

## On the Growth and Stability Effects of Habit Formation and Durability in Consumption

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This paper shows that a unique balanced growth monetary equilibrium exists in a transactions-based monetary endogenous growth model with habit formation or durability in consumption. An increase in the nominal money growth rate reduces the long-run output growth rate, wherein habit formation enforces the effectiveness of monetary policy while durability in consumption reduces it. We also show that while habit formation destabilizes the macroeconomy by making the balanced growth equilibrium exhibit local indeterminacy, durability in consumption maintains saddle-path stability of the balanced growth equilibrium. We find that the mechanism through which habit formation and durability impose different effects on both the growth-effect of money and the macroeconomic stabilizing properties is such that habit formation and durability influence the elasticity of intertemporal substitution in consumption in opposite directions.

*Key Words:* Habit formation; Durability; Superneutrality; Indeterminacy.

*JEL Classification Numbers:* E21, E52, O42.

### 1. INTRODUCTION

This paper explores the roles of two types of time-nonseparable preferences – habit formation and durability in consumption – in two of the important issues in monetary economics: the (non)superneutrality of money and the macroeconomic stabilizing properties. Habit formation means that the agent cares about its current level of consumption as compared to the stock of habit formed by past consumptions, which is used for in-

\* This paper is based on a revised chapter of my Ph.D. dissertation at National Taiwan University. I sincerely thank my advisor Ching-Chong Lai for his constant encouragement and invaluable guidance. Helpful comments and suggestions from the editor, Heng-Fu Zou, Jang-Ting Guo, Wen-Ya Chang, Hsueh-Fang Tsai, Been-Lon Chen, Mei-Lie Chu, and Juin-Jen Chang, are also greatly appreciated.

dexing the customary level of consumption. Durability, on the other hand, means that the agent consumes not only current consumption, but also a weighted average of past consumptions. An extensive empirical literature has demonstrated the importance of these two types of time nonseparability in consumption.<sup>1</sup> The habit formation model has also gained popularity in the last few decades because it is capable of accounting for some facts or puzzles arising in time additively-separable preferences models such as the excess smoothness of aggregate consumption (Campbell and Deaton 1989),<sup>2</sup> the observation of high aggregate income growth followed by high aggregate saving (Carroll et al. 2000), and the equity premium puzzle (Abel 1990; Constantinides 1990). In addition, Fuhrer (2000) and Letendre (2004) demonstrate that habit formation significantly improves the fit of the model by improving the dynamics of important macroeconomic variables. Mansoorian and Mohsin (2010) also demonstrate that durability has significant effects on the adjustments of important macroeconomic variables and can help account for some empirical facts.

In view of the relevant role of time nonseparability in consumption, many authors have been working on the macroeconomic policy implications of habit persistence and durability. Among them, Amato and Laubach (2004) and Mansoorian and Michelis (2005) focus on the issue of monetary policy rules; Ikeda and Gombi (1998) and Guo and Krause (2011) work on fiscal policy issues; Mansoorian (1996) and Uribe (2002) study exchange rate policies; and Faria (2001) and Mansoorian and Mohsin (2010) examine the effect of domestic inflation. It still leaves an open question regarding whether or not and how habit formation and durability in consumption will influence the (non)superneutrality result in an endogenous growth setting where the central bank adopts an exogenous money growth rule. The first purpose of this paper is thus to fill this gap in the literature.

The second purpose of this paper is to contribute to the literature by identifying how habit formation and durability in consumption govern the local stability properties of the monetary economy's balanced growth path. Our exploration of the macroeconomic (in)stability implication of durability is new in the literature. Regarding the implication of habit formation, mixed conclusions are reached in the literature. In particular, Weder (2000)

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<sup>1</sup>In this huge literature, for example, Constantinides (1990) and Fuhrer (2000) find evidence of habit persistence, while Eichenbaum et al. (1988) and Dynan (2000) find little evidence of habit formation. Studies supporting the durability of consumption expenditures include Hayashi (1985) and Eichenbaum and Hansen (1990), among others. Ferson and Constantinides (1991) find that habit persistence dominates the effect of durability in monthly, quarterly, and annual data. Heaton (1993) finds evidence that the data are consistent with time nonseparable preferences if consumption goods are durable and if individuals develop habits over the flow of services from the good.

<sup>2</sup>Luo, Smith, and Zou (2009*a, b*) show that the spirit of capitalism provides another explanation for the excess smoothness of aggregate consumption.

shows that habit formation increases the degree of productive externalities required for the emergence of equilibrium indeterminacy in a two-sector model of real business cycles. Auray et al. (2004) find that habit formation cannot cause equilibrium indeterminacy in a money-in-the-utility-function model. Auray et al. (2005) then show in a cash-in-advance economy that real indeterminacy of equilibrium occurs with sufficiently high degrees of habit persistence.

