Asset pricing and the Modigliani-Miller theorem with the spirit of capitalism

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Abstract: This paper investigates an intertemporal general equilibrium theory of capital asset pricing with the spirit of capitalism. It is an attempt to put together ideas from the modern finance literature and the literature on stochastic growth models. Using methods adopted in Brock (1982), an equivalent characterization of equilibrium by first-order conditions for optimal growth is investigated. We also consider the Modigliani-Miller theorem with the spirit of capitalism.

Keywords: Spirit of capitalism; equilibrium; Optimal growth; Social-status.

JEL Classification: G1, G10,G11,G12.

1 Introduction

In growth and asset-pricing models, wealth accumulation is often taken to be solely driven by one's desire to increase consumption rewards. The representative agent chooses his consumption path to maximize his/her discounted utility, which is defined only on consumption. Whereas this motive is important for wealth accumulation, it is, however, not the only motive. As social animals, people accumulate wealth also to gain prestige, social status, and power in the society. Possession of wealth is, to considerable degree, a measure and standard of a person's success in a society. There is a recent literature that has paid attention to this motive which cogently argues that concern for social status is instrumental in obtaining nonmarket goods; see Cole et al. (1992), Zou (1994,1995), Bakshi and Chen (1996), Carroll (2000) and Fancis (2009). In these wealth-is-status models, the representative agent accumulates wealth not only for consumption but also for wealth-induced status. Another interpretation of these models is in line with the spirit of capitalism (SOC) in the sense of Weber (1958) and Keynes (1971): capitalists accumulate wealth for the sake of wealth. To quote Weber (1958):

Man is dominated by the making of money, by acquisition as the ultimate purpose of his life. Economic acquisition is no longer subordinated to man as the means for the satisfaction of his material needs. This reversal relationship, so irrational from a naive point of view, is evidently a leading principle of capitalism.

The SOC hypothesis is formalized by including wealth into the utility function. Concerns about the relative social position arise endogenously in a model with intergenerational transmission of cultural traits (Bisin and Verdir, 1998) and in an evolutionary model with incomplete environmental information (Samuelson, 2004). The financial literature justifies direct preferences for wealth on different grounds. Behavioral experiments demonstrate that the degree of risk aversion varies with financial wealth. Financial models with utility flows from the level or changes in the level of financial wealth help to explain various stylized facts about financial market (Barberis et al., 2001; Levy and Levy, 2004; McQueen and Vorkink, 2004).

Lucas (1978) studied asset pricing theory and equilibrium in a pure exchange economy. The equilibrium for a production economy was investigated by Brock (1982). The current paper introduces the SOC hypothesis into a neoclassical growth model. The equilibrium for the model is defined and characterized by the first-order conditions for optimal growth problem.

The empirical estimates support the existence of the intrinsic desire for wealth. This is a contribution to the literature on the spirit of capitalism, since many studies in this area tend to be theoretically oriented. Karnizova (2010) introduces the SOC hypothesis into a neoclassical growth model with capital adjustment costs and demonstrates how the SOC hypothesis is one mechanism through which news shocks can lead to boom and busts. In this paper, we give a theoretical proof of the existence of equilibrium. We will characterize equilibrium by the first-order conditions of optimal growth model, and then prove the existence of equilibrium.

Modigliani and Miller(1958) prove that if markets are complete, then firms are indifferent between debt and equity financing, so the debt-equity ratio is indeterminate. In this paper, we will consider the Modigliani-Miller theorem for SOC model.

The rest of this paper is organized as follows. Section 2 describes the SOC model. Section 3 gives an equivalent characterization of equilibrium. Section 4 consider the Modigliani-Miller theorem with social-status concern. Section 5 an explicit example of the model is solved. We conclude this paper in Section 6. The Appendix contains the proofs.

2 The spirit-of-capitalism model

2.1 The household's problem

The economy is populated by identical households who maximize their lifetime utility

$$E_0\Big[\sum_{t=0}^{\infty}\beta^t u(c_t,x_t,l_t)\Big],\quad 0<\beta<1, \tag{1}$$

defined over consumption c_t, the index of status x_t, and the labor l_t. The notation E_0 corresponds to the expectation conditional upon the information available in period zero, and β is the discount factor. The utility function $u:\mathbb{R}^3_+\to\mathbb{R}^3_+$ is twice continuously differentiable, the assumption is equivalent to

$$u_c>0,\ u_x>0,\ u_l<0,\ u_{cc}<0,\ u_{xx}<0,\ u_{ll}>0.$$

In addition, $u(c_t,x_t,l_t)$ satisfies the Inada conditions

$$\lim_{c\to 0}u_c=\infty,\ \lim_{c\to\infty}u_c=0,\ \lim_{x\to 0}u_x=\infty,\ \lim_{x\to\infty}u_x=0.$$

The representative household faces a sequence of budget constraints

$$c_t+q_t z_{t+1}+b_{t+1}\le w_t l_t+(q_t+d_t)z_t+(1+r_t)b_t,\quad t=0,1,\cdots, \tag{2}$$

where w_t is the hourly wage, z_{t+1} is the share of a representative firm owned by the household at the end of period t, q_t is the period t price of a claim to future profits of the firm, d_t are the period t dividends, b_t and $1+r_t$ are the quantity and the gross return on risk-free one period bonds. The return $1+r_t$ is determined in period t, but is paid off in $t+1$. Status is defined as the value of financial assets held at the end of each period:

$$x_t=q_t z_{t+1}+b_{t+1}. \tag{3}$$

The representative household chooses consumption and asset holdings to maximize the lifetime utility subject to the definition of status and the budget constraints. The first conditions for the household's problem are

$$u_c(c_t,x_t,l_t)=u_x(c_t,x_t,l_t)+\beta E_t[u_c(c_{t+1},x_{t+1},l_{t+1})r^e_{t+1}], \tag{4}$$

$$u_c(c_t,x_t,l_t)=u_x(c_t,x_t,l_t)+\beta(1+r_t)E_t[u_c(c_{t+1},x_{t+1},l_{t+1})], \tag{5}$$

$$u_l(c_t,x_t,l_t)=w_t u_c(c_t,x_t,l_t). \tag{6}$$

The variable $r^e_{t+1}=(q_{t+1}+d_{t+1})/q_t$ defines the return from holding the firm for one period.

