + (J_1 - u_{cc}J_2/u_{cm})mu_{cm}\delta/u_{cc} \tag{A.5}$$

In (A5), we only need to show that the term in the bracket is positive:

$$(J_1 - u_{cc}J_2/u_{cm}) = \frac{1}{u_c^2 u_{cm}}(u_{mc}u_{cm} - u_{cc}u_{mm})u_c > 0 \tag{A.6}$$

From (A.2) and (A.4), it is very easy to see that there exist only one negative root because the existence of three negative roots will contradict (A.4). Our system has one state variable k and two jumping variables (c and m), so there exists a unique perfect foresight path converging to the steady state.

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