# Endogenous Timing in a Mixed Duopoly

# with Endogenous Vertical Differentiation

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**Abstract**

We consider a game of endogenous timing with observable delay in a mixed duopoly with endogenous vertical differentiation in the context of sequential quality and price choice. We find that a simultaneous play in the first opportunity at each stage turns out to be the unique subgame perfect Nash equilibrium, which contrasts with the endogenous timing in a purely private duopoly.

**Keywords:** Endogenous timing, public firm, private firm, vertical differentiation

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## 1. Introduction

Mixed oligopolies exist in many developing countries, especially in (fore-) communism countries. There are also industries in most western economies which can be seen as mixed oligopolies. In some industries, differentiated products are provided. However, most of the literature on mixed oligopolies has focused on the case of homogeneous goods. Cremer et al. (1991), Delbono et al. (1996), Ba′rcena-Ruiz (2007), Ishibashi and Kaneko (2008), among others, are some exceptions. Cremer et al. (1991) investigated under what conditions the social planner can obtain the first best in a horizontally differentiated mixed oligopoly setting. Delbono et al. (1996) analysed the endogenous quality choice in mixed duopoly using a vertical differentiation model and found that the public firm will choose to provide a product with higher quality than the private firm if it acts as a leader in the quality stage. On the contrary, using a Hotelling model of mixed duopoly with a quality dimension of product differentiation, Ishibashi and Kaneko (2008) showed that the public firm should provide lower quality to maximise social welfare. Using a model of exogenous product differentiation, Ba′rcena-Ruiz (2007) analysed endogenous timing in a Bertrand mixed duopoly with exogenous product differentiation and found that firms choose prices simultaneously.

In this paper, we will investigate the issue of endogenous timing in a mixed duopoly with endogenous product differentiation. We are interested in this issue for two reasons. First, in Delbono et al. (1996), the timing is exogenous. The authors first obtained two subgame perfect equilibria assuming firms act simultaneously in both quality stage and price stage and then showed that the public firm chooses a higher quality if it moves first in the quality stage and both firms move simultaneously in the price stage. What should the timing be when it is endogenized? Second, the issue of endogenous timing in a purely private duopoly with endogenous product differentiation has been examined by Lambertini (1996). He found that, in the non full market coverage situation, firms simultaneous playing in both the quality stage and the price stage is the unique subgame perfect equilibrium in pure strategies.[1] Note that Lambertini (1996) introduced only one pre-play stage. In our opinion, doing so is inappropriate since it cannot ensure subgame perfection. To ensure subgame perfection, two pre-play stages should be introduced. Using the same method as in this paper, we find that the subgame perfect equilibrium requires simultaneity in the quality stage and sequentiality in the price stage.[2] What we are interested in is whether firms' mixed objectives affect the distribution of roles and how if so.

For our purpose, a two-stage game of endogenous timing with observable delay (a la Hamilton and Slutsky (1990)) is considered here in the context of sequential quality and price choice. The sequential choice of quality and price is adopted not only because this is standard in the literature such as Shaked and Sutton (1982), Cremer et al. (1991), Delbono et al. (1996), Lambertini (1996), etc. but also because by nature quality is a long-run variable while price is a short-run variable.[3] It is well known that firms can strategically choose product characteristics in order to soften price competition. Once product quality is chosen (determined by technology), it is fixed in quite a long time, while price is much easier to change. After introducing two pre-play stages, we have a four-stage game. In stage one (the first pre-play stage), firms simultaneously announce in which period they will choose their qualities and are committed to this announcement. In stage two, firms choose their qualities knowing when the other firm chooses its quality level. In the third stage (the second pre-play stage), after observing each firm's quality

[1] Lambertini (1996) distinguished two situations, full and non full market coverage. We consider only the latter situation since it is more realistic. With the given utility function, the consumers with low $\theta$ do not purchase any good.

[2] The work is available upon request from the authors. We also proved that introducing only one pre-play stage does not ensure subgame perfection.

[3] Similarly, in models with horizontal product differentiation, firms choose locations first and then compete in prices or quantities.

level, firms simultaneously announce with commitment in which period they will choose their prices. Finally in stage four, firms choose their prices knowing when the other firm chooses its price level. We find that in both the quality stage and the price stage, moving in the first opportunity is a strictly dominant strategy and thus the unique subgame perfect equilibrium of the game with observable delay is characterized by simultaneous play in the first opportunity in both stages.