To address the two issues, we incorporate habit formation and durability in consumption into Jha et al.'s (2002) transactions-based monetary growth model. We view this as a good starting point since Jha et al. (2002) is by far the only work that simultaneously examines the issues of the (non)superneutrality of money and the macroeconomic stabilizing properties. The habit formation and durability specification of our model closely follows that developed by Carroll et al. (2000), which is also adopted by Fuhrer (2000), Faria (2001), and Alonso-Carrera et al. (2005), among many others.

For the (non)superneutrality result, we find that our model economy has a unique balanced growth monetary equilibrium. An increase in the nominal money growth rate reduces the long-run output growth rate, where-in habit formation strengthens the effectiveness of monetary policy while durability in consumption reduces it. To provide the economic intuition, we first notice that under our specification of the instantaneous utility function, a higher degree of habit formation leads to a higher intertemporal elasticity of substitution in consumption, meaning that the individual enjoys the fluctuations in intertemporal consumption more. This implies that, as pointed out by Alonso-Carrera et al. (2005, p.1667), "...the introduction of habits...raises the consumers' willingness to shift consumption from the present to the future." By contrast, a higher degree of durability leads to a lower intertemporal elasticity of substitution, meaning that the individual dislikes the fluctuations in intertemporal consumption more. In response to a higher inflation rate caused by a higher money growth rate, the agent decreases its holdings of real money balances. In the transactions cost model, the decline in the agent's money holdings leads to a larger fraction of real output devoted to transactions services. This decreases the net marginal product of capital and hence discourages the agent's investment. As a result, the rate of output growth declines. With a higher degree of habit formation, the agent will reduce its investment even further, and hence the reduction in the output growth rate will be deepened. In the case of durability in consumption, since the agent dislikes fluctuations in intertemporal consumption, the reduction in its investment will be smaller in magnitude. As a result, durability in consumption makes monetary policy less effective.

With regard to the macroeconomic stabilizing properties, we find that habit formation destabilizes the macroeconomy by making the balanced growth equilibrium exhibit local indeterminacy. Durability in consumption, on the other hand, maintains the saddle-path stability of the balanced growth equilibrium. Obviously, our result for the case of habit formation is consistent with that derived in Auray et al.'s (2005) cash-in-advance model, but runs in sharp contrast to the results obtained in Auray et al.'s (2004) money-in-the-utility-function model and Weder's (2000) two-sector real business cycle model.

The intuition underlying our (in)determinacy result is as follows. When the agent expects a higher future return on capital, it will increase its demand for capital. This will result in a rise in the price of capital, thereby reducing the net rate of return on capital. On the other hand, when expecting a higher future return on capital, the agent will also reduce consumption and increase investment today in exchange for higher future consumption. This in turn expands the supply of capital, hence lowering its price, and raises the net rate of return on capital. Recall that with habit formation the agent is more willing to shift consumption from the present to the future. Due to the enhanced intertemporal substitution effect, under habit persistence the rate of return on capital rises. As a result, the agent's initial optimistic expectations become self-fulfilling. By contrast, with durability in consumption, the weak intertemporal substitution effect leads to a decline in the rate of return on capital, thus preventing the agent's expectations from becoming self-fulfilling.

The remainder of this paper is organized as follows. Section 2 describes a transactions-based monetary endogenous growth model with habit formation and durability in consumption. Section 3 analyzes the existence and number of the economy's balanced growth paths, together with the associated growth effect of money and the local stability properties. Section 4 concludes.

## 2. THE ECONOMY

We slightly modify the instantaneous utility function of Carroll et al. (2000) into one that can describe both the cases of habit formation and durability in consumption, and then incorporate it into the transactions-based monetary endogenous growth model of Jha et al. (2002). The economy is populated by a unit measure of identical infinitely-lived households, each of which has perfect foresight and maximizes a discounted stream of utilities over its lifetime

$$U = \int_0^{\infty} \frac{(c_t S_t^\eta)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \quad (1)$$

where  $c_t$  is consumption,  $S_t$  represents services formed by past consumptions (Heaton, 1993),  $\rho \in (0, 1)$  denotes the subjective discount rate, and  $\sigma > 1$  is the inverse of relative risk aversion.<sup>3</sup> The parameter  $\eta$  indexes the importance of past consumptions. When  $\eta = 0$ , we return to time-separable preferences where the representative household cares only about its current level of consumption. Non-zero values of  $\eta$  indicate that the household cares also about its past consumptions, thereby giving rise to nonseparability over time.

