RARE-DISASTERS, THE SPIRIT OF CAPITALISM, OVERSAVING, AND ASSET PRICING

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Abstract

Rare disasters become increasingly important for understanding asset pricing, and the spirit of capitalism has been successful in explaining various issues in economic growth, savings behavior, and asset pricing. However, pervious studies don't reveal the connection of these two ideas. This paper tries to fill the gap in the literature and demonstrates the similarities in consumption decision making and savings behavior implied by models of rare disasters and spirit of capitalism, and analyzes interesting implications of this connection in asset pricing.

Keywords: *rare-disasters, spirit of capitalism, oversaving, asset pricing.*

1. Introduction

According to the life-cycle theory of consumption, wealth should decumulate after retirement. But many empirical studies find the so-called "saving puzzle". That is, wealth holdings tend to increase with age, even after retirement, which is hard to explain using usual consumption-saving model and asks us to go beyond the conventional wisdom of economics.

It is generally admitted that successful explanations of economic performance needs to encompass political and social forces in addition to measures of economic variables. Studies of Barro (2004) [3] and Barro and McCleary (2003, 2007) [7, 8] argue that explanations for economic growth should go further to include a nation's culture, of which religion is one important dimension. Based on Weber's (1905) [15] argument, Zou (1994) [16] explains the differences in per capita consumption, per capita stock, and endogenous growth rates by introducing the "capitalist spirit" in the sense of Weber which is modeled mathematically as a class of utility functions defined on both consumption and capital. In order to understand the "savings puzzle", Zou (1995) [17] builds a two-periods overlapping-generations model which also incorporates the "the spirit of capitalism": A typical agent of the i-th generation

We appreciate comments from Robert Barro, Danyang Xie, and Xiaojun Zhao.

maximizes the following utility function

$$u(c_{i,1}) + \alpha v(w_{i,1}) + \frac{u(c_{i,2}) + \alpha v(w_{i,2})}{1+\rho}, \tag{1.1}$$

where $u(.)$ is the utility derived from consumption and $\alpha v(.)$ is the utility derived from wealth accumulation itself. The parameter α is used to measure the so-called "capitalist spirit", and can take any positive value. (So the case that money is a nuisance is excluded by this assumption.) When $\alpha = 0$, it becomes the standard utility function as in Diamond (1965) [11]; when the term $\alpha v(w_{i,1})$ is dropped for the first period but retained for the second, then it becomes the standard bequest models like Blinder (1973) [10]. It is further assumed that both functions $u(.)$ and $v(.)$ are twice differentiable and satisfy: $u'(.) > 0$, $v'(.) > 0$, $u''(.) < 0$, and $v''(.) < 0$.

Instead of inspecting the saving problem with a two-period model, we modify the model in Zou (1995) [17] into a discrete time infinite horizon model of savings in which a general utility function is adopted. Except the introduction of the spirit of capitalism, namely, including wealth in the utility function, all the other aspects of the modified model are the same as the classical infinite horizon model of savings. However, we find that, in the deterministic setting, people's savings behavior can be surprisingly different with and without the spirit of capitalism.

In classical infinite horizon consumption and growth models, the transversality condition (TVC) tells us that it would be suboptimal for the agent to accumulate too much unconsumed assets, and she can raise her utility by consuming these assets in finite time. But the TVC may not hold in the model with the spirit of capitalism. Meanwhile, the consumption-to-wealth ratio should never approach zero in a classical savings model, but this can be the case with the presence of the spirit of capitalism. Based on these findings, we take the limit of the present value of unconsumed wealth in period t as t goes to infinity, and call it the *Index of Absolute Oversaving*; we take limit of the consumption-to-wealth ratio in period t as t goes to infinity, and call it the *Index of Relative Oversaving*. These two indexes clearly characterize the differences between the savings behavior with and without the spirit of capitalism.