This result is different from the one in a purely private duopoly.[4] The intuition is as follows. In the price stage, the public firm wants to move in the first opportunity to prevent the private firm from charging a high price as a leader since the public firm cares about social welfare; the private firm also wants to move in the first opportunity since otherwise it would have to set a low price as a response to the public firm's low price. On the contrary, in a purely private duopoly, firms want to move sequentially since given one firm sets price in the first opportunity, the other firm is better off by choosing a high price in the second opportunity (since prices are strategic complements and the first-mover chooses a high price). In the quality stage, regardless of a mixed duopoly setting or a purely private duopoly setting, each firm prefers moving in the first opportunity to avoid being a lower-quality producer.

The rest of the paper is organized as follows. In the next section, the model is described. In Section 3, we analyse the model and derive the subgame perfect equilibrium. Section 4 concludes with some remarks.

## 2. The Model

[4] This result is actually the same as what Lambertini (1996) found in a purely private duopoly. However, as we have emphasized, Lambertini introduced only one pre-play stage, while we introduce two to ensure subgame perfection. Ba′rcena-Ruiz (2007) also shows that in a private duopoly the endogenous timing requires firms to choose prices sequentially, though the author adopts a different model of product differentiation.

The setup of the model is similar to the one in Delbono et al. (1996) except that now our focous is on endogenous timing. A vertically differentiated mixed duopoly market is considered with one public firm, called firm A, and one private firm, called firm B. The public firm's objective is the maximisation of the social surplus (welfare) which is defined as the sum of consumer surplus and both firms' profits, while the private firm is a profit-maximiser. Production costs are $C(q,x)=q^2x$, where $x$ denotes quantity and $q$ denotes quality.[5]

On the demand side of the industry, there is a continuum of consumers whose total number is normalized to one. The good is indivisible and each consumer buys at most one unit. The utility of a consumer labeled $\theta$ is given by

$$U=\begin{cases}\theta q_i - p_i & \text{if the consumer buys one unit good with quality } q_i \text{ at price } p_i\\ 0 & \text{otherwise.}\end{cases}$$

Consumers are heterogeneous and $\theta$ is uniformly distributed over the interval $[0,1]$. The parameter $\theta$ can be interpreted as the marginal willingness to pay for quality.

By finding the marginal consumers, one can easily obtain firms' demand function: $x_H = 1-(p_H - p_L)/(q_H - q_L)$, and $x_L = (p_H - p_L)/(q_H - q_L) - p_L/q_L$, where the subscripts $H$ and $L$ denote the firm producing the higher quality product and the one producing the lower quality product respectively.

The public firm aims at maximising social welfare and the objective function is

[5] We use the specific form of cost function for technical tractability. This function is also used in Delbono et al. (1996). One could use general cost function $C(q,x)=q^{\alpha}x$, where $\alpha>1$. However, introducing parameter $\alpha$ makes the analysis very messy. Note that the cost function $C(q,x)=qx$ cannot be used in this model, since an infinitesimal increment in the quality level $dq$ chosen by a firm is valued $\theta dq$ by the consumer labeled $\theta$. Hence it is valued at most $dq$. On the other hand, the increase in marginal cost is $dq$. Hence no firm would find it profitable to produce any product with $q>0$. A similar argument also suggests that no firm would want to choose $q$ above 1/2 (the increase in marginal cost $2qdq$ must be less than $dq$).

$$W = \int_{p_L/q_L}^{(p_H - p_L)/(q_H - q_L)} \theta q_L d\theta + \int_{(p_H - p_L)/(q_H - q_L)}^{1} \theta q_H d\theta \\ - q_L^2 \left( \left( p_H - p_L \right) / \left( q_H - q_L \right) - p_L / q_L \right) - q_H^2 \left( 1 - \left( p_H - p_L \right) / \left( q_H - q_L \right) \right), \quad (1)$$

and the private firm's objective function is

$$\pi_i = \left( p_i - q_i^2 \right) x_i = \begin{cases} \left( p_H - q_H^2 \right) \left( 1 - \left( p_H - p_L \right) / \left( q_H - q_L \right) \right) & \text{if } i = H \\ \left( p_L - q_L^2 \right) \left( \left( p_H - p_L \right) / \left( q_H - q_L \right) - p_L / q_L \right) & \text{if } i = L \end{cases}. \quad (2)$$

We consider a four-stage game as described in the Introduction. Firms choose qualities and prices sequentially and there is one pre-play stage for the quality stage and one for the price stage. Our objective is to solve for the subgame perfect Nash equilibrium (or equilibria) of this extended game using backward induction.