2.2 The firm's problem

A perfectly competitive representative firm produces aggregate output from capital and labor inputs using a function that satisfies constant returns to scale

$$y_t = f(k_t, l_t, \theta_t), \tag{7}$$

where $f_k > 0$, $f_l > 0$, $f_{kk} < 0$, $f_{ll} < 0$ and $\theta_t : \Omega \to \Theta = [\underline{\theta}, \overline{\theta}]$ is the technology shock that is i.i.d with stationary distribution function G. The function G has the properties that $G(\theta) = 0$ for $\theta \leq \underline{\theta}$ and $G(\theta) = 1$ for $\theta \geq \overline{\theta}$. Also $dG > 0$ and dG is continuous.

The firm hires labor and makes investment to maximize its value to the owners. The firm dose not issue new shares, and the amount of the existing shares is normalized to one. The period t value of the firm equals to the present value of its current and future dividends

$$E_t(D_t + S_t) = D_t + E_t\Big\{\sum_{i=1}^{\infty}\Big[\Big(\prod_{j=1}^{i}\frac{1}{r^e_{t+i+j}}\Big)D_{t+i+j}\Big]\Big\},$$

where $D_t = y_t - w_t l_t - i_t$, and i_t is the investment of period t. The firm's capital stock obeys an accumulation equation

$$k_{t+1} = (1-\delta)k_t + i_t, \quad k_0 \text{ is given.}$$

The first order conditions for the profit maximization are

$$f_l(k_t, l_t, \theta_t) = w_t; \tag{8}$$

$$1 = E_t\{\frac{1}{r^e_{t+1}}[f_k(k_{t+1}, l_{t+1}, \theta_{t+1}) + (1-\delta)]\}. \tag{9}$$

Constant returns to scale in output production implies that the present value of the expected future dividends per unit of capital is equal to 1, that is

$$1 = \frac{1}{k_{t+1}}\sum_{j=1}^{\infty}\Big[\Big(\prod_{i=1}^{j}\frac{1}{r^e_{t+j}}\Big)N_{t+j}\Big] = \frac{q_t}{k_{t+1}}.$$

Thus, at an optimum the period t market value of the firm is $q_t = k_{t+1}$. The value of the firm is equal to the capital it owns. Assumptions also imply that the return on owning the firm for one period coincides with the return on owning a unit of capital. Hence an alternative expression for the risky return is

$$r^e_{t+1} = f_k(k_{t+1}, l_{t+1}, \theta_{t+1}) + (1-\delta). \tag{10}$$

2.3 Equilibrium

Definition 1. *A rational expectation equilibrium is the sequences of prices* $\{w_t, r_t^e, q_t, r_t\}_{t=0}^{\infty}$ *and allocations* $\{l_t, c_t, x_t, i_t, z_{t+1}, b_{t+1}, y_t, k_{t+1}, d_t\}_{t=0}^{\infty}$ *such that* (1) *allocations are optimal given prices (that is, households maximize utility and firms maximize profits) and* (2) *market for the goods and assets clear, given* k_0 *and the technology processes* $\{\theta_t\}$, *that is* (*i*) $z_{t+1} \leq 1$ *and* $q_t = 0$ *if* $z_{t+1} < 1$; (*ii*) $c_t + i_t = f(k_t, l_t, \theta_t)$; (*iii*) $b_{t+1} = 0$.

3 Asset Pricing and growth with SOC

3.1 A pricing function with SOC

It would be nice if the first order conditions $(4)-(6)$ characterize consumer optima. But it is well known that a "transversality condition" as infinity is needed in addition to completely characterize optima. In fact, we can prove

Lemma 1. *Assume that* $W(y_t, t) \to 0, \ \ t \to \infty$, *where* $W(y_t, t)$ *is defined by*

$$W(y_t, t) = \max E_0\big[\sum_{s=t}^{\infty} \beta^s u(c_s, x_s, l_s)\big] \tag{11}$$

subject to (2). *Then, given* $\{w_t\}_{t=0}^{\infty}$, $\{q_t\}_{t=0}^{\infty}$, $\{d_t\}_{t=0}^{\infty}$, *optimum solutions* $\{l_t\}_{t=0}^{\infty}$, $\{c_t\}_{t=0}^{\infty}$, $\{z_{t+1}\}_{t=0}^{\infty}$, $\{b_{t+1}\}_{t=0}^{\infty}$ *to the consumer's problem* (1) *subject to* (2) *are characterized by* $(4)-(6)$ *and*

$$\lim_{t\to\infty} E_0\{\beta^t u_c(c_t, x_t, l_t) q_t z_{t+1}\} = 0, \tag{12}$$

$$\lim_{t\to\infty} E_0\{\beta^t u_c(c_t, x_t, l_t) b_{t+1}\} = 0. \tag{13}$$

and

$$\lim_{t\to\infty} E_0\{\beta^t u_x(c_t, x_t, l_t) x_t\} = 0. \tag{14}$$

In view of (4), we get

$$\begin{aligned} u_c(c_t, x_t, l_t) q_t &= u_x(c_t, x_t, l_t) q_t + \beta E_t[u_c(c_{t+1}, x_{t+1}, l_{t+1})(q_{t+1} + d_{t+1})] \\ &= \frac{u_x(c_t, x_t, l_t)}{u_c(c_t, x_t, l_t)} u_c(c_t, x_t, l_t) q_t + \beta E_t[u_c(c_{t+1}, x_{t+1}, l_{t+1}) d_{t+1}] \\ &\quad + \beta E_t[u_c(c_{t+1}, x_{t+1}, l_{t+1}) q_{t+1}]. \end{aligned}$$

