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Social Security and Early Retirement in an Overlapping-Generations Growth Model*

Philippe Michel

IUP and GREQAM, Marseille, France

and

Pierre Pestieau

CREPP, Liège, Belgium; CORE, Louvain-la-Neuve, Belgium and DELTA Paris, France
E-mail: p.pestieau@ulg.ac.be

This paper explains why workers retire earlier, and earlier at the same time as society becomes more and more indebted through increasing pay-as-you-go pension liabilities. To do so, we extend the standard two-overlapping-generations growth model to allow for endogenous labor participation in the later period of life. We show that the rate of participation declines as the size of social security system increases. We also show that mandatory early retirement many be socially desirable in case of underaccumulation.

1. INTRODUCTION

Over the last decades the effective retirement age has dramatically decreased in Europe. By the mid-1990 the labor force participation of men 60 to 64 had fallen to below 20 percent in Belgium, Italy, France and the Netherlands, whereas in the early 1960s it has been above 70%. At the same time the level of intergenerational transfers, mainly pension liabilities, was rapidly increasing. It is tempting to relate these two trends.

The purpose of this paper is to do so by presenting an overlapping generations growth model with endogenous retirement age and pay-as-you-go

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pensions. It will appear that pension rights have a negative effect on the labor participation of the elderly workers. But before proceeding, let us look at the evidence of declining labor force participation of older persons at times when individuals live longer and the pressure on the financial viability of the social security systems is enormous.

What are the factors explaining the striking decline in labor force participation of the elderly workers ? In addition to causes linked to changes in preferences and to growth-related income effects, the standard explanation for this decline is that the social security provisions themselves provide an enormous incentive to leave the labor force early. This idea was forcefully developed more than two decades ago by Feldstein (1974) and Boskin (1977), who focused on two key parameters of social security systems : the income guarantee and the implicit tax on earnings. For example, Boskin shows that in the United States a decrease in the implicit tax rate on earnings from one-half to one-third reduces the annual probability of retirement by fifty percent. This idea was recently further developed by Gruber and Wise (1998) in their international study of social security and retirement. For 11 OECD countries they calculate the implicit tax imposed on a person over 54 who decides to work one more year, in other words the change in the present value of future benefts. They then relate this implicit tax to the so-called unused labor force capacity between ages 55 and 65. The result is a strong correspondence between the tax on retirement postponement and the unused capacity of the labor force.

Blondal and Scarpetta (1998) pursue the same objective as Gruber and Wise with the use of a pooled cross-country time series regression on 15 OECD countries over the period 1961-95. They also show that the variation in participation rates can be explained by various features of old age pensions systems, as well as by unemployment insurance, early retirement schemes and labor market conditions.

These two studies, like many others, focus on the implicit marginal tax that social security provisions impose on working one more year. In other words, their argument focuses on the substitution effect of social security provisions in a partial equilibrium and static setting.

This paper by contrast uses a general equilibrium approach and a dynamic framework. To do so, we rely on the overlapping-generations growth model of Samuelson (1958) and Diamond (1965). It is a model that has been widely used to study various aspects of social security (see, e.g., Burbidge, 1983). Paradoxically, with the sole exception of Hu (1979), it has not been used to study the interaction between social security and retirement, the reason for the omission being in the analytical difficulty of studying the dynamics of overlapping-generations models with endogenous labor supply. Because of this difficulty we use a particular specification, namely, a lognormal utility function and a Cobb-Douglas production function.

Our paper, in its main conclusions, reaches the same finding as those based on partial equilibrium : that is, that by its size and its structure the social security system exacerbates the very financial problems that it purports to solve.

An objection to be raised to this kind of work is that in most European countries early retirement is most often compulsory. For example, in Belgium wage earners can normally retire between the ages of 60 and 65. When they retire earlier, whatever route they take (early retirement scheme, disability insurance, unemployment compensations,...), they do not have much choice about changing their option and thus there is no link between retirement decisions and benefits received. But this is not entirely true, as the benefits offered within these alternative schemes are designed in such a way as to collectively attract support. Somehow, individual rationality is replaced by group rationality. This distinction is similar to that used in macroeconomics between individually involuntary unemployment and collectively voluntary unemployment.