## 3. Analysis

### 3.1 Endogenzing the Timing in the Price Stage

Using backward induction, we obtain each firm's payoff in fixed-timing games, namely, the simultaneous-move, public-leader, and public-follower games. After that, we can then obtain the endogenous timing in the price stage. Since we do not know which firm produces higher-quality product, we need to distinguish two cases, namely, the case of the public firm providing lower-quality product and the case of the public firm providing higher-quality product.[6] The detailed analysis is put in the Appendix.

The endogenous timing in the price stage is summarized in the following proposition.

[6] Needless to say, which firm produces higher-quality product is endogenously determined. It will be addressed later.

**Proposition 1:** *Both the public firm and the private firm setting prices in the first opportunity is the unique strictly dominant strategy equilibrium in the price stage no matter which firm provides higher quality product.*

**Proof**: Let $a$ and $b$ denote the qualities of the products provided by the public firm and the private firm respectively. Let the superscripts "S", "F" and "L" denote "simultaneous-mover", "follower" and "leader" respectively.

In the case of the public firm providing lower-quality product, $a < b$. In the appendix, we present each firm's payoff in fixed-timing games. Clearly, we have $W^L > W^S$ and $\pi_b^L > \pi_b^S$ since a leader can always choose its simultaneous-move equilibrium price while the leader actually chooses a different price. These inequalities mean that a firm wants to act as a leader if the other firm moves in the second period. We also find $W^S > W^F$ and $\pi_b^S > \pi_b^F$. These inequalities mean that a firm wants to act as a simultaneous-mover if the other firm moves in the first period.

In the case of the public firm providing higher-quality product, $a > b$. In the appendix, we present each firm's payoff in fixed-timing games. Again, we have $W^L > W^S$, $\pi_b^L > \pi_b^S$, $W^S > W^F$, and $\pi_b^S > \pi_b^F$.

Q.E.D

This result is the same as the one in Ba′rcena-Ruiz (2007) who also investigated endogenous timing in a differentiated mixed duopoly when firms compete in prices but the product differentiation in his model is exogenous and the demand function is derived from a representative consumer's utility maximisation problem. On the contrary, when firms compete in quantities in a homogeneous product market, the public firm and the private firm(s) producing simultaneously cannot be sustained as a subgame perfect equilibrium outcome, as demonstrated by Pal (1998) and Lu (2006), among others.

Intuitively, the public firm does not want to be a follower since if so the private firm, as a leader, would choose a high price level, which would lower the social welfare level. So the public firm will choose price level in the first opportunity. The private firm wants to do the same thing since being a follower means that it has to set a low price as a response to the public firm's low price, yielding low profit level.

### 3.2 Endogenzing the Timing in the Quality Stage

Having found the unique Nash equilibrium in the price stage, we can now investigate endogenous timing in the quality stage. Again, we will first obtain each firm's payoff in fixed-timing games.

Since the equilibrium requires simultaneity in the price stage, the same as assumed in Delbono et al. (1996), we can use the results obtained by Delbono et al. (1996) directly. Delbono et al. (1996) investigated quality choice in a vertically differentiated mixed duopoly assuming firms set prices simultaneously. Assuming firms choose qualities simultaneously, they found two subgame perfect Nash equilibria in pure strategies, one involving the public firm providing lower quality product and one involving the public firm providing higher quality product. In the former equilibrium, $a = 0.259$, $b = 0.380$, and the associated payoffs are $W^S = 0.07755$ and $\pi_b^S = 0.00907$. In the latter equilibrium, $a = 0.390$, $b = 0.260$, and the associated payoffs are $W^S = 0.07792$ and $\pi_b^S = 0.00741$. They also found the Stackelberg equilibrium with the public firm as a leader. In this equilibrium, $a = 0.374$, $b = 0.249$, and the associated payoffs are

$W^L = 0.07801$ and $\pi_b^F = 0.00652$.[7] They did not consider the Stackelberg equilibrium with the public firm as a follower to which we now turn.

When $b > a$, it can be shown that the private firm's maximum profit is attained at $b = 0.350$ (which implies that the best reply of the public firm is $a = 0.253$). The maximum profit level is $\pi_b = 0.00934$ and the associated welfare level is $W = 0.07693$. When $b < a$, it can be shown that the private firm's maximum profit is attained at $b = 0.287$ (which implies that the best reply of the public firm is $a = 0.395$). The maximum profit level is $\pi_b = 0.00756$ and the associated welfare level is $W = 0.07738$. So in the Stackelberg equilibrium with the public firm as a follower, $a = 0.253$ and $b = 0.350$, and the associated payoffs are $W^F = 0.07693$ and $\pi_b^L = 0.00934$.

The endogenous timing in the quality stage can be summarized in the following proposition.