Therefore,

$$\begin{aligned}
u_c(c_t, x_t, l_t)q_t &= \big(1 - \frac{u_x(c_t, x_t, l_t)}{u_c(c_t, x_t, l_t)}\big)^{-1}\beta E_t[u_c(c_{t+1}, x_{t+1}, l_{t+1})d_{t+1}] \\
&\quad + \big(1 - \frac{u_x(c_t, x_t, l_t)}{u_c(c_t, x_t, l_t)}\big)^{-1}\beta E_t[u_c(c_{t+1}, x_{t+1}, l_{t+1})q_{t+1}].
\end{aligned}$$

If we assume $d_t \geq 0$, then $1 - \frac{u_x(c_t,x_t,l_t)}{u_c(c_t,x_t,l_t)} \geq 0$, that is $u_x(c_t, x_t, l_t) \leq u_c(c_t, x_t, l_t)$. Therefore, the equilibrium attains only on the set

$$\Big\{\{c_t, x_t, l_t\}_{t=0}^{\infty} | u_x(c_t, x_t, l_t) \leq u_c(c_t, x_t, l_t),\ t \geq 0\Big\}.$$

For any $0 < \alpha < 1$, define

$$\Lambda_\alpha = \Big\{y \in [0, \infty) \times [0, \infty) \times [0, \infty) | 0 < \frac{\beta}{1 - \frac{u_2(y)}{u_1(y)}} \leq \alpha\Big\}.$$

Let

$$G(y_t) = \big(1 - \frac{u_x(y_t)}{u_c(y_t)}\big)^{-1}\beta E_t[u_c(y_{t+1})d_{t+1}]$$

for $y_t = (c_t, x_t, l_t)$, and Z_α be the space of bounded continuous functions defined on Λ_α with the norm defined by $\|F\| = \sup_{y\in\Lambda_\alpha} |F(y)|$, for $F \in Z_\alpha$, define

$$(TF)(y) = G(y) + \frac{\beta}{1 - \frac{u_2(y)}{u_1(y)}} E_t[F(y')].$$

It is easy to know $(Z_\alpha, \|\cdot\|)$ is a Banach space, and we can prove

Lemma 2. *$T : Z_\alpha \to Z_\alpha$ is a contraction with modulus α.*

Proof. We must show that for any two elements F_1, $F_2 \in Z_\alpha$,

$$\|TF_1 - TF_2\| \leq \alpha\|F_1 - F_2\|.$$

This can be prove by the definition of T easily. □

Theorem 1. *There exists exactly one asset pricing function of the form $P(y)$ where $P \in Z_\alpha$.*

The proof of Theorem 1 is a simple application of contraction mapping theorem.

Remark 1. *If we let $\alpha = \beta$, then Theorem 1 is the case of Brock (1982).*

3.2 Optimal growth model with SOC

Consider the model which is given by

$$\max E_0\big[\sum_{t=0}^{\infty}\beta^t u(c_t, k_{t+1}, l_t)\big], \tag{15}$$

subject to

$$c_t + k_{t+1} \leq f(k_t, l_t, \theta_t) + (1-\delta)k_t, \tag{16}$$

where f is the production function and k_0, θ_0 are historically given.

The first-order conditions for the optimal growth model are follows:

$$u_c(c_t, k_{t+1}, l_t) = u_k(c_t, k_{t+1}, l_t) + \beta E_t[u_c(c_{t+1}, k_{t+2}, l_{t+1})(f_k(k_{t+1}, l_{t+1}, \theta_{t+1}) + (1-\delta))]; \tag{17}$$

$$u_l(c_t, k_{t+1}, l_t) = -u_c(c_t, k_{t+1}, l_t) f_l(k_t, l_t, \theta_t). \tag{18}$$

Lemma 3. *(i) Let $X = \{c_t, k_t, l_t\}_{t=0}^{\infty}$ solve the optimal growth problem* (15). *If define*

$$w_t = f_l(k_t, l_t, \theta_t),\;\; i_t = k_{t+1} - (1-\delta)k_t,\;\; x_t = q_t = k_{t+1}; \tag{19}$$

and

$$\begin{aligned} &d_t = f(k_t, l_t, \theta_t) - f_k(k_t, \theta_t)k_t,\; 1 + r_t = f_k(k_t, l_t, \theta_t) + (1-\delta),\; r_t^e = \frac{q_t + d_t}{q_t}, \\ &z_{t+1} = 1,\; b_{t+1} = 0, \end{aligned} \tag{20}$$

then $(w_t, q_t, d_t, r_t, r_t^e, l_t, x_t, i_t, y_t, k_{t+1}, c_t, z_{t+1}, b_{t+1})$ is an equilibrium.

(ii) Let $(w_t, s_t, d_t, r_t^f, r_t^e, l_t, x_t, i_t, y_t, k_{t+1}, c_t, z_t, b_t)$ be an equilibrium. Then $X = \{c_t, k_t, l_t\}_{t=0}^{\infty}$ solves the optimal growth problem (15)*, where $k_{t+1} = q_t$.*

4 The Modigliani-Miller Theorem with SOC

The Modigliani-Miller theorem says that if markets are complete, then firms are indifferent between debt and equity financing, so the debt-equity ratio is indeterminate. In this section, we consider the Modigliani-Miller theorem for SOC model. To ease the process of derivation and comparisons, we follow the notations of Altug and Labadie (2008).