The rest of this paper is organized as follows. Section 2 presents the basic model but without social security, while section 3 derives the results when social security is introduced; they pertain to the market solution, the first-and the second-best optimum. In the concluding section, we discuss the main findings of the paper.

2. THE BASIC MODEL WITHOUT SOCIAL SECURITY

2.1. The household sector

The total population of the economy grows at a constant rate n. It consists of two generations, the young and the old, of size $N_t = (1+n)N_{t-1}$ and N_{t-1} respectively. At time t, each person belonging to generation t lives for two periods and is capable of providing one unit of labor per period. In the first period he works full time earning a wage income of w_t which is devoted to either first period consumption c_t or to saving s_t:

$$w_t = c_t + s_t$$

During the second period, he works a fraction of the time equal to z_{t+1} and then retires. Consumption in the second period, d_{t+1}, therefore consists of a wage income and the proceeds from saving $R_{t+1}s_t$ where $R_{t+1} \equiv 1 + r_{t+1}$ and r_{t+1} is the rate of interest. Thus,

$$d_{t+1} = R_{t+1}s_t + w_{t+1}z_{t+1}.$$

The lifetime utility of an individual of generation t depends upon consumption in both periods and the length of retirement in the second period.

Using a loglinear form, we write such a utility as:

$$u_t = u_t(c_t, d_{t+1}, 1 - z_{t+1}) = \log c_t + \beta(\log d_{t+1} + \gamma \log(1 - z_{t+1})) \qquad (1)$$

where β is a factor of time preference, γ is the parameter measuring preference for leisure or retirement, and $1 \geq z_{t+1} \geq 0$[1]. Substituting the above constraints, we have:

$$\begin{aligned} u_t &= \log(w_t - s_t) + \beta[\log(R_{t+1}s_t + w_{t+1}z_{t+1}) \\ &\quad + \gamma \log(1 - z_{t+1})]. \end{aligned}$$

We denote by $s(w_t, w_{t+1}, R_{t+1})$ and $z(w_t, w_{t+1}, R_{t+1})$ the optimal choices of saving and labor supply. These functions are characterized by the first order conditions (FOC) for a maximum of u_t. Thus we get :

$$\frac{\partial u_t}{\partial s_t} = -\frac{1}{w_t - s_t} + \frac{\beta R_{t+1}}{R_{t+1}s_t + w_{t+1}z_{t+1}} = 0 \qquad (2)$$

and

$$\frac{\partial u_t}{\partial z_{t+1}} = \frac{\beta w_{t+1}}{R_{t+1}s_t + w_{t+1}z_{t+1}} - \frac{\beta\gamma}{1 - z_{t+1}} \leq 0 (= 0) \text{ for } z_{t+1} = 0 (> 0). \qquad (3)$$

Two cases are distinguished. First, when $z_{t+1} = 0$, saving is equal to :

$$s_t = \frac{\beta}{1 + \beta} w_t. \qquad (4)$$

On the contrary, when $z_{t+1} > 0$, the solutions for both saving and labor are:

$$s_t = s(w_t, w_{t+1}, R_{t+1}) = \frac{\beta(1 + \gamma)w_t - w_{t+1}/R_{t+1}}{1 + \beta + \beta\gamma}. \qquad (5)$$

and

$$z_{t+1} = z(w_t, w_{t+1}, R_{t+1}) = \frac{1 + \beta - \beta\gamma w_t R_{t+1}/w_{t+1}}{1 + \beta + \beta\gamma}. \qquad (6)$$

2.2. The production sector

Consider a competitive economy in which the production function is given by $Y_t = F(K_t, L_t)$ where Y_t is the level of output, K_t is capital input

[1]We define γ as meaning preference for leisure or retirement. Strickly speaking $\frac{\partial u_t}{\partial \gamma} \leq 0$ as $\log(1 - z_{t+1}) \leq 0$. To avoid this outcome, we would have to add a large number to $\log(1 - z_{t+1})$ so that $\frac{\partial u_t}{\partial \gamma} > 0$.

and L_t is aggregate labor supply consisting of N_t young workers supplying one unit and N_{t-1} old workers, each supplying z_t. That is:

$$L_t = N_t + N_{t-1}z_t.$$

Assuming constant returns to scale, maximization of profit $F(K_t, L_t) - w_tL_t - R_tK_t$ leads to the standard equality between wage rate and marginal productivity of labor, and between the rate of interest and the marginal productivity of capital (with total depreciation assumed).

$$\begin{aligned} w_t &= F'_L(k_t, 1) \\ R_t &= F'_K(k_t, 1) \end{aligned}$$

where $k_t = K_t/L_t$.