**Proposition 2:** *Both the public firm and the private firm choosing qualities in the first opportunity is the unique strictly dominant strategy equilibrium in the quality stage.*

**Proof***:* Clearly, regardless of which equilibrium arises in the simultaneous-move game, that is, regardless of $\left(W^S, \pi_b^S\right) = \left(0.07755, 0.00907\right)$ or $\left(W^S, \pi_b^S\right) = \left(0.07792, 0.00741\right)$, we have $\pi_b^L > \pi_b^S > \pi_b^F$ (0.00934>max{0.00907, 0.00741}>0.00652) and $W^L > W^S > W^F$ (0.07801>max{0.07755, 0.07792}>0.07693). So choosing quality level in the first opportunity is each firm's strictly dominant strategy.

Q.E.D.

[7] The authors did not compute the private firm's profit. It is straightforward to obtain it.

Intuitively, the public firm does not want to be a follower since if so the private firm would choose a low quality level (but still high enough so that the public firm will choose an even lower quality level), which would lower the social welfare level. So the public firm will choose quality level in the first opportunity. The private firm wants to do the same thing since being a follower means being a lower-quality provider (since the public firm chooses higher quality), yielding low profit level.

It should be noted that Lambertini (1999) showed that in a private duopoly firms also choose qualities simultaneously in the first opportunity given firms' simultaneous price competition. It turns out producing higher quality is a strictly dominant strategy for any firm regardless of the firm's ownership.

### 3.3 Subgame Perfect Equilibrium

Combining propositions 1 and 2, we summarize the main result in the form of a theorem.

**Theorem:** *In a quality-then-price game of endogenous timing with observable delay in a mixed duopoly with endogenous vertical differentiation, the unique subgame perfect equilibrium in pure strategies is characterized by simultaneous play in the first opportunity in both the quality stage and the price stage.*

Compared with endogenous timing in the same setting in a purely private duopoly, the timing in the price stage is different while the timing in the quality stage is the same. We have explained why it is so in the Introduction section. Because a public firm cares about social welfare, it prefers moving in the first opportunity in the price stage and the private firm has the same preference knowing that it would have to choose low price if moving in the second opportunity. However, this is not the case in the purely private duopoly. In the purely private duopoly, each firm prefers followership to simultaneous play in the price stage, sequentiality is thus required. In the quality stage, each firm prefers moving in the first opportunity to avoid being a lower-quality producer.

## 4. Concluding Remarks

In this paper, we consider endogenous order of moves in the observable delay game of Hamilton and Slutsky (1990) in the context of a "two-stage" game of quality and price setting in a mixed duopoly. We find that the unique subgame perfect equilibrium in pure strategies is characterized by simultaneous play in the first opportunity in both stages. The result is different from the one in a purely private duopoly which is characterized by simultaneous play in the quality stage and sequential play in the price stage.

The subgame perfect equilibrium obtained in our analysis is the consequence of the interaction between the public firm and the private firm. Since the public firm aims at maximising social welfare, it wants to move in the first opportunity in both the quality stage and the price stage so that the overall product quality is high and the price is low. Anticipating the public firm's behavior, the private firm does not want to be a follower either.

## Appendix

In this appendix, we show how to derive each firm's payoff in fixed-timing games when firms compete in prices, namely, the simultaneous-move, public-leader, and private-leader games. Let $a$ and $b$ denote the qualities of the products provided by the public firm and the private firm respectively. Let the superscripts "S", "F" and "L" denote "simultaneous-mover", "follower" and "leader" respectively.

### A.1 The case of the public firm providing lower-quality product

In this case, $a < b$.

In this case, (1) becomes

$$W = \int_{p_a/a}^{(p_b - p_a)/(b-a)} \theta a d\theta + \int_{(p_b - p_a)/(b-a)}^{1} \theta b d\theta \\ - a^2\left(\left(p_b - p_a\right)/\left(b-a\right) - p_a/a\right) - b^2\left(1 - \left(p_b - p_a\right)/\left(b-a\right)\right), \tag{3}$$

and (2) becomes

$$\pi_b = \left(p_b - b^2\right)\left(1 - \left(p_b - p_a\right)/\left(b-a\right)\right). \tag{4}$$

In the simultaneous-move game, we first obtain each firm's reaction function as the following:

$$p_a = a\left(p_b - b^2 + ab\right)/b, \tag{5}$$

$$p_b = \left(p_a + b - a + b^2\right)/2. \tag{6}$$

Once the reaction functions are obtained, one can then solve for the equilibrium prices in the three fixed-timing games. Once the equilibrium prices are derived, it is then straightforward to get each firm's payoff in each fixed-timing game which is presented below.