4.1 The Modigliani-Miller theorem for SOC model

The gross profit of the firm equals total sales minus its wage bill, or

$$\pi_t = f(k_t, l_t, \theta_t) - w_t l_t.$$

The receipts π_t are disbursed in various ways: either paid out as dividends, $d_t z_t$, as payments on bonds $(1 + r_t)b_t$, or held as retained earnings, RE_t. The following accounting identity holds:

$$\pi_t = RE_t + d_t z_t + (1 + r_t)b_t.$$

A firm can finance investment by: issuing new bonds b_{t+1}; issuing new equity shares $q_t(z_{t+1} - z_t)$; or its retained earnings, RE_t. Hence, the firm's investment, $k_{t+1} - (1 - \delta)k_t$, satisfies:

$$k_{t+1} - (1 - \delta)k_t = b_{t+1} + (z_{t+1} - z_t)q_t + RE_t.$$

Define net cash flow from the firm to households as

$$N_t = \pi_t - k_{t+1} + (1 - \delta)k_t,$$

which is the gross profit net of investment. By the definition of π_t,

$$N_t = d_t z_t + (1 + r_t)b_t + q_t(z_t - z_{t+1}) - b_{t+1}.$$

Thus, the net cash flow to households equals the sum of dividend and interest payments on outstanding shares and debt minus new share and debt issues.

The ex-dividend value of the firm, or the value of the firm at the end of the period after all dividend and debt payments have been made, is defined as the value of its equity shares, $q_t z_{t+1}$, plus the value of its outstanding debt, b_{t+1}. $W_t^e = q_t z_{t+1} + b_{t+1}$. We can also define the ex-dividend value as the total claims of shareholders and debtholders on the firm. The value of the firm at the beginning of period t is the sum of net cash flow in period t and the ex-dividend value

$$W_t = N_t + W_t^e = N_t + q_t z_{t+1} + b_{t+1}.$$

Let $m_{t+1} = \beta E_t\big[\frac{u_c(c_{t+1},x_{t+1},l_{t+1})}{u_c(c_t,x_t,l_t)}\big]$, then

$$\begin{aligned}
W_t^e &= \big[1 - \frac{u_x(c_t,x_t,l_t)}{u_c(c_t,x_t,l_t)}\big]^{-1} E_t\big\{m_{t+1}\big[(q_{t+1}+d_{t+1})z_{t+1} + (1+r_t)b_{t+1}\big]\big\} \\
&= \big[1 - \frac{u_x(c_t,x_t,l_t)}{u_c(c_t,x_t,l_t)}\big]^{-1} E_t\{m_{t+1}[W_{t+1}^e + d_{t+1})z_{t+1} + (1+r_{t+1})b_{t+1} \\
&\quad +q_{t+1}(z_{t+1} - z_{t+2} - b_{t+2}]\} \\
&= \big[1 - \frac{u_x(c_t,x_t,l_t)}{u_c(c_t,x_t,l_t)}\big]^{-1} E_t[m_{t+1}(W_{t+1}^e + N_{t+1})] \\
&= \big[1 - \frac{u_x(c_t,x_t,l_t)}{u_c(c_t,x_t,l_t)}\big]^{-1} E_t\Big[\sum_{i=1}^{\infty}\Big(\prod_{j=1}^{i} m_{t+i+j}\Big)N_{t+i+j}\Big],
\end{aligned}$$

assuming that the discounted value

$$\lim_{i\to\infty} E_t[(\prod_{j=1}^{i} m_{t+i+j})W_{t+i}^e] \to 0.$$

At the beginning of period t, the firm solves

$$W_t = \max_{\{k_{j+1},l_j\}_{j=t}^{\infty}} \big[1 - \frac{u_x(c_t,x_t,l_t)}{u_c(c_t,x_t,l_t)}\big]^{-1}\Big\{N_t + E_t\Big[\sum_{i=1}^{\infty}\Big(\prod_{j=1}^{i} m_{t+i+j}\Big)N_{t+i+j}\Big]\Big\},$$

subject to the law of motion for capital $k_{t+1} = (1-\delta)k_t + i_t$, given the expression for net cash flow $N_t = f(k_t,l_t,\theta_t) - w_t l_t - i_t$ and the initial capital stock k_t.

Under the assumption of constant returns to scale, gross profits are equal to $\pi_t = f'(k_t,l_t,\theta_t)k_t$, or the value of capital which is the marginal product of capital times the capital stock per capita. Hence, the net cash flow is

$$N_t = f'(k_t,l_t,\theta_t)k_t - [k_{t+1} - (1-\delta)k_t].$$

Therefore, the firm's cash flow does not depend on the financing decisions made by the firm. In particular, it does not depend on the amount of equity issued, the debt-equity ratio, nor on the amount of retained earnings. If there is no SOC hypothesis, then $\frac{u_x(c_t,x_t,l_t)}{u_c(c_t,x_t,l_t)} = 0$, this is the Modigliani-Miller theorem. With SOC hypothesis, although $\frac{u_x(c_t,x_t,l_t)}{u_c(c_t,x_t,l_t)} \neq 0$, the Modigliani-Miller theorem also holds.

4.2 Taxes and the debt-equity ratio with SOC

It is well known that the Modigliani-Miller theorem does not hold with taxes, we hope to offer further insights on this issue with SOC in this section. The government assesses a proportional income tax equal to τ_y on households so that a household's budget constraint becomes

$$c_t + q_t z_{t+1} + b_{t+1} \leq (1-\tau_y)(w_t l_t + r_t b_t + d_t z_t) + b_t + q_t z_t. \tag{21}$$