We here use a Cobb-Douglas, $Y = AK^{\alpha}L^{1-\alpha}$, so that:

$$w_t = (1-\alpha)Ak_t^{\alpha},$$

and

$$R_t = \alpha A k_t^{\alpha-1}.$$

2.3. Market equilibrium and dynamics

In period t, one has an equilibrium consisting of three sets of relations.

- equality between demand and supply of factors:

$$L_t = N_{t-1}(1 + n + z_t) \text{ and } K_t = N_{t-1}s_{t-1};$$

- factor prices:

$$w_t = F'_L(k_t, 1) \equiv \omega(k_t) \text{ and } R_t = F'_K(k_t, 1) \equiv \rho(k_t);$$

- consumers' choices:

$$\begin{aligned} c_t &= w_t - s_t, \qquad s_t = s(w_t, w_{t+1}, R_{t+1}), \\ z_{t+1} &= z(w_t, w_{t+1}, R_{t+1}), \text{ and } d_{t+1} = R_{t+1}s_t + w_{t+1}z_{t+1}. \end{aligned}$$

We now write the equation for capital accumulation with the Cobb-Douglas functions for the case $z_{t+1} = 0$:

$$(1+n)k_{t+1} = \frac{\beta}{1+\beta}(1-\alpha)Ak_t^{\alpha}.$$

For the case of $z_{t+1} > 0$, we combine equations (5) and (6) to obtain:

$$(1 + n + z_{t+1})k_{t+1} = \frac{\beta(1+\gamma)(1-\alpha)Ak_t^\alpha - \frac{(1-\alpha)}{\alpha}k_{t+1}}{1+\beta+\beta\gamma}. \tag{7}$$

We can now use (6) and (7) to solve for z_{t+1} and k_{t+1}.

$$z_{t+1} \equiv \overline{z} = \frac{1-\alpha-\alpha\gamma(1+\alpha)}{1-\alpha+\gamma}, \tag{8}$$

and

$$Bk_{t+1} = \frac{\beta}{1+\beta}(1-\alpha)Ak_t^\alpha, \tag{9}$$

with

$$B \equiv 1 + n + \frac{(1+\alpha\beta)(1-\alpha-\gamma(1+n)\alpha)}{\alpha(1+\beta)(1-\alpha+\gamma)}.$$

It thus appears that the level of participation is time invariant. Indeed, we can derive the threshold value for γ below which workers start to work. Namely, we have $\overline{z} > 0$ iff

$$\gamma < \overline{\gamma} = \frac{1-\alpha}{\alpha}\frac{1}{1+n}. \tag{10}$$

We can also note that for $z = 0$, B just reduces to $1 + n$. The dynamics of k_t for $z_t > 0$ is given by (9). The steady-state market equilibrium value of labor supply in the second period of live is simply $z^* = \overline{z}$, when we use a $*$ to denote the steady-state market solutions. That of the capital stock k^* is from (9):

$$k^* = \left(\frac{\beta(1-\alpha)A}{(1+\beta)B}\right)^{\frac{1}{1-\alpha}} \tag{11}$$

Parameter γ plays a crucial role. Indeed one can define k^* as a function of γ. As it appears in Figure 1, for $\gamma < \overline{\gamma}$, k^* increases with γ and for $\gamma \geq \overline{\gamma}$, k^* being the value obtained in the original Diamond model. Namely:

$$k_D^* - \left[\frac{\beta(1-\alpha)A}{(1+\beta)(1+n)}\right]^{\frac{1}{1-\alpha}}.$$

Capital accumulation is smaller the lower the γ as long as $\gamma < \overline{\gamma}$; when the preference for leisure decreases, and hence the labor supply increases in the second period, the need to save for retirement becomes less stringent. In the limit case, when $\gamma \to 0$, $z \to 1$, the stock of capital reaches its floor level denoted by k_0^*.