Zou (1995) [17] attributes the source of the spirit of capitalism to some ethical reason or "the principle of frugality" which "comes with us from the womb, and never leaves us till we go into the grave". (See Smith (1776) [13, pp. 324–325].) However, we think that the source of the capitalist spirit, or the (possibly monomaniacal) principle to save, is rooted in the fear of uncertainty in the future which may lead to catastrophic consequences. Recent research builds on the rare-disasters idea of Rietz (1988) [12] to explain the long-term average equity premium (Barro (2006) [4], Barro and Ursua (2008) [9], Barro and Jin (2011) [6]) and related issues. We find that a rare-disasters model of savings which is modified from the aforementioned models can also derive the distinctive savings behavior which presents in the model with the capitalist spirit.

From the discussion of the rare-disasters model of savings, we see that the coefficient of relative risk aversion γ, the disaster probability p, and the tail exponent α are crucial to the agent's saving behavior. Among the three parameters, p and α are about the distribution of disasters. It is obvious that people's utility can be enhanced if there is some way to reduce the probability p of disasters. But there is one thing that needs to point out, even if reducing disaster probability p is not possible. When people make savings decisions, what matters is what people believe, instead of what is the true distribution of disasters. Although rational people should have the right expectations about the real world in the long run, it is possible that people are under delusions during a specific period of time. When people overestimate the possibility or severity of disasters, they will get a suboptimal outcome of oversaving and undergo welfare loss.

2. A Classical Model of Savings: Deterministic Case

First, let's consider a deterministic discrete time infinite horizon model of savings. A typical agent faces the following utility maximization problem

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{2.1}$$

such that

$$\begin{aligned} & c_t \in \Gamma^C(w_t) \equiv [0, w_t] \\ & w_{t+1} \in \Gamma^W(w_t, c_t, R_t, y_t) \equiv \{R_t(w_t - c_t) + y_t\} \\ & w_0 = \bar{w}_0 \geq 0 \end{aligned} \tag{2.2}$$

where $\beta \in (0, 1)$ is the time discount factor, c_t and w_t are real consumption and wealth level in period t, respectively, and $R_t(\geq 0)$ and $y_t(\geq 0)$ are gross return rate of savings and real labor income in period t, respectively. It is assumed that R_t and y_t are functions of wealth w_t and c_t:

$$R_t = R(w_t, c_t) \text{ and } y_t = y(w_t, c_t).\text{[1]} \tag{2.6}$$

[1]In fact, this is a very general setting, of which the standard Neoclassical Growth Model can be a special case. To see this, let W_t and C_t be the total amount of wealth and consumption in period t, respectively, then the amount of capital $K_t = W_t - C_t$. Let N_t be the size of population (or labor) in period t. For simplicity, the population growth rate is set to be 0, i.e., $N_t \equiv N$ for all t, but the nonzero growth rate of population can also be modeled in this setting. Let w_t, c_t and k_t be the per capita amount of wealth, consumption and capital, respectively, then $k_t = w_t - c_t$. Consider the neoclassical production function

The instant utility function $u(.)$ is twice differentiable and satisfies $u'(.) > 0$ and $u''(.) < 0$. For technical simplicity, it is assumed that $u(.)$ satisfies the Inada condition. The Euler equation for problem (2.1) is that

$$\begin{aligned} u'(c_t) &= \beta R_t u'(c_{t+1}), \text{ if } c_t < w_t \\ u'(c_t) &\geq \beta R_t u'(c_{t+1}), \text{ if } c_t = w_t \end{aligned} \tag{2.7}$$

and there is the following theorem.

Theorem 2.1. *Problem (2.1) has a solution. If sequence $\{c_t\}_{t=0}^{\infty}$ that is feasible (i.e., satisfying constraints (2.2)) is a solution to problem (2.1), then it satisfies the Euler equation (2.7) and the TVC*

$$\liminf_{t\to\infty} \left(\prod_{s=0}^{t-1} R_s^{-1} \right) (w_t - c_t) = 0. \tag{2.8}$$