The first-order conditions are

$$u_c(c_t, x_t, l_t) = u_x(c_t, x_t, l_t) + \beta E_t[u_c(c_{t+1}, x_{t+1}, l_{t+1}) \frac{q_{t+1} + (1-\tau_y)d_{t+1}}{q_t}], \tag{22}$$

$$u_c(c_t, x_t, l_t) = u_x(c_t, x_t, l_t) + \beta(1 + (1-\tau_y)r_t)E_t[u_c(c_{t+1}, x_{t+1}, l_{t+1})], \tag{23}$$

$$u_l(c_t, x_t, l_t) = (1-\tau_y)w_t u_c(c_t, x_t, l_t). \tag{24}$$

We assume constant returns to scale in production for convenience, the firm's gross profits are

$$\pi_t = f(k_t, l_t, \theta_t) - w_t l_t.$$

The gross profits of a firm are distributed as

$$(1-\tau_p)\pi_t = RE_t + d_t z_t + (1+r_t)b_t - \tau_p r_t b_t.$$

In this expression, the term $\tau_p r_t b_t$ reflects the deduction of interest payments on debt and $\tau_p \pi_t$ is the amount of tax paid by the firm. Define the after-tax net cash flow as N_t^τ by

$$\begin{aligned} N_t^\tau &= (1-\tau_p)\pi_t - [k_{t+1} - (1-\delta)k_t] \\ &= d_t z_t + (1+r_t)b_t - \tau_p r_t b_t + q_t(z_t - z_{t+1}) - b_{t+1}. \end{aligned}$$

Let $m_{t+1} = \beta E_t\big[\frac{u_c(c_{t+1}, x_{t+1}, l_{t+1})}{u_c(c_t, x_t, l_t)}\big]$. Then

$$\begin{aligned} W_t^e &= q_t z_{t+1} + b_{t+1} \\ &= \big[1 - \frac{u_x(c_t, x_t, l_t)}{u_c(c_t, x_t, l_t)}\big]^{-1} E_t\big\{m_{t+1}\big[(q_{t+1} + (1-\tau_y)d_{t+1})z_{t+1} \\ &\quad +(1 + r_{t+1}(1-\tau_y))b_{t+1}\big]\big\} \\ &= \big[1 - \frac{u_x(c_t, x_t, l_t)}{u_c(c_t, x_t, l_t)}\big]^{-1} E_t\{m_{t+1}[W_{t+1}^e + (1-\tau_y)d_{t+1})z_{t+1} \\ &\quad +(1 + r_{t+1}(1-\tau_y))b_{t+1} + q_{t+1}(z_{t+1} - z_{t+2} - b_{t+2}]\} \\ &= \big[1 - \frac{u_x(c_t, x_t, l_t)}{u_c(c_t, x_t, l_t)}\big]^{-1} E_t\Big\{m_{t+1}\big[((\tau_p - \tau_y)r_{t+1}b_{t+1} \\ &\quad -\tau_y d_{t+1}z_{t+1} + N_{t+1}^\tau + W_{t+1}^e\big]\Big\}. \end{aligned}$$

Define the debt-equity ratio and the dividend-price ratio by

$$D_t = \frac{b_{t+1}}{q_t z_{t+1}}, \quad \Psi_{t+1} = \frac{d_{t+1}}{q_t},$$

then, by first-order condition (23),

$$q_t z_{t+1} = \frac{W_t^e}{1+D_t}, \quad e_t[m_{t+1} r_{t+1}] = \frac{1 - \frac{u_x(c_t,x_t,l_t)}{u_c(c_t,x_t,l_t)} - E_t[m_{t+1}]}{1-\tau_y}.$$

Therefore,

$$\begin{aligned} W_t^e &= \big[1 - \frac{u_x(c_t,x_t,l_t)}{u_c(c_t,x_t,l_t)}\big]^{-1}\Big\{\frac{\tau_p - \tau_y}{1-\tau_y}[1 - \frac{u_x(c_t,x_t,l_t)}{u_c(c_t,x_t,l_t)} - E_t[m_{t+1}]]\frac{W_t^e D_t}{1+D_t} \\ &\quad + E_t\Big[m_{t+1}\Big(-\tau_y \frac{W_t^e \Psi_{t+1}}{1+D_t} + N_{t+1}^{\tau} + W_{t+1}^e\Big)\Big] \\ &= E_t[\Omega_{t+1}(N_{t+1}^{\tau} + W_{t+1}^e)], \end{aligned}$$

where

$$\Omega_{t+1} = \frac{m_{t+1}}{1 - \frac{u_x(c_t,x_t,l_t)}{u_c(c_t,x_t,l_t)} + \frac{D_t}{1+D_t}\frac{\tau_y - \tau_p}{1-\tau_y}(1 - \frac{u_x(c_t,x_t,l_t)}{u_c(c_t,x_t,l_t)} - E_t[m_{t+1}]) + \frac{\tau_y}{1+D_t}E_t[m_{t+1}\Psi_{t+1}]}.$$

Notice that if $\tau_y = \tau_p = 0$, then $\Omega_{t+1} = m_{t+1}$. The cost of capital to the firm, defined as $\rho = 1/\Omega - 1$ when there is taxation and $\rho = 1/m - 1$ when there is not, increases with distortionaty taxation. The cost of capital with distortionary taxation can be expressed by

$$\begin{aligned} \rho_{t+1} &= \frac{1}{m_{t+1}}\Big(1 - \frac{u_x(c_t,x_t,l_t)}{u_c(c_t,x_t,l_t)} + \frac{b_{t+1}}{W_t^e}\frac{\tau_y - \tau_p}{1-\tau_y}(1 - \frac{u_x(c_t,x_t,l_t)}{u_c(c_t,x_t,l_t)} - E_t[m_{t+1}]) \\ &\quad + \frac{\tau_y q_t z_{t+1}}{W_t^e}E_t[m_{t+1}\Psi_{t+1}] - m_{t+1}\Big), \end{aligned}$$

which is a weighted average of the cost of debt capital and the cost of equity capital. So, as long as the costs of debt and equity are not equal, the cost of capital of the firm will depend on how much is financed with debt and equity. The Modigliani-Miller theorem no longer holds because clearly the discount rate now depends on the financing decisions made by firm.