Formally,

◊ if $\gamma \geq \overline{\gamma}$, $k^* \equiv k_D^* = \left[\frac{\beta(1-\alpha)A}{(1+\beta)(1+n)}\right]^{\frac{1}{1-\alpha}}$;

◊ if $\gamma < \overline{\gamma}$, $k^* = \left[\frac{\beta(1-\alpha)A}{(1+\beta)B}\right]^{\frac{1}{1-\alpha}}$, with $\frac{\partial B}{\partial \gamma} < 0$;

◊ if $\gamma \to 0$, $k^* \to k_0^* = \left[\frac{\beta(1-\alpha)A}{(1+\beta)(1+n)+\beta+1/\alpha}\right]^{\frac{1}{1-\alpha}}$.

2.4. Golden rule

To obtain the golden rule one maximizes the steady-state utility of an individual subject to the resource constraint:

$$c + \frac{1}{1+n}d = \left(1 + \frac{z}{1+n}\right)[Ak^\alpha - (1+n)k]. \tag{12}$$

After subsituting for d, we maximize $u(c, d, 1-z)$ with respect to k and c, and we obtain:

$$\alpha A\hat{k}^{\alpha-1} = 1 + n = \frac{\hat{d}}{\beta\hat{c}}, \tag{13}$$

and

$$\hat{k} = \left[\frac{\alpha A}{1+n}\right]^{\frac{1}{1-\alpha}}, \tag{14}$$

where the ˆ denotes optimal value.

Equations (12) give both the golden rule of capital accumulation and the optimal rule of choice between present and future consumption. Now maximizing $u(c, d, 1-z)$ with respect to z gives:

$$\frac{1-\hat{z}}{\gamma} = \frac{\hat{d}}{\hat{w}}, \tag{15}$$

where we assume $\hat{z} > 0$ and use the equality $\hat{w} = A\hat{k}^\alpha - (1+n)\hat{k}$.

Using (13) the resource constraint (12) becomes:

$$\frac{\hat{d}}{1+n}\frac{1+\beta}{\beta} = \left(1 + \frac{\hat{z}}{1+n}\right)\hat{w}.$$

From (15), we derive the golden rule optimal value of the retirement variable $\hat{z}$:

$$\hat{z} = \frac{1+\beta-\beta\gamma(1+n)}{1+\beta+\beta\gamma}. \tag{16}$$

Consequently, the optimal value of z is $\hat{z} > 0$ given by (16) iff

$$\gamma < \frac{1+\beta}{\beta(1+n)}. \tag{17}$$

And, $\hat{z} = 0$ iff

$$\gamma \geq \frac{1+\beta}{\beta(1+n)}.$$

If we want to have the equality between k^* and $\hat{k}$, the market and the golden rule levels of capital, it suffices to have $\frac{\beta(1+n)}{(1+\beta)B} = \frac{\alpha}{1-\alpha}$. This also implies the equality between $\hat{z}$ and z^* as it appears from comparing (8) and (16).

Note that $\hat{k}$ does not depend on the parameters of the utility functions. We face three standard possibilities: overaccumulation, underaccumulation and the golden rule. On Figure 1, we illustrate the case of $k_D^* \geq \hat{k} \geq k_0^*$. The values k_0^* and k_D^* correspond to the market equilibrium for $\gamma \to 0$ and the Diamond market equilibrium with $\gamma > \overline{\gamma}$ respectively.

◊ Case 1. $\hat{k} \leq k_0^*$

There is overaccumulation. After substitution it follows that:

$$\frac{1}{1+n} \leq \frac{\beta(1-2\alpha)-\alpha}{1+\alpha\beta},$$

which is excluded for $\beta(1-2\alpha) \leq \alpha$. For $\alpha = 1/3$ and $\beta \leq 1$, this case is indeed excluded.