Furthermore, under certain conditions, (e.g., $R_t \geq r$ for all $t \geq 0$ and some $r > 1$,) the following expression holds

$$\limsup_{t\to\infty} \frac{c_t}{w_t} > 0. \tag{2.9}$$

Remark 2.2. *The above theorem is not difficult to prove, readers may either refer to some standard text, for example, Stokey and Lucas with Prescott (1989)* [14], *or show it by yourselves. But the converse is a tricky question: Whether the Euler equation and the TVC form a sufficient condition for a feasible sequence $\{c_t\}_{t=0}^{\infty}$ to be the solution to problem (2.1)? This involves the problem of indeterminacy. Note that under some mild restrictions, we can show that problem (2.1) has a unique solution and the first order condition (FOC)*[2] *and the TVC form a sufficient condition for a feasible sequence $\{c_t\}_{t=0}^{\infty}$ to be the solution to problem (2.1). But the FOC involves the value function $V(.)$ which*

$F(K_t, N_t)$ and let

$$f(k_t) = F(k_t, 1) + (1-\delta)k_t, \tag{2.4}$$

where $\delta \in [0,1]$ is the depreciation rate of capital, then $f(k_t) = w_{t+1}$ is the total amount of goods and capital available per person at the beginning of period t. So in this case,

$$R_t = R(w_t, c_t) = f'(k_t) = f'(w_t - c_t) \text{ and} \tag{2.5}$$

$$y_t = y(w_t, c_t) = f(k_t) - R_t \cdot (w_t - c_t) = f(w_t - c_t) - R_t \cdot (w_t - c_t). \tag{2.6}$$

Another special case is that R_t and y_t are constant.

[2]For problem (2.1), define the value function as

$$V(w_0) = \max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{2.10}$$

such that constraints in (2.2) are satisfied, and we have the following Bellman equation

$$g(w_t) = \max_{\substack{c_t \in \Gamma^C(w_t) \\ w_{t+1} \in \Gamma^W(w_t, c_t, R_t, y_t)}} \{u(c_t) + \beta g(w_{t+1})\}. \tag{2.11}$$

is, although theoretically well-defined, hard to calculate. So the FOC is much more difficult to check than the Euler equation.

The intuition of the TVC and expression (2.9) is clear: It would be suboptimal for the agent to accumulate too much unconsumed wealth (assets), because her utility would increase if these assets were consumed in finite time instead.

3. **A Capitalist-spirit Model of Savings: Deterministic Case**

In this section, we modify the capitalist-spirit model in Zou (1995) [17] into a discrete time infinite horizon model of savings in which a general utility function is adopted. A typical agent then faces the following utility maximization problem

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \breve{u}(c_t, w_t) \tag{3.1}$$

such that constraints (2.2) is satisfied. It is assumed that the utility functions $\breve{u}(c,w)$ is concave, twice differentiable and satisfy: $\frac{\partial \breve{u}}{\partial c} > 0$, $\frac{\partial \breve{u}}{\partial w} \geq 0$, where the case that $\frac{\partial \breve{u}}{\partial w} \equiv 0$ corresponds to the standard utility function in which the wealth level w plays no role, i.e., the utility function occurred in problem (2.1). For technical simplicity, it is assumed that $\breve{u}(c,w)$ satisfies the Inada condition with respect to c.

The Euler equation for problem (3.1) is that

$$\begin{array}{ll} \frac{\partial \breve{u}}{\partial c}(c_t, w_t) = \beta R_t \left[\frac{\partial \breve{u}}{\partial c}(c_{t+1}, w_{t+1}) + \frac{\partial \breve{u}}{\partial w}(c_{t+1}, w_{t+1})\right], & \text{if } c_t < w_t \\ \frac{\partial \breve{u}}{\partial c}(c_t, w_t) \geq \beta R_t \left[\frac{\partial \breve{u}}{\partial c}(c_{t+1}, w_{t+1}) + \frac{\partial \breve{u}}{\partial w}(c_{t+1}, w_{t+1})\right], & \text{if } c_t = w_t \end{array} \tag{3.2}$$

which contains an extra term $\frac{\partial \breve{u}}{\partial w}$, the marginal utility of wealth, compared with Euler equation (2.7) of problem (2.1).

The following example shows that the introduction of w into the instant utility function $\breve{u}(c,w)$ is not trivial, i.e., the solution to problem (3.1) characterizes a savings behavior different from that determined by the solution to problem (2.1).