Brock and Turnovsky (1981) study the firm's optimization problem and show that the optimal dividend policy and optimal capital structure will involve a corner solution: either all debt financing or all equity financing. With SOC hypothesis, we have

$$\begin{aligned}
sgn\frac{\partial\rho_{t+1}}{\partial D_t} &= sgn\Big\{\frac{1}{m_{t+1}^2}\beta E_t[\frac{u_c(c_{t+1},x_{t+1},l_{t+1})u_{cx}(c_t,x_t,l_t)q_tz_{t+1}}{u_c(c_t,x_t,l_t)^2}] \\
&[1-\frac{u_x(c_t,x_t,l_t)}{u_c(c_t,x_t,l_t)}+\frac{D_t}{1+D_t}\frac{\tau_y-\tau_p}{1-\tau_y}(1-\frac{u_x(c_t,x_t,l_t)}{u_c(c_t,x_t,l_t)}-E_t[m_{t+1}])] \\
&+\frac{\tau_y}{1+D_t}E_t[m_{t+1}\Psi_{t+1}]+\frac{1}{m_{t+1}} \\
&[-\frac{u_c(c_t,x_t,l_t)u_{xx}(c_t,x_t,l_t)q_tz_{t+1}-u_x(c_t,x_t,l_t)u_{cx}(c_t,x_t,l_t)q_tz_{t+1}}{u_c^2(c_t,x_t,l_t)} \\
&+\frac{1}{(1+D_t)^2}\frac{\tau_y-\tau_p}{1-\tau_y}(1-\frac{u_x(c_t,x_t,l_t)}{u_c(c_t,x_t,l_t)}-E_t[m_{t+1}]) \\
&+\frac{D_t}{1+D_t}\frac{\tau_y-\tau_p}{1-\tau_y}(1-\frac{u_c(c_{t+1},x_{t+1},l_{t+1})u_{cx}(c_t,x_t,l_t)q_tz_{t+1}}{u_c(c_t,x_t,l_t)^2}-E_t[m_{t+1}]) \\
&+\frac{D_t}{1+D_t}\frac{\tau_y-\tau_p}{1-\tau_y}(1-\frac{u_x(c_t,x_t,l_t)}{u_c(c_t,x_t,l_t)} \\
&-\beta E_t[\frac{u_c(c_{t+1},x_{t+1},l_{t+1})u_{cx}(c_t,x_t,l_t)q_tz_{t+1}}{u_c(c_t,x_t,l_t)^2}])-\frac{\tau_y}{(1+D_t)^2} \\
&E_t[m_{t+1}\Psi_{t+1}]+\frac{\tau_y}{1+D_t}E_t[\frac{u_c(c_{t+1},x_{t+1},l_{t+1})u_{cx}(c_t,x_t,l_t)q_tz_{t+1}}{u_c(c_t,x_t,l_t)^2}\Psi_{t+1}]]\Big\},
\end{aligned}$$

$$sgn\frac{\partial\rho_{t+1}}{\partial\Psi_{t+1}} = sgn\{\tau_yE_t[m_{t+1}]\}.$$

Compare with Altug and Labadie (2008) (P276), SOC hypothesis makes the sign of $\frac{\partial\rho_{t+1}}{\partial D_t}$ much more difficult to be decided. In other words, it is very difficult to minimizes firm's cost of capital by choosing the optimal debt-equity ratio D_t.

5 An Example

In this section, we present a solved example. Let

$$u(c_t,x_t,l_t) = (1-\omega)\ln c_t + \omega\ln x_t - \eta_t\frac{l_t^{1+\psi}}{1+\psi},$$

where $0<\omega<1$ and $\eta_t>0,\ \ \psi\geq 0$. The production function is given by

$$f(k_t,l_t,\theta_t) = \theta_t k_t^{\alpha} l_t^{1-\alpha}$$

for $0<\alpha<1$ and the production shock θ_t.

We consider the following problem:

$$\max E_0\Big[\sum_{t=0}^{\infty}\beta^t u(c_t, k_{t+1}, l_t)\Big]$$

subject to

$$c_t + k_{t+1} \le f(k_t, l_t, \theta_t).$$

First-order conditions are

$$\frac{1-\omega}{c_t} = \frac{\omega}{k_{t+1}} + \alpha\beta E_t\Big\{\frac{\omega}{c_{t+1}}\big[\theta_{t+1}k_{t+1}^{\alpha-1}l_{t+1}^{1-\alpha}\big]\Big\} \tag{25}$$

and

$$\eta_t l_t^{\psi} = \frac{1-\omega}{c_t}\theta_t(1-\alpha)k_t^{\alpha}l_t^{-\alpha}. \tag{26}$$

Let $y_t = f(k_t, l_t, \theta_t) + (1-\delta)k_t$ and $c_t = \lambda y_t$. Then $k_{t+1} = (1-\lambda)y_t$. Following from (25),

$$\begin{aligned}
\frac{1-\omega}{\lambda y_t} &= \frac{\omega}{k_{t+1}} + \alpha\beta E_t\Big\{\frac{1-\omega}{\lambda y_{t+1}}\big[\theta_{t+1}k_{t+1}^{\alpha-1}l_{t+1}^{1-\alpha}\big]\Big\}\\
&= \frac{\omega}{k_{t+1}} + \alpha\beta E_t\Big\{\frac{(1-\omega)\big[\theta_{t+1}k_{t+1}^{\alpha-1}l_{t+1}^{1-\alpha}\big]}{\lambda\big[\theta_{t+1}k_{t+1}^{\alpha}l_{t+1}^{1-\alpha}\big]}\Big\}\\
&= \frac{\omega}{k_{t+1}} + \frac{\alpha\beta(1-\omega)}{\lambda k_{t+1}} = \frac{\lambda\omega + \alpha\beta(1-\omega)}{\lambda k_{t+1}}.
\end{aligned}$$

This proves that

$$k_{t+1} = \big[\frac{\omega}{1-\omega}\lambda + \alpha\beta\big]y_t = (1-\lambda)y_t.$$

So $\lambda = (1-\omega)(1-\alpha\beta)$, that is

$$c_t = (1-\omega)(1-\alpha\beta)y_t, \text{ and } k_{t+1} = \big((1-\alpha\beta)\omega + \alpha\beta\big)y_t.$$

By (26),

$$l_t = \left(\frac{(1-\omega)(1-\alpha)\theta_t k_t^{\alpha}}{\eta_t c_t}\right)^{\frac{1}{\phi+\alpha}}.$$

Then, by Lemma 3, we get a equilibrium.