◊ Case 2. $k_0^* < \hat{k} < k_D^*$

The second inequality ($\hat{k} < k_D^*$) is standard and equivalent to $\frac{\alpha}{1-\alpha} < \frac{\beta}{1+\beta}$. This case occurs when the horizontal line $\hat{k}$ crosses the curvy segment of k^* for a value of γ between 0 and $\overline{\gamma}$ and denoted $\tilde{\gamma}$. For $\gamma < \tilde{\gamma}$, there is underaccumulation and for $\gamma > \tilde{\gamma}$, there is overaccumulation.

◊ Case 3. $\hat{k} = k_D^*$

In this limit case $\frac{\alpha}{1-\alpha} = \frac{\beta}{1+\beta}$; there is underaccumulation if $\gamma < \overline{\gamma}$ and the golden rule are satisfied if $\gamma \geq \overline{\gamma}$.

◊ Case 4. $\hat{k} > k_D^*$

This inequality is equivalent to that found in Diamond's model: $\frac{\alpha}{1-\alpha} > \frac{\beta}{1+\beta}$. In this case there is always underaccumulation.

As we know quite well the desirability of public debt and pay-as-you-go social security depends on the existence of excess accumulation with respect to the modified golden rule. In Case 2, as the retirement parameter

γ increases, overaccumulation becomes likelier. This is pretty intuitive. As γ increases, z_{t+1} decreases and eventually falls to zero. Then the need for retirement saving increases, and the possibility of overinvestment becomes likelier.

3. THE MODEL WITH SOCIAL SECURITY

3.1. Pay-as-you-go social security

We now introduce pay-as-you-go social security. Social security consists of two parameters : the payroll tax rate τ_t and the (per unit) benefit level p_t. This implies the following for the levels of consumption:

$$c_t = (1 - \tau_t)w_t - s_t \tag{18}$$

and

$$d_{t+1} = R_{t+1}s_t + (1 - \tau_{t+1})w_{t+1}z_{t+1} + p_{t+1}(1 - z_{t+1}). \tag{19}$$

Note that total retirement benefit depends on both p_{t+1} and the length of retirement, $(1 - z_{t+1})$.

With a pay-as-you-go system the revenue constraint is:

$$p_{t+1}(1 - z_{t+1}) = \tau_{t+1}(1 + n + z_{t+1})w_{t+1} \tag{20}$$

Note that with this relationship we can rewrite (19):

$$d_{t+1} = R_{t+1}s_t + w_{t+1}(z_{t+1} + (1 + n)\tau_{t+1}).$$

Using this constraint, we can also express the net value of transfers received by a member of generation t:

$$T_t = -\tau_t w_t + \frac{(1 + n)\tau_{t+1}w_{t+1}}{R_{t+1}}.$$

In the steady-state $T = \tau w \frac{n-r}{1+r} < 0$ for $r > n$, which is standard.

This retirement system is not the only one. We adopt this specification because it fits well to what is happening in a number of countries (See Gruber and Wise, 1999) where there is a double tax on continued work: the payroll tax $\tau_{t+1}w_{t+1}\Delta z_{t+1}$ and the forgone pension benefits $p_{t+1}\Delta z_{t+1}$.

An alternative specification would be a system such that continued work would not have these unpleasant implications. In other words, net retirement benefits would be unvariant to the value of z_{t+1}. In that case equation (19) would become:

$$d_{t+1} = R_{t+1}s_{t+1} + w_{t+1}z_{t+1} + \overline{p}_{t+1} \tag{19'}$$

and the revenue constraint

$$\overline{p}_{t+1} = \tau_{t+1}(1+n)w_{t+1} \tag{20'}$$

With this neutral scheme there is no gap between the market choice of retirement and the optimal one, as will become clear further on.

3.2. Market equilibrium

The competitive market conditions $w_t = (1-\alpha)Ak_t^{\alpha}$ and $R_t = \alpha A k_t^{\alpha-1}$ hold. On the consumer side we have the FOC:

$$\frac{1}{w_t(1-\tau_t) - s_t} - \frac{\beta R_{t+1}}{R_{t+1}s_t + w_{t+1}(z_{t+1} + (1+n)\tau_{t+1})} = 0$$

and

$$\frac{-\gamma}{1-z_{t+1}} + (1-\tau_{t+1}-\rho_{t+1})\frac{w_{t+1}}{d_{t+1}} \leq 0 \ (= 0 \text{ if } z_{t+1} > 0).$$