Example 3.1. *Let $\breve{u}(c,w) = \log c + w$, $\beta = 0.96$, $w_0 = 3$, $R_t = 1.01$ and $y_t = 3$ for all $t \geq 0$. Then by Euler equation (3.2), we see that the solution $\{c_t\}_{t=0}^{\infty}$ must satisfy*

$$c_t < (0.96 \cdot 1.01)^{-1} \approx 1.031 \text{ for all } t \geq 0, \tag{3.3}$$

Then the FOC is for problem (2.1) is as follows:

$$\begin{array}{ll} u'(c_t) = \beta R_t V'(w_{t+1}), & \text{if } c_t < w_t \\ u'(c_t) \geq \beta R_t V'(w_{t+1}), & \text{if } c_t = w_t \end{array} \tag{2.12}$$

which implies that in each period, at most one third of her labor income will be consumed by the agent. So it is not hard to check that

(3.4)
$$\liminf_{t\to\infty}\left(\prod_{s=0}^{t-1}R_s^{-1}\right)(w_t-c_t) > \sum_{t=0}^{\infty}1.01^{-t}(3-(0.96\cdot 1.01)^{-1}) = 303-\frac{100}{0.96}\approx 198.83$$

and

$$\limsup_{t\to\infty}\frac{c_t}{w_t}=0, \tag{3.5}$$

which also implies

$$\lim_{t\to\infty}\frac{c_t}{w_t}=0. \tag{3.6}$$

So Theorem 2.1 doesn't hold for problem (3.1) here, although it is valid for problem (2.1). Expression (3.3) and (3.5) clearly shows that with the capitalist spirit, the agent will have a motivation to accumulate wealth even if she would never consume it in any finite time. The quantity $\liminf_{t\to\infty}\left(\prod_{s=0}^{t-1}R_s^{-1}\right)(w_t-c_t)$ and $\limsup_{t\to\infty}\frac{c_t}{w_t}$ are crucial to characterize this distinction in agent's savings behavior with and without the capitalist spirit. We call

$$\liminf_{t\to\infty}\left(\prod_{s=0}^{t-1}R_s^{-1}\right)(w_t-c_t)$$

the *Index of Absolute Oversaving* and denote it by I_A, and we call

$$\limsup_{t\to\infty}\frac{c_t}{w_t}$$

the *Index of Relative Oversaving* and denote it by I_R.[3]

Remark 3.2. *Note that in usual economic applications where irregular or "pathological" examples are excluded, the policy function for consumption $c(w)$ is a concave function of w, which implies that when $\lim_{t\to\infty}w_t=\infty$,*

$$\limsup_{t\to\infty}\frac{c_t}{w_t}=\lim_{t\to\infty}\frac{c_t}{w_t}=\lim_{w\to\infty}\frac{c(w)}{w}. \tag{3.7}$$

Readers may argue that in the previous example, the crucial reason for $I_A>0$ and $I_R=0$ is that $\check{u}(c,w)$ satisfies the Inada condition with respect to c, but not with respect to w, and the utility function used there is not even strictly concave with respect to w. The following example shows that the Inada condition and the strict concaveness is not essential in determining the signs of I_A and I_R.

[3]The term "oversaving" here has a different meaning from that in classical growth models, e.g., the Solow-Swan model and the overlapping-generation (OLG) model. For more details, please refer to some standard text, for instance, Barro and Sala-i-Martin (2004) [2].