As claimed by Ferson and Constantinides (1991, p. 200), "...Habit persistence implies that the coefficients on the lagged expenditures are negative, whereas durability implies positive coefficients." Accordingly, we refer to  $\eta < 0$  as habit formation in consumption. In this case, the household cares about its current level of consumption as compared to its customary level of consumption, and consumption is thus complementary over time. In addition, decreases in the (negative) value of  $\eta$  represent increases in the degree of habit formation. By contrast, we refer to  $\eta > 0$  as durability in consumption, whereby the representative household consumes a weighted average of past consumptions and thus consumption is substitutable over time. Increases in the (positive) value of  $\eta$  then represent increases in the degree of durability. We further follow Carroll et al. (2000), Fuhrer (2000), and Alonso-Carrera et al. (2005) in imposing  $\eta > -1$  so as to guarantee that utility is strictly increasing in consumption along the balanced growth path.

The consumption stock  $S_t$  in (1) is a weighted average of past consumptions:

$$S_t = \beta \int_{-\infty}^t c_v e^{-\beta(t-v)} dv; \quad (2)$$

or equivalently,

$$\dot{S}_t = \beta(c_t - S_t), \quad S_0 > 0 \text{ given}, \quad (3)$$

where  $\beta > 0$  determines the relative weights of consumption at different times.

As in Jha et al. (2002), we consider pecuniary costs associated with transactions. Let  $m_t \equiv M_t/P_t$  denote real money balances, where  $M_t$  and  $P_t$  respectively represent nominal money balances and the price level. The real resource costs required to facilitate transactions services in the economy are given by  $\phi_t y_t$ , where  $y_t$  is real output,  $\phi_t = \phi(m_t/c_t)$  is the fraction of real output devoted to transactions which is assumed to be twice

<sup>3</sup>We follow Carroll et al. (2000), Fuhrer (2000), and Alonso-Carrera et al. (2005) in assuming  $\sigma > 1$ , where Fuhrer (2000) obtains that both the Full Information Maximum Likelihood estimate and the Generalized Method of Moments estimate of  $\sigma$  for the U.S. are much bigger than one over the sample period 1966:1-1995:4.

continuously differentiable and to satisfy  $\phi'_t < 0$ ,  $\phi''_t \geq 0$ ,  $\lim_{m_t/c_t \rightarrow 0} \phi_t = 1$ , and  $\lim_{m_t/c_t \rightarrow 1} \phi_t = \bar{\phi} \in (0, 1)$ . To simplify the analysis and to focus on the role of habit persistence and durability in consumption, in what follows we adopt Jha et al.'s (2002) suggestion of the linear transactions cost technology:  $\phi(m_t/c_t) = 1 - a(m_t/c_t)$ , where  $0 < a < 1$ .

Under the transactions cost technology, the budget constraint faced by the representative household is given by

$$\dot{k}_t + \dot{m}_t = (1 - \phi_t)y_t - c_t - \pi_t m_t + \tau_t, \quad k_0 > 0 \text{ given}, \quad (4)$$

where  $k_t$  is the household's capital stock,  $\pi_t$  is the inflation rate, and  $\tau_t$  represents real lump-sum transfers that households receive from the monetary authority. Following Chen and Guo (2005), we assume that output  $y_t$  is produced using the linear technology:

$$y_t = Ak_t, \quad A > 0. \quad (5)$$

On the monetary side of the economy, nominal money supply is postulated to evolve according to

$$M_t = M_0 e^{\mu t}, \quad M_0 > 0 \text{ given}, \quad (6)$$

where  $\mu \neq 0$  is the constant money growth rate, and the resulting seigniorage is returned to households as a lump-sum transfer, hence  $\tau_t = \mu m_t$ .

The first-order conditions for the representative household with respect to the indicated variables and the associated transversality conditions (TVC) are<sup>4</sup>

$$c_t : \quad c_t^{-\sigma} S_t^{\eta(1-\sigma)} = -\beta \lambda_{1t} + \left(1 + \frac{A a m_t k_t}{c_t^2}\right) \lambda_{2t}, \quad (7)$$

$$S_t : \quad \eta c_t^{1-\sigma} S_t^{\eta(1-\sigma)-1} - \beta \lambda_{1t} = -\dot{\lambda}_{1t} + \rho \lambda_{1t}, \quad (8)$$

$$m_t : \quad \left(\frac{A a k_t}{c_t} - \pi_t\right) \lambda_{2t} = -\dot{\lambda}_{2t} + \rho \lambda_{2t}, \quad (9)$$