After some substitution, we obtain for $z_{t+1} > 0$ and k_{t+1}:

$$z_{t+1} = \frac{1-\alpha-\alpha\gamma(1+n) - \tau_{t+1}(1-\alpha)(2+n+\gamma(1+n))}{1-\alpha+\gamma} \tag{21}$$

and

$$k_{t+1}\left(\alpha(1+n) + \frac{1+\alpha\beta}{1+\beta}z_{t+1} + \tau_{t+1}\frac{(1-\alpha)(1+n)}{1+\beta}\right) = (1-\alpha)(1-\tau_t)\frac{\alpha\beta}{1+\beta}Ak_t^{\alpha}. \tag{22}$$

Note that z_{t+1} depends on just τ_{t+1}. In the steady-state, when $\tau_t = \tau$, we obtain the following values for the capital stock and the rate of participation.

$$k^*(\tau) = \left[\frac{(1-\alpha)(1-\tau)\alpha\beta A}{(1+\beta)\alpha(1+n) + (1+\alpha\beta)z^*(\tau) + \tau(1-\alpha)(1+n)}\right]^{\frac{1}{1-\alpha}} \tag{23}$$

and

$$z^*(\tau) = \frac{1-\alpha-\alpha\gamma(1+n) - \tau(1-\alpha)(2+n+\gamma(1+n))}{1-\alpha+\gamma} \tag{24}$$

We are particularly interested by the effect of τ on labor participation. As it clearly appears from (24), τ and γ have an unambiguous negative effect on $z^*(\tau)$. From (24), we can derive the value of γ under which individuals decide to work. We have $z^*(\tau) > 0$ if:

$$\gamma < \overline{\gamma}(\tau) \equiv \frac{(1-\alpha)(1-\tau(2+n))}{(1+n)(\alpha+(1-\alpha)\tau)}.$$

Both $z^*(\tau)$ and $\overline{\gamma}(\tau)$ are decreasing functions of τ, which is is quite expected. (Finding 1).

When $z^*(\tau) = 0$, that is in Diamond's case, we have

$$k_D^*(\tau) = \left[\frac{\beta(1-\tau)(1-\alpha)A}{(1+n)((1+\beta)+\frac{1-\alpha}{\alpha}\tau)}\right]^{\frac{1}{1-\alpha}}.$$

Not surprisingly $\frac{dk_D^*}{d\tau}$ is negative. However, when the participation rate is positive, namely when $z^*(\tau) > 0$, the effect of τ on k^* becomes ambiguous; it can turn positive for particular values of our parameters.

3.3. Golden rule and optimal policy

We now turn to the policy part of this paper. We will consider three problems: the maximum utility in the steady-state when the planner controls both the payroll tax rate and the retirement age; the maximum utility in the steady-state when the only instrument is the payroll tax; the maximum utility when the planner can only determine the retirement age for a given payroll tax.

In this subsection, for the sake of generality but also of exposition, the analysis is conducted in terms of general utility, $u(c, d, 1-z)$, and production, $F(K, L)$, functions.

The maximum utility in the steady-state is obtained by choosing the values of z, k and d that maximize the following expression:

$$u = u\left(F(k,1)\left(1+\frac{z}{1+n}\right) - k(1+n+z) - d/(1+n), d, 1-z\right).$$

This yields the following optimality conditions:

$$F'_K = (1+n), \tag{25}$$

$$\frac{\partial u}{\partial c}\frac{F'_L}{1+n} = \frac{\partial u}{\partial(1-z)}, \quad \text{(if) } z > 0 \tag{26}$$

and

$$\frac{\partial u}{\partial c} = (1+n)\frac{\partial u}{\partial d}. \tag{27}$$

The first condition (25) yields the golden rule capital stock, which is given by (14) for the Cobb-Douglas.

Equation (26) gives the condition for the optimal retirement age to be contrasted with that of the market equilibrium. As it will appear below, when social security benefits are positive, the market solution implies a lower age of retirement than the optimal one.