Example 3.3. *Let* $\check{u}(c,w) = \log(\log(c+1)) + 100 \cdot \log w$, $\beta = 0.96$, $w_0 = 300$, $R_t = 1.01$ *and* $y_t = 3$ *for all* $t \geq 0$. *Note that in this example,* $\check{u}(c,w)$ *satisfies the Inada condition with respect to both* c *and* w *and is strictly concave. Then by Euler equation (3.2), we see that the solution* $\{c_t\}_{t=0}^{\infty}$ *must satisfy*

$$\frac{1}{\log(c_t+1)} \frac{1}{c_t+1} > 100 \cdot \beta R_t \frac{1}{w_{t+1}} \text{ for all } t \geq 0, \tag{3.8}$$

i.e.,

$$\frac{c_t+1}{w_{t+1}} - \frac{1}{100 \cdot \beta R_t \log(c_t+1)} < 0. \tag{3.9}$$

The left hand side of (3.9) is increasing with c_t, *and from* $w_0 = 300$, $R_t = 1.01$ *and* $y_t = 3$ *for all* $t \geq 0$, *it is not difficult to show that*

$$c_t < 0.009 \cdot w_t \tag{3.10}$$

for all $t \geq 0$. *Expression (3.10) tells us that the agent's consumption in period* t *will be less than the interest accrued from her remaining wealth,* $w_t - c_t$. *So* $\{w_t\}_{t=0}^{\infty}$ *is an increasing sequence, and*

$$\lim_{t \to \infty} w_t = \infty. \tag{3.11}$$

By (3.9), (3.11), $R_t = 1.01$ *and* $y_t = 3$ *for all* $t \geq 0$, *we can show that*

$$\limsup_{t \to \infty} \frac{c_t}{w_t} = 0, \tag{3.12}$$

i.e., in this example, $I_R = 0$. *(3.11) and (3.12) together are very close to the conclusion that* $I_A > 0$, *as we only need to exclude the case that*

$$\liminf_{t \to \infty} \left(\prod_{s=0}^{t-1} R_s^{-1} \right) w_t = 0.$$

It takes some work to show that $I_A > 0$. *Please see the Appendix for the proof of this fact.*

4. **A Rare-Disasters Model of Savings**

Here, we modify the model of Barro (2006) [4] to set up a rare-disasters model of savings. In this setting, a typical agent faces the following utility maximization problem

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{4.1}$$

such that the following constraints are satisfied:

$$
\begin{aligned}
& c_t \in \Gamma^C(w_t) \equiv [0, w_t] \\
& w_{t+1} \in \Gamma^W(w_t, c_t, \tilde{R}_t, \tilde{y}_t) \equiv \{\tilde{R}_t(w_t - c_t) + \tilde{y}_t\} \\
& w_0 = \bar{w}_0 \geq 0
\end{aligned}
\tag{4.2}
$$

Problem (4.1) is the same as problem (2.1) except that the gross return rate of savings $\tilde{R}_t$ and labor income $\tilde{y}_t$ are now subject to uncertainty in the future.

Note that up to now, the setting of problem (4.1) is just a savings model with uncertainty. If we assume that the distributions of $\tilde{R}_t$ and $\tilde{y}_t$ follow normal distributions, like the case in many other models, then the corresponding model won't imply a similar savings behavior as in the model with the spirit of capitalism. So the distribution of $\tilde{R}_t$ is crucial here.[4]

In this model, rare and large macroeconomic disasters may happen in a period with a constant probability $p \geq 0$. If no disaster occurs, then $\tilde{R}_t = R_t$ and $\tilde{y}_t = y_t$ as described previously.[5] If a disaster occurs in period t, then there will be a significant wealth loss at the beginning of period $t+1$ which is represented by $\tilde{R}_t = (1-b)R_t$ and a drop in $\tilde{y}_t$. More clearly, at the beginning of period t, w_t is already fixed; when the agent decides how much to consume, she doesn't know whether disasters will occur later in that period, i.e., she doesn't know the values of $\tilde{R}_t$ and $\tilde{y}_t$. Depending on the setting of the model, the real labor income may or may not contract in the same way. For simplicity, here we assume that they contract by the same fraction $b \in (0,1)$. The disaster size, b, follows a certain probability distribution, which is studied in Barro and Ursua (2008) [9] and Barro and Jin (2011) [6] by using long-term historical data. Let $d(.)$ denote the distribution density of b, then we have

$$d(b) \geq 0 \text{ for any } b \in [b_0, 1), \text{ and } d(b) = 0 \text{ when } b \notin [b_0, 1),$$

where $b_0 \in (0,1)$ is the lower threshold of disaster sizes.