<sup>4</sup>Since the instantaneous utility function in (1) is not jointly concave with respect to  $c_t$  and  $S_t$ , for the household's first-order necessary conditions to also be sufficient for optimization, a concavity condition should be imposed on the Hamiltonian. See the Appendix for the proof.

$$k_t : \quad \frac{Aam_t}{c_t} \lambda_{2t} = -\dot{\lambda}_{2t} + \rho \lambda_{2t}, \quad (10)$$

$$\text{TVC}_1 : \quad \lim_{t \rightarrow \infty} e^{-\rho t} \lambda_{1t} S_t = 0, \quad (11)$$

$$\text{TVC}_2 : \quad \lim_{t \rightarrow \infty} e^{-\rho t} \lambda_{2t} k_t = 0, \quad (12)$$

$$\text{TVC}_2 : \quad \lim_{t \rightarrow \infty} e^{-\rho t} \lambda_{2t} m_t = 0, \quad (13)$$

where  $\lambda_{1t}$  and  $\lambda_{2t}$  represent the shadow values of the consumption stock and wealth, respectively. Equations (7) and (8) equate the marginal benefit with the marginal cost of current consumption and the consumption stock, respectively. Equations (9) and (10) govern the evolution of the shadow value of wealth, which together imply that the inflation rate is

$$\pi_t = \frac{Aa(k_t - m_t)}{c_t}. \quad (14)$$

Clearing in the goods and money markets implies that

$$\dot{k}_t = (1 - \phi_t)y_t - c_t, \quad (15)$$

and

$$\dot{m}_t = (\mu - \pi_t)m_t. \quad (16)$$

### 3. BALANCED GROWTH PATH

We focus on the economy's balanced growth path (BGP) along which output, consumption, capital, real money balances, and the consumption stock exhibit a common, positive constant growth rate denoted by  $\theta$ . We can also obtain that along the BGP the shadow prices  $\lambda_{1t}$  and  $\lambda_{2t}$  grow at the same rate  $-\Sigma\theta$ , where  $\Sigma \equiv \sigma + \eta(\sigma - 1) > 0$  is the inverse of the intertemporal elasticity of substitution in consumption. Recall that  $\eta$  is negative/positive when consumption exhibits habit formation/durability and that  $\sigma > 1$ . Therefore, the intertemporal elasticity of substitution in the case of habit persistence/durability is bigger/smaller than the inverse of relative risk aversion  $\frac{1}{\sigma}$ .<sup>5</sup> Moreover, in the case of habit formation ( $\eta < 0$ ), as the degree of habit formation increases ( $\eta$  gets lower), the intertemporal

<sup>5</sup>As documented by Carroll et al. (2000, p. 347), "...The infinite-horizon intertemporal elasticity of substitution in the model of habit formation is strictly greater than the inverse of the coefficient of relative risk aversion."

elasticity of substitution increases (since  $\Sigma$  decreases), meaning that the individual enjoys the fluctuations in intertemporal consumption more. This implies, as pointed out by Alonso-Carrera et al. (2005, p.1667), that "...the introduction of habits...raises the consumers' willingness to shift consumption from the present to the future." By contrast, in the case of durability ( $\eta > 0$ ), as the degree of durability increases ( $\eta$  gets higher), the intertemporal elasticity of substitution decreases (since  $\Sigma$  increases), meaning that the individual has a greater dislike for the fluctuations in intertemporal consumption.

Based on the aforementioned properties of the BGP, to facilitate the analysis of perfect-foresight dynamics, we make the following transformation of variables:  $x_t \equiv \frac{c_t}{k_t}$ ,  $w_t \equiv \frac{m_t}{k_t}$ ,  $s_t \equiv \frac{S_t}{k_t}$ , and  $\varphi_t \equiv \frac{\lambda_{2t}}{\lambda_{1t}}$ . With this transformation, the model's equilibrium conditions can be expressed as the following differential equations:

$$\dot{x}_t = \left[ \frac{\alpha_t A a \varphi_t}{\beta x_t^2} \dot{w}_t + \frac{\eta(1-\sigma)}{s_t} \dot{s}_t + \frac{\alpha_t}{\varphi_t} \dot{\varphi}_t - \rho - \frac{(\Sigma-1) A a w_t}{x_t} + \Sigma x_t \right] \frac{x_t}{\Psi_t \sigma}, \quad (17)$$

$$\dot{w}_t = \left( \mu - \frac{A a}{x_t} + x_t \right) w_t, \quad (18)$$

$$\dot{s}_t = \left[ \beta \left( \frac{x_t}{s_t} - 1 \right) - \frac{A a w_t}{x_t} + x_t \right] s_t, \quad (19)$$