6 Summary

In this paper, we have proved following results: (1) the existence of equilibrium asset price with SOC ; (2)the relation between equilibrium and optimal growth with SOC; (3)the M-M theorem holds with SOC; (4)The M-M theorem does not hold with taxes for SOC.

7 Appendix

Proof of Lemma 1. Suppose $\{\bar{c}_t\}_{t=0}^{\infty}$, $\{\bar{z}_{t+1}\}_{t=0}^{\infty}$, $\{\bar{b}_{t+1}\}_{t=0}^{\infty}$ are characterized by $(4)-(6)$, $(12)-(14)$ and let $\{c_t\}_{t=0}^{\infty}$, $\{z_{t+1}\}_{t=0}^{\infty}$, $\{b_{t+1}\}_{t=0}^{\infty}$ be any stochastic process satisfying the same initial conditions. Compute for each T an upper bound to the shortfall:

$$\begin{aligned}
& E_0\left\{\sum_{t=0}^{T}\beta^t\big(u(c_t,x_t,l_t)-u(\bar{c}_t,\bar{x}_t,\bar{l}_t)\big)\right\} \\
= \;& E_0\left\{\sum_{t=0}^{T}\beta^t\big[u_c(\bar{c}_t,\bar{x}_t,\bar{l}_t)(c_t-\bar{c}_t)+u_x(\bar{c}_t,\bar{x}_t,\bar{l}_t)(x_t-\bar{x}_t)+u_l(\bar{c}_t,\bar{x}_t,\bar{l}_t)(l_t-\bar{l}_t)\big]\right\} \\
= \;& E_0\left\{\sum_{t=0}^{T}\beta^t\big[u_c(\bar{c}_t,\bar{x}_t,\bar{l}_t)\big(w_tl_t+(q_t+d_t)z_t+(1+r_t)b_t-q_tz_{t+1}-b_{t+1}\right. \\
& -w_t\bar{l}_t-(q_t+d_t)\bar{z}_t-(1+r_t)\bar{b}_t+q_t\bar{z}_{t+1}+\bar{b}_{t+1}\big)+u_x(\bar{c}_t,\bar{x}_t,\bar{l}_t)(x_t-\bar{x}_t) \\
& \left.+u_l(\bar{c}_t,\bar{x}_t,\bar{l}_t)(l_t-\bar{l}_t)\big]\right\} \\
= \;& E_0\left\{\sum_{t=0}^{T}\beta^t\{-u_l\bar{c}_t,\bar{x}_t,\bar{l}_t)l_t+\beta^{-1}u_c(\bar{c}_{t-1},\bar{x}_{t-1},\bar{l}_{t-1})(q_{t-1}z_t+b_t)\right. \\
& -\beta^{-1}u_x(\bar{c}_{t-1},\bar{x}_{t-1},\bar{l}_{t-1})(q_{t-1}z_t+b_t)-u_c(\bar{c}_t,\bar{x}_t,\bar{l}_t)(q_tz_{t+1}+b_{t+1}) \\
& +u_l\bar{c}_t,\bar{x}_t,\bar{l}_t)\bar{l}_t-\beta^{-1}u_c(\bar{c}_{t-1},\bar{x}_{t-1},\bar{l}_{t-1})(q_{t-1}\bar{z}_t+\bar{b}_t) \\
& +\beta^{-1}u_x(\bar{c}_{t-1},\bar{x}_{t-1},\bar{l}_{t-1})(q_{t-1}\bar{z}_t+\bar{b}_t)+u_c(\bar{c}_t,\bar{x}_t,\bar{l}_t)(q_t\bar{z}_{t+1}+\bar{b}_{t+1}) \\
& \left.+u_x(\bar{c}_t,\bar{x}_t,\bar{l}_t)(x_t-\bar{x}_t)+u_l(\bar{c}_t,\bar{x}_t,\bar{l}_t)(l_t-\bar{l}_t)\big]\}\right\} \\
= \;& E_0\left\{\sum_{t=0}^{T}\{\beta^{t-1}u_c(\bar{c}_{t-1},\bar{x}_{t-1},\bar{l}_{t-1})\big(q_{t-1}z_t+b_t)-\beta^{t-1}u_x(\bar{c}_{t-1},\bar{x}_{t-1},\bar{l}_{t-1})\right. \\
& \big(q_{t-1}z_t+b_t)-\beta^tu_c(\bar{c}_t,\bar{x}_t,\bar{l}_t)(q_tz_{t+1}+b_{t+1})+\beta^tu_x(\bar{c}_t,\bar{x}_t,\bar{l}_t)(q_tz_{t+1}+b_{t+1}) \\
& -\beta^{t-1}u_c(\bar{c}_{t-1},\bar{x}_{t-1},\bar{l}_{t-1})\big(q_{t-1}\bar{z}_t+\bar{b}_t)+\beta^{t-1}u_x(\bar{c}_{t-1},\bar{x}_{t-1},\bar{l}_{t-1}) \\
& \left.\big(q_{t-1}\bar{z}_t+\bar{b}_t)+\beta^tu_c(\bar{c}_t,\bar{x}_t,\bar{l}_t)(q_t\bar{z}_{t+1}+\bar{b}_{t+1})-\beta^tu_x(\bar{c}_t,\bar{x}_t,\bar{l}_t)(q_t\bar{z}_{t+1}+\bar{b}_{t+1})\right\} \\
= \;& E_0\left\{\beta^Tu_c(\bar{c}_T,\bar{x}_T,\bar{l}_T)(q_{T-1}\bar{z}_T+\bar{b}_T)-\beta^Tu_x(\bar{c}_T,\bar{x}_T,\bar{l}_T)(q_{T-1}\bar{z}_T+\bar{b}_T)\right. \\
& \left.-\beta^Tu_c(\bar{c}_T,\bar{x}_T,\bar{l}_T)(q_{T-1}z_T+b_T)+\beta^Tu_x(\bar{c}_T,\bar{x}_T,\bar{l}_T)(q_{T-1}z_T+b_T)\right\} \\
\leq \;& E_0\left\{\beta^Tu_c(\bar{c}_T,\bar{x}_T,\bar{l}_T)(q_{T-1}\bar{z}_T+\bar{b}_T)-\beta^Tu_x(\bar{c}_T,\bar{x}_T,\bar{l}_T)(q_{T-1}\bar{z}_T+\bar{b}_T)\right\}\to 0
\end{aligned}$$