1) Simultaneous move

$$p_a^S = a\left(b - a + 2ab - b^2\right)/\left(2b - a\right), \quad p_b^S = b\left(b - a + a^2 + b^2 - ab\right)/\left(2b - a\right),$$

$$\pi_b^S = b^2\left(b-a\right)\left(1 - a - b\right)^2/\left(2b-a\right)^2,$$

$$W^S = \frac{b}{2\left(2b-a\right)^2}\left(3b^4 + 5a^3b - 5a^2b^2 - 2a^4 + 2a^3 - 4a^2b + 6ab^2 - 6b^3 + 3b^2 - 2ab\right).$$

2) Public leader

$$p_a^L = a\left(b-a+4ab-2a^2-b^2\right)\Big/\left(4b-3a\right),$$

$$p_b^F = \left(a^2+2b^2-3ab+2b^3-a^3+2a^2b-2ab^2\right)\Big/\left(4b-3a\right),$$

$$\pi_b^F = \left(2b-a\right)^2\left(b-a\right)\left(1-a-b\right)^2\Big/\left(4b-3a\right)^2,$$

$$W^L = \frac{1}{2\left(4b-3a\right)}\left(3b^4+ab^3+3a^3b-5a^2b^2-a^4+2a^3-2a^2b+4ab^2-6b^3+3b^2-ab-a^2\right).$$

3) Public follower

$$p_a^F = a\left(1+a-b\right)/2,\quad p_b^L = b\left(1-a+b\right)/2,\ \pi_b^L = b\left(1-a-b\right)^2\Big/4,$$

$$W^F = \frac{b}{8}\left(3b^2-5a^2+2ab+2a-6b+3\right).$$

Clearly, we have $W^L > W^S$ and $\pi_b^L > \pi_b^S$ since a leader can always choose its simultaneous-move equilibrium price while the leader actually chooses a different price. We can also find

$$W^S - W^F = ab\left(4b-3a\right)\left(1-a-b\right)^2\Big/8\left(2b-a\right)^2 > 0,$$

$$\pi_b^S - \pi_b^F = a\left(b-a\right)^2\left(1-a-b\right)^2\left(a^2+8b^2-7ab\right)\Big/\left(2b-a\right)^2\left(4b-3a\right)^2 > 0.$$

### A.2 The case of the public firm providing higher-quality product

In this case, $a > b$. Following the same procedure as in Subsection A.1, we get each firm's payoff in each fixed-timing game.

1) Simultaneous move

$$p_a^S = a\left(2a^2-b^2\right)\Big/\left(2a-b\right),\quad p_b^S = b\left(a^2+ab-b^2\right)\Big/\left(2a-b\right),$$

$$\pi_b^S = a^3 b\left(a-b\right)\Big/\left(2a-b\right)^2 \ ,$$

$$W^S = \frac{a}{2\left(2a-b\right)^2}\left(4a^4 - b^4 - a^3 b - 5a^2 b^2 + 4ab^3 - 8a^3 + 8a^2 b - 2ab^2 + \left(2a-b\right)^2\right).$$

2) Public leader

$$p_a^L = \left(4a^3 - 2a^2 b - ab^2\right)\Big/\left(4a-3b\right), \quad p_b^F = b\left(ab + 2a^2 - 2b^2\right)\Big/\left(4a-3b\right),$$

$$\pi_b^F = ab\left(a-b\right)\left(2a-b\right)^2\Big/\left(4a-3b\right)^2 \ ,$$

$$W^L = \frac{a}{2\left(4a-3b\right)}\left(4a^3 + 2b^3 - 5ab^2 + 6ab - 8a^2 + 4a - 3b\right).$$

3) Public follower

$$p_a^F = a\left(2a+b\right)/2 \ , \quad p_b^L = b\left(a+2b\right)/2 \ , \quad \pi_b^L = a^2\, b/4 \ ,$$

$$W^F = \frac{a}{8}\left(4a^2 - 4b^2 + 3ab - 8a + 4\right).$$

Clearly, we have $W^L > W^S$ and $\pi_b^L > \pi_b^S$ . We can also find

$$W^S - W^F = a^2 b^2\left(4a-3b\right)\Big/8\left(2a-b\right)^2 > 0 \ ,$$

$$\pi_b^S - \pi_b^F = ab^2\left(a-b\right)^2\left(8a^2 + b^2 - 7ab\right)\Big/\left(2a-b\right)^2\left(4a-3b\right)^2 > 0 \ .$$

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