$$\dot{\varphi}_t = \left\{ -\frac{A a w_t}{x_t} - \beta - \frac{\eta x_t}{s_t} \left[ \beta - \varphi_t \left( 1 + \frac{A a w_t}{x_t^2} \right) \right] \right\} \varphi_t, \quad (20)$$

where  $\alpha_t \equiv \frac{-\beta}{-\beta + \varphi_t(1 + A a w_t/x_t^2)} = -\frac{\beta \lambda_{1t}}{c_t^{-\sigma} S_t^{\eta(1-\sigma)}} < 0$  and  $\Psi_t \equiv 1 - \frac{2\alpha_t A a w_t \varphi_t}{\sigma \beta x_t^2} > 0$ .

Given the above dynamical system (17)-(20), the BGP equilibrium is characterized by positive real numbers  $(x^*, w^*, s^*, \varphi^*)$  that satisfy  $\dot{x}_t = \dot{w}_t = \dot{s}_t = \dot{\varphi}_t = 0$ . It is straightforward to show that  $x^*$  is the solution to the following quadratic equation:

$$\mu - \frac{A a}{x^*} + x^* = 0. \quad (21)$$

We can then obtain the expressions of  $w^*$ ,  $s^*$  and  $\varphi^*$  as

$$\begin{aligned} w^* &= \frac{(\Sigma x^* - \rho)x^*}{(\Sigma - 1)A a}, \quad s^* = \frac{\beta x^*}{\frac{x^* - \rho}{\Sigma - 1} + \beta} \quad \text{and} \\ \varphi^* &= \frac{\frac{(\Sigma + \eta)x^* - (1 + \eta)\rho}{\Sigma - 1} + (1 + \eta)\beta}{\frac{\eta[(2\Sigma - 1)x^* - \rho]}{\beta(\Sigma - 1)x^*} \left( \frac{x^* - \rho}{\Sigma - 1} + \beta \right)}. \end{aligned} \quad (22)$$

In addition, the common (positive) rate of economic growth  $\theta$  is

$$\theta = \frac{\frac{A a w^*}{x^*} - \rho}{\Sigma} = \frac{x^* - \rho}{\Sigma - 1}, \quad (23)$$

where  $\frac{Aaw^*}{x^*}$  is the real interest rate. Given that  $\Sigma - 1 = (1 + \eta)(\sigma - 1) > 0$ , (23) implies that the BGP's growth rate is positively related to the consumption-capital ratio  $x^*$ :  $\frac{d\theta}{dx^*} > 0$ .

To examine the existence and number of the economy's balanced growth paths in a transparent manner, we let  $f(x^*) \equiv \frac{Aa}{x^*} - \mu$  from (21). Therefore, the equilibrium  $x^*$  will be located from the intersection of  $f(x^*)$  and the 45-degree line. We then obtain that

$$f' = -\frac{Aa}{(x^*)^2} < 0, \quad f'' = \frac{2Aa}{(x^*)^3} > 0, \quad f(0) \rightarrow \infty \text{ and } f(\infty) \rightarrow 0. \quad (24)$$

Figure 1 depicts the above features, which shows that  $f(x^*)$  is a downward-sloping and convex curve that intersects the 45-degree line once in the positive quadrant. Therefore, the economy exhibits a unique balanced growth path.

FIG. 1.

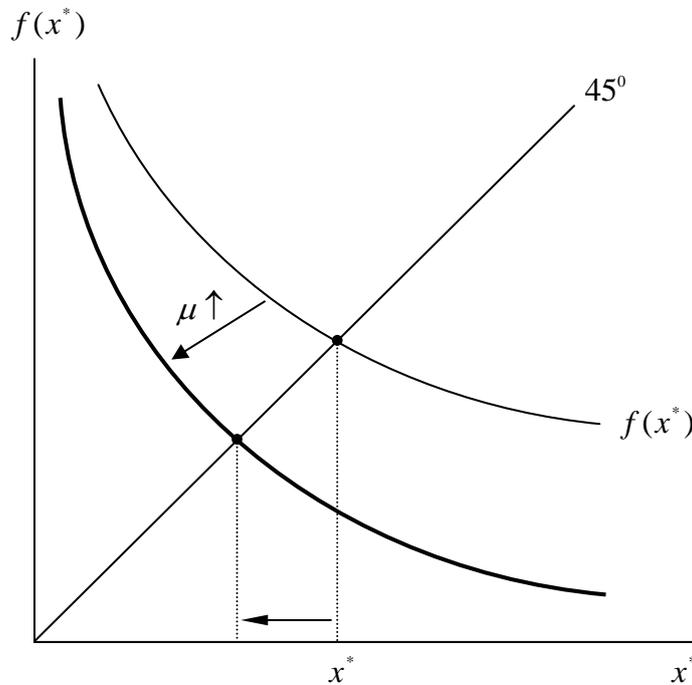


Figure 1 also shows that a higher nominal money growth rate shifts the  $f(x^*)$  locus downward such that  $\frac{dx^*}{d\mu} < 0$ , which then results in a lower BGP growth rate since  $\frac{d\theta}{dx^*} > 0$ . Mathematically, we obtain from (21) and