According to Barro and Jin (2011) [6], the size distribution of macroeconomic disasters are well fitted with power law distribution, especially a special power law—the double-power law distribution. Since the single power law distribution already has the essence of the size distribution of macroeconomic disasters, we adopt the single power law distribution here for simplicity. And the extension to the double power law case is immediate. Concretely speaking,

[4]The distribution of $\tilde{y}_t$ is less important, as the labor income in a given period is usually much less than the wealth holdings. Also note that in this simplified model, there is only one asset in the economy which is either risk-free or risky, depending on whether the gross return rate $\tilde{R}_t$ is constant or not.

[5]This assumption can be generalized, that is, the gross return rate $\tilde{R}_t$ and the real labor income $\tilde{y}_t$ can be random variables which fluctuate around their mean values, when there are no disasters.

we define the transformed disaster size as follows:

$$z \equiv \frac{1}{1-b}, \tag{4.3}$$

which in the rare-disaster models is the normal to disaster consumption or GDP. The distribution of b is characterized by the density function of z as

$$f(z) = \begin{cases} 0, & \text{if } z < z_0 \\ Az^{-\alpha}, & \text{if } z \geq z_0 \end{cases} \tag{4.4}$$

where $z_0 = \frac{1}{1-b_0} > 1$ is the transformed lower threshold of disaster sizes. The condition that the density integrate to 1 implies

$$A = (\alpha - 1) \cdot z_0^{\alpha-1}. \tag{4.5}$$

The key parameter that describes the power-law density is the exponent $\alpha > 1$.

In general, the Euler equation for problem (4.1) is as follows:

$$\begin{array}{l} u'(c_t) = \beta E_t \tilde{R}_t u'(c_{t+1}), \text{ if } c_t < w_t \\ u'(c_t) \geq \beta E_t \tilde{R}_t u'(c_{t+1}), \text{ if } c_t = w_t \end{array} \tag{4.6}$$

We assume that the agent has a constant-relative-risk-aversion (CRRA) utility function

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}, \tag{4.7}$$

where $\gamma > 0$, and the case that $\gamma = 1$ corresponds to the log utility. For the case here, the Euler equation becomes

$$\begin{array}{l} c_t^{-\gamma} = \beta E_t \tilde{R}_t c_{t+1}^{-\gamma}, \text{ if } c_t < w_t \\ c_t^{-\gamma} \geq \beta E_t \tilde{R}_t c_{t+1}^{-\gamma}, \text{ if } c_t = w_t \end{array} \tag{4.8}$$

Let $\underline{w}_{t+1}$ denote the wealth level when there are no disasters, then

$$\underline{w}_{t+1} = R_t(w_t - c_t) + y_t;$$

let $\breve{w}_{t+1}$ denote the wealth level when disasters occur in period t, then at the beginning of period $t+1$, $\breve{w}_{t+1}$ is given by

$$\breve{w}_{t+1} = (1-b)R_t(w_t - c_t) + (1-b)y_t = (1-b)\underline{w}_{t+1} = \frac{1}{z}\underline{w}_{t+1}.$$

Let RHS denote the right hand side of the above Euler equation, then we have

$$\begin{array}{rcl} RHS & = & \beta\left[(1-p)R_t c_{t+1}^{-\gamma} + p\int_{z_0}^{\infty} c_{t+1}^{-\gamma}(1-\frac{1}{z})Az^{-\alpha}dz\right] \\ & \geq & \beta\left[(1-p)R_t c_{t+1}^{-\gamma} + pA\int_{z_0}^{\infty} \breve{w}_{t+1}^{-\gamma}(1-\frac{1}{z})z^{-\alpha}dz\right] \\ & = & \beta\left[(1-p)R_t c_{t+1}^{-\gamma} + pA\int_{z_0}^{\infty} z^{\gamma}\underline{w}_{t+1}^{-\gamma}(1-\frac{1}{z})z^{-\alpha}dz\right] \\ & \geq & \beta\left[(1-p)R_t c_{t+1}^{-\gamma} + pA\underline{w}_{t+1}^{-\gamma}\int_{z_0}^{\infty}(1-\frac{1}{z})z^{\gamma-\alpha}dz\right] \\ & \geq & \beta\left[(1-p)R_t c_{t+1}^{-\gamma} + pA\underline{w}_{t+1}^{-\gamma}(1-\frac{1}{z_0})\int_{z_0}^{\infty} z^{\gamma-\alpha}dz\right] \end{array} \tag{4.9}$$