We now want to see how to achieve this first-best allocation by using two instruments, a social security scheme characterized by $\hat{\tau}$ and a mandatory retirement age characterized by $\hat{z}$. To do so, we start from our market equilibrium and derive the optimal value of those two instruments. These two instruments are needed to satisfy conditions (25) and (26); condition (27) is met as soon as $R = (1+n)$. We will use the indirect utility function:

$$v(\tau, z) = u(c(\tau, z), d(\tau, z), 1 - z). \tag{28}$$

One can easily obtain the following relations:

$$\begin{aligned} \frac{\partial c}{\partial \tau} &= -w + (1-\tau)\frac{\partial w}{\partial \tau} - \frac{\partial s}{\partial \tau} \\ \frac{\partial d}{\partial \tau} &= R\frac{\partial s}{\partial \tau} + s\frac{\partial R}{\partial \tau} + (1+n)w + (z + (1+n)\tau)\frac{\partial w}{\partial \tau}. \end{aligned}$$

We have $\frac{\partial R}{\partial \tau} = F''_{KK}(k,1)\frac{\partial k}{\partial \tau}$ and $\frac{\partial w}{\partial \tau} = F''_{LK}(k,1)\frac{\partial k}{\partial \tau}$. In addition, $F''_{LK} = -kF''_{KK} = \frac{-s}{1+n+z}F''_{KK}$. Thus, we can write:

$$s\frac{\partial R}{\partial \tau} = -(1+n+z)\frac{\partial w}{\partial \tau}.$$

Hence, using $\frac{\partial u}{\partial c} = R\frac{\partial u}{\partial d}$ we have:

$$\frac{\partial v}{\partial \tau} = \frac{\partial u}{\partial c}\left(\frac{1+n}{R} - 1\right)\left(w - (1-\tau)\frac{\partial w}{\partial \tau}\right), \tag{29}$$

which is equal to zero only at the golden rule.[2]

[2] For the loglinear utility and Cobb-Douglas production function, one can easily find the value of τ which leads to the golden rule for a given value of z.

$$\hat{\tau} = \frac{(1+n)[\beta(1-\alpha) - (1+\beta)\alpha] - z(1+\alpha\beta)}{(1+n)(1-\alpha)(1+\beta)}$$

One sees right away that $\frac{d\hat{\tau}}{dz} < 0$. Namely, the need for social security decreases as z increases. Also, one can find for $z = 0$, the optimal tax rate in Diamond's case :

$$\hat{\tau}_D = \frac{\beta(1-\alpha) - (1+\beta)\alpha}{(1-\alpha)(1+\beta)}.$$

We now turn to the determination of retirement age. We dierentiate the indirect utility function with respect to z:

$$\begin{aligned}\frac{\partial v}{\partial z} &= \frac{\partial u}{\partial c}\left(\frac{\partial c}{\partial z}+\frac{1}{R}\frac{\partial d}{\partial z}\right)-\frac{\partial u}{\partial(1-z)} \\ &= \frac{\partial u}{\partial c}\left((1-\tau)\frac{\partial w}{\partial z}\left(1-\frac{1+n}{R}\right)+\frac{w}{R}\right)-\frac{\partial u}{\partial(1-z)}.\end{aligned} \tag{30}$$

In equation (30), $\frac{\partial v}{\partial z}=0$ yields the optimal value of z for $R=1+n$ (or $\tau=\hat{\tau}$). It is the solution of:

$$\frac{\partial u}{\partial c}\frac{w}{R}=\frac{\partial u}{\partial(1-z)}, \tag{31}$$

and can be compared with the market solution given by:

$$\frac{\partial u}{\partial c}\frac{w}{R}\left(1-\frac{\tau(2+n)}{1-z}\right)=\frac{\partial u}{\partial(1-z)}. \tag{32}$$

Comparing (31) and (32), it is clear that both market and optimal solution coincide only when $\hat{\tau}=0$. Further,

$$z^*(\hat{\tau}) \gtreqless \hat{z} \text{ if } \hat{\tau} \lesseqgtr 0.$$

In other words, it is only when the optimal level of social security is 0 that the laisser-faire participation rate coincides with the optimal one. If the optimal level of social security is negative, then the market participation rate exceeds the optimal one.

Note however that if instead of (19) we have to use (19'), the market solution would coïncide with (31). When the social security system provides net benefits that are invariant to z, the market solution is optimal for $\tau=\hat{\tau}$.