(23) that

$$\frac{d\theta}{d\mu} = -\frac{(x^*)^2}{(\Sigma - 1)[Aa + (x^*)^2]} < 0. \quad (25)$$

Thus, a higher nominal money growth rate lowers the long-run economic growth rate.

To provide an explanation of the growth rate-retarding result in (25), we derive from (21), (22), and the transactions cost technology  $\phi^* = 1 - \frac{aw^*}{x^*}$  that:

$$\frac{d(\frac{m_t^*}{c_t^*})}{d\mu} = \frac{d(\frac{w^*}{x^*})}{d\mu} = -\frac{\Sigma(x^*)^2}{Aa(\Sigma - 1)[Aa + (x^*)^2]} < 0, \quad (26)$$

$$\frac{d\phi^*}{d\mu} = \frac{a\Sigma(x^*)^2}{Aa(\Sigma - 1)[Aa + (x^*)^2]} > 0. \quad (27)$$

Equations (26) and (27) indicate that in the long run a permanent rise in the money growth rate lowers the real balances-consumption ratio and raises the transactions cost. Intuitively, a higher inflation rate resulting from an increase in the money growth rate discourages the agent's holdings of real money balances. Equations (26) and (27) indicate that this will lead to a reduction in the real balances-consumption ratio and subsequently a larger fraction of real output being devoted to transactions services. The increased transactions cost lowers the net marginal product of capital ( $A(1 - \phi^*) = \frac{Aaw^*}{x^*}$ ), which in turn suppresses private investment. As a consequence, the rate of output growth declines, as shown in (25).

The role of habit formation and durability in consumption here is that it affects the effectiveness of monetary policy. Specifically, from (25) we obtain that

$$\frac{d}{d\eta} \left| \frac{d\theta}{d\mu} \right| = \frac{1}{1 + \eta} \cdot \frac{d\theta}{d\mu} < 0. \quad (28)$$

The above equation indicates that in the case of habit formation, as the degree of habit formation increases ( $\eta$  gets lower), the effectiveness of monetary policy is enforced; by contrast, in the case of durability, as the degree of durability increases ( $\eta$  gets higher), the effectiveness of monetary policy is reduced.

To provide the economic intuition for (28), recall first what we mentioned in the beginning of this section which is that with a higher degree of habit persistence the individual enjoys the fluctuations in intertemporal consumption more and is more willing to shift consumption from the present to the future. Thus, in response to a reduction in the net marginal product of capital resulting from a higher money growth rate, the individual with habit formation in consumption will further reduce its investment,

and hence the reduction in the output growth rate will be enhanced. In the case of durability in consumption, the individual dislikes fluctuations in intertemporal consumption. Therefore, in the face of a lower net marginal product of capital, the individual's reduction in its investment will be less in magnitude. As a result, durability in consumption makes monetary policy less effective.

In terms of the BGP's local stability properties, we linearize the dynamical system (17)-(20) around the steady state to obtain the following linear system:

$$\begin{bmatrix} \dot{x}_t \\ \dot{w}_t \\ \dot{s}_t \\ \dot{\varphi}_t \end{bmatrix} = J \begin{bmatrix} x_t - x^* \\ w_t - w^* \\ s_t - s^* \\ \varphi_t - \varphi^* \end{bmatrix}, \tag{29}$$

where  $J$  is the  $4 \times 4$  Jacobian matrix of partial derivatives of the dynamical system (17)-(20) evaluated at the steady state. The arguments in  $J$ ,  $J_{ij}$ ,  $i = 1, \dots, 4$ ,  $j = 1, \dots, 4$ , are given by