as $T \to \infty$.

Here equations (4) – (6) were used to telescope out the middle terms in the series and get

$$\beta^T u_c(\bar{c}_T, \bar{x}_T, \bar{l}_T)(q_{T-1} z_T + b_T) \geq \beta^T u_x(\bar{c}_T, \bar{x}_T, \bar{l}_T)(q_{T-1} z_T + b_T),$$

the last equation follows from (12) – (14).

Now let $\{\bar{c}_t\}_{t=0}^{\infty}$, $\{\bar{z}_{t+1}\}_{t=0}^{\infty}$, $\{\bar{b}_{t+1}\}_{t=0}^{\infty}$,$\{\bar{l}_t\}_{t=0}^{\infty}$ be optimal given $\{w_t, q_t, d_t, r_t\}$. Since $u_c(0, x_t, l_t) = +\infty$ implies that $\bar{(c)}_t > 0$ and W is differentiable at $\bar{y}_t$, by concavity of W, and $u \geq 0$,

$$\begin{aligned}
W(y_t, t) &\geq W(y_t, t) - W(y_t/2, t) \geq W'(y_t, t) y_t/2 \\
&= \beta^t \big[u_c(c_t, x_t, l_t) + u_x(c_t, x_t, l_t) + u_l(c_t, x_t, l_t) \frac{1}{w_t}\big] y_t/2.
\end{aligned}$$

Therefore,

$$E_0[W(y_t, t)] \to 0, \;\; t \to \infty$$

implies

$$E_0 \big\{\beta^t \big[u_c(c_t, x_t, l_t) + u_x(c_t, x_t, l_t) + u_l(c_t, x_t, l_t) \frac{1}{w_t}\big]\big\} y_t \to 0, \;\; t \to \infty.$$

But $y_t = w_t l_t + q_t z_t + d_t z_t + r(1 + r_t) b_t$, so that

$$\begin{aligned}
&E_0 \big\{\beta^t \big[u_c(c_t, x_t, l_t) + u_x(c_t, x_t, l_t) + u_l(c_t, x_t, l_t) \frac{1}{w_t}\big]\big\} \\
&(w_t l_t + q_t z_t + d_t z_t + (1 + r_t) b_t)\big\} \\
\geq\; &E_0 \big\{\beta^{t-1} [u_c(c_{t-1}, x_{t-1}, l_{t-1}) q_{t-1} z_t\big\} + E_0 \big\{\beta^{t-1} [u_c(c_{t-1}, x_{t-1}, l_{t-1}) b_t\big\} \\
&\quad + E_0 \big\{\beta^t u_x(c_t, x_t, l_t) x_t\big\}.
\end{aligned}$$

Hence

$$\lim_{t \to \infty} E_0 \{\beta^t U_c(c_t, x_t, l_t) q_t z_{t+1}\} = 0,$$
$$\lim_{t \to \infty} E_0 \{\beta^t U_c(c_t, x_t, l_t) b_{t+1}\} = 0.$$
$$\lim_{t \to \infty} E_0 \{\beta^t U_c(c_t, x_t, l_t) x_t\} = 0.$$

This gives the proof of Lemma 1. □

Proof of Lemma 3. (i)It is obvious that X satisfies the first-order necessary conditions for an equilibrium by its very definition. What is at issue is (12) – (14). Put

$$V(k_t, t) = \max E_0 \Big\{ \sum_{s=t+1}^{\infty} \beta^s u(c_s, k_{s+1}, l_s) \Big\}$$

subject to $c_s + k_{s+1} = f(k_s, l_s, \theta_s) + (1-\delta)k_s$. Then

$$\begin{aligned} V(k_t,t) &\geq V(k_t,t) - V(k_t/2,t) \geq V'(k_t,t)k_t/2 \\ &\geq E_0\big\{\beta^{t+1}u_c(c_{t+1},k_{t+2},l_{t+1})[f_k(k_{t+1},l_{t+1},\theta_{t+1}) + (1-\delta)]k_t/2\big\}k_t/2\big\} \geq 0. \end{aligned}$$

Since $\lim_{t\to\infty} V(k_t,t) = 0$,

$$\begin{aligned} & E_0\big\{\beta^t u_c(c_t,k_{t+1},l_t)k_{t-1}\big\} = E_0\big\{\beta^t u_c(c_t,k_{t+1},l_t)x_t\big\} \\ = \; & E_0\big\{\beta^t u_c(c_t,k_{t+1},l_t)(q_t z_{t+1} + b_{t+1})\big\} \to 0. \end{aligned}$$

Hence (12) $-$ (14) hold. This establishes the implication (i) implies (ii).

In showing (ii) implies (i) it is clear that the first-order necessary conditions for the quantity side of an equilibrium boil down to the first-order conditions for the optimal growth problem. Similarly as the proof of Lemma 1, we can prove X is a solution of the optimal growth problem. This ends the proof of Lemma 3. □

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