We have just compared the participation rate freely chosen by individuals given a payroll tax $\hat{\tau}$ and the optimal participation rate. Clearly, this means that to reach the first best one needs a mandatory retirement age (Finding 2)[3].

This leads us to the next question: if τ is the only instrument available, what is the optimal (second-best) rule? We would like to know whether this second-best value denoted $\tilde{\tau}$ is lower or higher than the golden rule value $\hat{\tau}$. To check this, we look at the derivation of v with respect to τ at the point $\hat{\tau}$:

$$\left.\frac{dv}{d\tau}\right|_{\tau=\hat{\tau}}=\left(\frac{w}{R}\frac{\partial u}{\partial c}-\frac{\partial u}{\partial(1-z)}\right)\frac{\partial z^*}{\partial\tau}.$$

[3] Again, it would suffice to modify the social security benefit system and make it neutral towards retirement age.

As $z^* > 0$, we have from the FOC for the consumer's optimum (32) that

$$\left.\frac{dv}{d\tau}\right|_{\tau=\hat{\tau}} = \frac{\hat{w}}{\hat{R}}\frac{\partial u}{\partial c}\hat{\tau}\left(\frac{2+n}{1-z}\right)\frac{\partial z^*}{\partial \tau},$$

which has the sign of $-\hat{z}$. Thus,

$$\left.\frac{dv}{d\tau}\right|_{\tau=\hat{\tau}} \lesseqgtr 0 \text{ and } \tilde{\tau} \lesseqgtr \hat{\tau} \text{ iff } \hat{\tau} \gtreqless 0.$$

This shows that for nonzero $\hat{\tau}$, one indeed needs two instruments to achieve the first-best. It also shows that for positive optimal social security, the second-best level is lower than the first-best and this implies overaccumulation (Finding 3). This finding is rather intuitive. Assume that $\hat{\tau} > 0$. We know that the laissez-faire retirement age is lower than its optimal value. To increase it, given that there is only one instrument, one has to adopt a tax rate lower than the first-best one.

Let us now take a more positive approach, that of tax reform. We assume that τ is given and not necessarily optimal from the steady-state viewpoint. For that level of social security benefits, there will be a market retirement age $z^*(\tau)$. Should it be desirable to impose a different age, particularly a lower retirement age?

As implied by equation (32), we know that as soon as $\tau > 0$, there will be "early" retirement relative to the optimal retirement given by (31). Using (30), we also see that when $\tau = 0$, $\frac{\partial v}{\partial z} < 0$ if there is underaccumulation ($R > 1 + n$) since $\frac{\partial k}{\partial z} < 0$. The intuition of this finding, which can be extended to positive but low values of τ, is that by forcing people to retire earlier than they would like to, the planner induces them to save more for retirement and this is good when there is underaccumulation (Finding 4). In general, however, we can expect τ to be positive and rather high. Then the opposite recommendation is likely.

4. CONCLUSION

We can sum up the main conclusions of this paper in a number of findings which have been proved to hold, either in general or in the particular example of a loglinear utility function and a Cobb-Douglas production function. Note that our normative conclusions apply only to the steady-state.

Finding 1: At each period of time increasing social security benefits reduces the market labor force participation rate.

Finding 2: To achieve the steady-state first-best optimum, one needs to control both an unrestricted pay-as-you-go social security tax and the retirement age.

Finding 3: If the only policy variable is the social security tax and if that tax is positive, then some overaccumulation is optimal.

Finding 4: A mandatory decrease in the retirement age may have a positive effect on the steady-state welfare when there is underaccumulation. and the social security tax is zero or not small.

Under the plausible assumptions that the economy is in a state of underaccumulation, and that the payroll tax is positive, we know that in general there is no Pareto-improving policy since a decline in tax which is desirable in the long run implies a loss in the welfare of the current generation. However, in the model studied here there is a basic distortion in the consumption-retirement choice. If the government could reform the social security scheme and make it unvariant to the age of retirement, then the current generation of retirees would be better off, as well as all future generations. Indeed, the gain in welfare could be neutralized to decrease the payroll tax, and thus there would be a double gain: no distortion in the consumption-retirement choice and more capital accumulation. This analysis is on our research agenda.

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