$$\begin{aligned} J_{11} &= \frac{x^*}{\Psi^* \sigma} \left[ \frac{(\Sigma - 1)Aaw^*}{(x^*)^2} + \Sigma + \frac{\alpha^* Aa\varphi^* J_{21}}{\beta x^{*2}} + \frac{\eta(1 - \sigma)J_{31}}{s^*} + \frac{\alpha^* J_{41}}{\varphi^*} \right], \\ J_{12} &= \frac{x^*}{\Psi^* \sigma} \left[ -\frac{(\Sigma - 1)Aa}{x^*} + \frac{\alpha^* Aa\varphi^* J_{22}}{\beta x^{*2}} + \frac{\eta(1 - \sigma)J_{32}}{s^*} + \frac{\alpha^* J_{42}}{\varphi^*} \right], \\ J_{13} &= \frac{x^*}{\Psi^* \sigma} \left[ \frac{\alpha^* Aa\varphi^* J_{23}}{\beta x^{*2}} + \frac{\eta(1 - \sigma)J_{33}}{s^*} + \frac{\alpha^* J_{43}}{\varphi^*} \right], \\ J_{14} &= \frac{x^*}{\Psi^* \sigma} \left[ \frac{\alpha^* Aa\varphi^* J_{24}}{\beta x^{*2}} + \frac{\eta(1 - \sigma)J_{34}}{s^*} + \frac{\alpha^* J_{44}}{\varphi^*} \right], \\ J_{21} &= \left[ \frac{Aa}{(x^*)^2} + 1 \right] w^*, \quad J_{22} = J_{23} = J_{24} = 0, \\ J_{31} &= \left[ \frac{\beta}{s^*} - \frac{Aaw^*}{(x^*)^2} + 1 \right] s^*, \quad J_{32} = -\frac{Aas^*}{x^*}, \quad J_{33} = -\frac{\beta x^*}{s^*}, \quad J_{34} = 0, \\ J_{41} &= \left[ \left( 1 - 2\frac{\eta\varphi^*}{s^*} \right) \frac{Aaw^*}{(x^*)^2} + \frac{\frac{Aaw^*}{x^*} + \beta}{x^*} \right] \varphi^*, \quad J_{42} = \left( \frac{\eta\varphi^*}{s^*} - w^* \right) \frac{Aa\varphi^*}{x^*}, \\ J_{43} &= -\left( \frac{Aaw^*}{x^*} + \beta \right) \frac{\varphi^*}{s^*}, \quad J_{44} = \frac{\eta x^* \varphi^*}{s^*} \left( 1 + \frac{Aaw^*}{(x^*)^2} \right), \end{aligned}$$

where  $\alpha^* \equiv \frac{-\beta}{-\beta + \varphi^* [1 + Aaw^*/(x^*)^2]} < 0$  and  $\Psi^* \equiv 1 - \frac{2\alpha^* Aaw^* \varphi^*}{\sigma \beta (x^*)^2} > 0$ .

The stability of the BGP is determined by comparing the eigenvalues of  $J$  that have negative real parts with the number of initial conditions in the dynamical system (17)-(20), which is one because  $x_t$ ,  $w_t$ , and  $\varphi_t$  are all jump variables, and  $s_t$  is pre-determined. As a result, the BGP displays

saddlepath stability and equilibrium uniqueness when three eigenvalues have positive real parts and one eigenvalue has a negative real part. If more than one eigenvalue has a negative real part, then the BGP is locally indeterminate (a sink) and can be exploited to generate endogenous growth fluctuations driven by agents' self-fulfilling expectations or sunspots. If all eigenvalues have positive real parts, then the BGP is a source.

Let  $v_1, v_2, v_3,$  and  $v_4$  denote the eigenvalues of  $J$ . It can be obtained that the trace and determinant of the Jacobian are given by

$$Tr = v_1 + v_2 + v_3 + v_4 = J_{11} + \underbrace{J_{22}}_{=0} + J_{33} + J_{44}, \quad (30)$$

$$Det = v_1 v_2 v_3 v_4 = \frac{(\Sigma - 1)Aaw^*}{\Psi^* \sigma} \left[ \frac{Aa}{(x^*)^2} + 1 \right] J_{33} J_{44}. \quad (31)$$

Obviously,  $J_{33}$  and  $J_{44}$  are two of the eigenvalues. We let  $v_1 = J_{33} < 0$  and  $v_2 = J_{44} > 0$ , as  $\eta > 0$ . The remaining two eigenvalues,  $v_3$  and  $v_4$ , have the properties that  $v_3 + v_4 = J_{11} > 0$  and  $v_3 v_4 = \frac{(\Sigma-1)Aaw^*}{\Psi^* \sigma} \left[ \frac{Aa}{(x^*)^2} + 1 \right] > 0$ . This indicates that both  $v_3$  and  $v_4$  have positive real parts. Thus, we reach the conclusion that under habit formation ( $\eta < 0$ ), the BGP is characterized by two positive roots and two negative roots, which indicates that the BGP is a sink. For the case of durability in consumption ( $\eta > 0$ ), the BGP is characterized by three positive roots and one negative root, which indicates that the BGP is a saddle.