From the above inequalities, we see that when $\alpha - \gamma > 1$ doesn't hold, we have $RHS \geq \infty$, which is impossible. So when $\alpha - \gamma > 1$ doesn't hold, there is no optimal solution to problem (4.1), as the agent has an insatiable desire to save. Recall that in Barro and Jin (2011) [6], there is a situation which is similar to this one in some sense: when $\alpha - \gamma > 1$ is not the case, the equity premium will be infinite. Now we assume $\alpha - \gamma > 1$, which also accords with the result in Barro and Jin (2011) [6]. Then we have

$$
\begin{aligned}
RHS &\geq \beta \left[(1-p)R_t c_{t+1}^{-\gamma} + pA\underline{w}_{t+1}^{-\gamma} \int_{z_0}^{\infty} (1 - \tfrac{1}{z}) z^{\gamma-\alpha} dz\right] \\
&\geq \beta \left[(1-p)R_t \underline{w}_{t+1}^{-\gamma} + pA\underline{w}_{t+1}^{-\gamma} \int_{z_0}^{\infty} (1 - \tfrac{1}{z}) z^{\gamma-\alpha} dz\right] \\
&= \beta \left[(1-p)R_t + pA\left(\tfrac{z_0^{\gamma-\alpha+1}}{\alpha-\gamma-1} - \tfrac{z_0^{\gamma-\alpha}}{\alpha-\gamma}\right)\right] \underline{w}_{t+1}^{-\gamma} \\
&= \beta \left[(1-p)R_t + p(\alpha-1)z_0^{\alpha-1}\left(\tfrac{z_0^{\gamma-\alpha+1}}{\alpha-\gamma-1} - \tfrac{z_0^{\gamma-\alpha}}{\alpha-\gamma}\right)\right] \underline{w}_{t+1}^{-\gamma} \\
&= \beta \left[(1-p)R_t + p(\alpha-1)\left(\tfrac{z_0^{\gamma}}{\alpha-\gamma-1} - \tfrac{z_0^{\gamma-1}}{\alpha-\gamma}\right)\right] \underline{w}_{t+1}^{-\gamma}.
\end{aligned} \tag{4.10}
$$

A lot of information is contained in Euler equation (4.8) and inequalities (4.9) and (4.10). In order to have a clear understanding of the concealed information, let's see a concrete example.

Example 4.1. *Suppose that $R(w_t, c_t)$ and $y(w_t, c_t)$ are constant functions, i.e., $R_t = R(w_t, c_t) > 1$ and $y_t = y(w_t, c_t) > 0$ are constants. Let $\xi = (\alpha - 1)\left(\frac{z_0^{\gamma}}{\alpha-\gamma-1} - \frac{z_0^{\gamma-1}}{\alpha-\gamma}\right)$, and assume that $\beta R_t < 1$ and $\xi \gg R_t^{\gamma}$. Then there exists some $p \in (0,1)$ such that the corresponding consumption policy function $c(w)$ satisfies*

$$
\lim_{w\to\infty} \frac{c(w)}{w} = 0. \tag{4.11}
$$

From Remark 3.2, we see that (4.11) is equivalent to the fact that the index of relative oversaving $I_R = 0$, if $\lim_{t\to\infty} w_t = \infty$ is the case. The proof of (4.11) takes some efforts. Please see the Appendix for the proof.

5. **Appendix**

Proof of $I_A > 0$ in Example 3.3. Suppose that the policy function for consumption is $c(.)$, i.e., $c_t = c(w_t)$. In this example, it is clear that $c(.)$ is a nondecreasing function.

Proof of (4.11) in Example 4.1. To be finished.

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