The intuition for this (in)determinacy result can be understood as follows. When the agent expects a higher future return on capital, it will increase its demand for capital. This will result in a rise in the price of capital, thereby reducing the net rate of return on capital. On the other hand, when expecting a higher future return on capital, the agent will also reduce consumption and increase investment today in exchange for higher future consumption. This in turn expands the supply of capital, hence lowering its price, and raises the net rate of return on capital. Recall that under habit formation the agent enjoys the fluctuations in intertemporal consumption more and is more willing to shift consumption from the present to the future. Due to the enhanced intertemporal substitution effect, under habit persistence the rate of return on capital rises. As a result, agents' initial optimistic expectations become self-fulfilling. On the contrary, with durability in consumption the agent dislikes fluctuations in intertemporal consumption. The weak intertemporal substitution effect thus prevents agents' expectations from becoming self-fulfilling.

#### 4. CONCLUSION

By incorporating habit formation and durability in consumption into Jha et al. (2002)'s transactions-based monetary growth model, this paper shows that habit formation enforces the effectiveness of monetary policy while durability reduces the effectiveness of monetary policy. We also show that habit formation destabilizes the macroeconomy by making the BGP exhibit local indeterminacy while durability maintains the saddle-path stability of the BGP. The determining factor for habit formation and durability to impose different effects on the growth-rate effect of money and the macroeconomic stability properties is that they influence the elasticity of intertemporal substitution in consumption in opposite directions.

Regarding possible extensions of our analyses, it would be worthwhile investigating a model with a "keeping up with the Joneses" and/or a "catching up with the Joneses" utility function,<sup>6</sup> a monetary model with the spirit of capitalism,<sup>7</sup> or a model with multiple production sectors, among others. In particular, we notice that recently an "inflation aversion" monetary model has been developed to account for the psychological effect of inflation on the time preference rate (Wang and Zuo 2001*a, b*; Zou 2001). The authors show that inflation aversion leads the Sidrauski (1967) model to deviate from the standard results of long-run money superneutrality and the optimality of the Fredman rule. With inflation aversion, a higher anticipated inflation raises the rate of time preference. Therefore, agents become less patient and are less willing to shift consumption from the present to the future. Inflation aversion will then work with habit formation and durability in governing the agents' intertemporal consumption behaviors. All these future research subjects will allow us to examine the robustness of our results, and to further identify other channels that can affect the growth effect of money and the local stability properties of the economy's balanced growth paths. We plan to pursue these research projects in the near future.

#### APPENDIX

The Hamiltonian is

$$\hat{H} = \underbrace{\frac{(c_t S_t^\eta)^{1-\sigma} - 1}{1-\sigma}}_{=u_t} + \lambda_{1t} [\beta(c_t - S_t)] + \lambda_{2t} [a(m_t/c_t)y_t - c_t - \pi_t m_t + \tau_t].$$

<sup>6</sup>See, for example, Ljungqvist and Uhlig (2000) and Guo (2005).

<sup>7</sup>See, for example, Zou (1994), Zou (1998), and Gong and Zou (2001).

A sufficiency theorem requires that  $\hat{H}$  be jointly concave in the state and control variables (see, for example, Seierstad and Sydsaeter (1977, p. 370)). This is certified when the Hessian

$$H = \begin{bmatrix} \hat{H}_{cc} & \hat{H}_{cS} & \cdot & \cdot \\ \hat{H}_{Sc} & \hat{H}_{SS} & & \\ \cdot & \cdot & 0 & \\ \cdot & \cdot & & 0 \end{bmatrix}$$

is negative definite. This in turn imposes the following restrictions on the feasible values of the structural parameters and their relationships with endogenous variables:

$$\begin{aligned} \hat{H}_{cc} &= u_{cc} + 2 \frac{Aam_t k_t \lambda_{2t}}{c_t^3} < 0, \\ \hat{H}_{SS} &= u_{SS} = [\eta(1-\sigma) - 1] \eta c_t^{1-\sigma} S_t^{\eta(1-\sigma)-2} < 0, \\ \hat{H}_{cc} \hat{H}_{SS} - \hat{H}_{cS}^2 &= u_{cc} u_{SS} - u_{cS}^2 + 2 \frac{Aam_t k_t \lambda_{2t}}{c_t^3} u_{SS} > 0. \end{aligned}$$

To satisfy  $\hat{H}_{SS} < 0$ ,  $\eta < -\frac{1}{\sigma-1}$  should be imposed for the case of habit formation ( $\eta < 0$ ).

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