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Strategic Trading of Informed Trader with Monopoly on Short- and Long-Lived Information

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In his seminal paper, Kyle (1985) analyzed a market model with an informed trader who has monopoly on long-lived information. We consider a market with the same participants as in Kyle's, but where the informed trader has monopoly on two types of information, long-lived one and short-lived one. A necessary and sufficient condition for a market equilibrium in the form of a difference equation system is found, and a method of calculating a solution is suggested. The strategic behavior of the informed trader is analyzed with the emphasis on the interactive effect of the two types of information.

Key Words: Kyle model; Informed trader; Short- and long-lived information; Market equilibrium.
JEL Classification Numbers: G14, D82.

1. INTRODUCTION

Traders buy and sell assets in financial markets based on their own information about future asset values, but the quality of information may differ from one trader to another. This variation in quality of information and its effect on subsequent behavior of traders are partially responsible for price changes. Modeling the effect of asymmetric information and analyzing strategic behavior of informed trader are important in understanding price dynamics in financial markets.

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Kyle (1985) introduced a discrete multi-period model in which there are three kinds of market participants; a risk-neutral trader with monopoly information, liquidity traders whose market orders are determined exogenously, and a market maker who sets the market clearing price. Before the first trading begins, informed trader receives private information about the future asset value which is known only to her for the entire trading period. She submits a market order to market maker at each trading period with the goal of maximizing her total gain. Kyle obtained a linear equilibrium and studied strategic trading of informed trader and how information about asset value is incorporated into market prices. Informed trader tends to restrict orders in early trading periods for a greater profit in the latter periods, and market becomes efficient gradually as informed trader releases private information.

Kyle's seminal work has been extended in various directions. Holden and Subrahmanyan (1992) and Foster and Viswanathan (1993, 1994, 1996) extended Kyle's model by allowing multiple and asymmetrically informed traders. Bernhardt and Miao (2004), Foster and Viswanathan (1990) and Back and Pederson (1998) provided models with long-lived information which is given to informed trader at every period or continuously. Holden and Subrahmanyan (1994) and Baruch (2002) studied behavior of risk-averse informed traders, while Spiegel and Subrahmanyan (1992) and Bernhardt and Massoud (1999) replaced irrational liquidity traders with rational price-sensitive risk-averse traders who determine their market order endogenously.

In contrast, Admati and Pfleiderer (1988) developed a model with informed trader whose monopoly on the information lasts only for one trading period and the information is released to public after the period. Hence, informed trader has no reason to save her private information for the later trade.

We consider a market with the same participants as in Kyle's, but there are two types of information, short-lived one and long-lived one. We assume that a single trader has monopoly on both types of information, partly because the person is likely to be the one with information advantage. In Section 2, we describe the basic model and introduce a necessary and sufficient condition for a linear equilibrium in the form of a difference equation system, and suggest a method of finding solutions. We examine market equilibrium under various initial amount of monopoly information in Section 3. Market efficiency contributed by informed trader is compared to the one when informed trader is given only one of short- or long-lived information. Section 4 concludes the paper.

2. MARKET EQUILIBRIUM

A single security is traded over N trading periods whose value is given by $v = \sum_{n=1}^{N} s_n + l$ where s_n's are independently drawn from a zero mean normal distribution with variance $\sigma_n^2 > 0$, $n = 1, \ldots, N$, and l is normally distributed with mean zero and variance $\tau^2 > 0$ and independent of s_n's. Thus, the market participant's prior belief of asset value is normally distributed with mean zero and variance $\sum_{n=1}^{N} \sigma_n^2 + \tau^2$.

There are three kinds of market participants; an informed trader, liquidity traders and a market maker. The informed trader receives two types of information l and s_1 at the beginning of the first trading period, and s_n at the start of each subsequent period $n = 2, \ldots, N$. At the end of each trading period n, s_n is revealed to public. However, l is kept from public until the last trade is completed. Thus the informed trader accumulates more precise information about the asset value as trade proceeds and has the exact value by the last trading period. Since s_n is given exclusively to the informed trader for only one period, we call it the *short-lived information*, while l is called the *long-lived information*. At each period n, $n = 1, 2, ..., N$, the informed trader determines her market order x_n based on her private information.

Liquidity traders submit market orders u_n, which are independently drawn from a zero mean normal distribution with variance η^2. We assume that u_n are independent of all other random variables in the model, implying that nothing can be learned about asset value from their orders.

Market maker sees total market order $y_n = x_n + u_n$, but *not* x_n and u_n separately, and releases it to public after setting the trading price. He determines the trading price so that it satisfies the *market efficiency condition*: The price is set by the conditional expectation of v, given the market order y_n and the public information available at the time, $\Phi_n = \{s_1, \ldots, s_{n-1}, y_1, y_2, \ldots, y_{n-1}\}$, making market maker's expected profit zero. Hence, the n-th trading price p_n is given by

$$\begin{aligned} p_n &= E[v|\Phi_n,\ y_n] \\ &= E\Big[\sum_{t=1}^{N} s_t \big|\Phi_n,\ y_n\Big] + E[l|\Phi_n,\ y_n] \\ &= \sum_{t=1}^{n-1} s_t + E[s_n|y_n] + E[l|y_1, \ldots, y_n]. \end{aligned} \tag{1}$$

The first two terms in (1) are from the facts that s_t, $t < n$, are public information when the market maker sets the price, that $s_{n+1}, \ldots, s_N$ are independent of $y_1, \ldots, y_n$, and that s_n is independent of $y_1, \ldots, y_{n-1}$. The last term follows from independence of s_t's and l.

Let $\pi_n = \sum_{t=n}^{N}(v-p_t)x_t$ be the profit of the informed trader by strategic trading at periods $n,\ldots,N$. In equilibrium, she would determine her market order $x_n,\ldots,x_N$ in order to maximize the expected value of π_n given her own information $\Phi_n^I = \Phi_n \cup \{l, s_n\}$.

Given the above structure, we shall find the necessary and sufficient condition for linear equilibrium. We first conjecture the informed trader's value function $V_n = \max E[\pi_n|\Phi_n^I]$ at the n-th trading period in the form

$$V_n = \xi_{n-1}s_n^2 + \alpha_{n-1}(l-\bar{l}_{n-1})^2 + \mu_{n-1}s_n(l-\bar{l}_{n-1}) + \psi_{n-1}, \tag{2}$$

where $\bar{l}_0 = 0$, $\bar{l}_{n-1} = E[l|y_1,y_2,\ldots,y_{n-1}]$ for $n = 2,\ldots,N$, and ξ_{n-1}, α_{n-1}, μ_{n-1} and ψ_{n-1} are some parameters to be determined. We then find a linear equilibrium of the form

$$x_n = \beta_{1n}s_n + \beta_{2n}(l-\bar{l}_{n-1}), \tag{3}$$

for some β_{1n} and β_{2n}, and

$$E[s_n|y_n] = \lambda_{1n}y_n \quad \text{and} \quad \bar{l}_n = \bar{l}_{n-1} + \lambda_{2n}y_n, \tag{4}$$

for some λ_{1n} and λ_{2n}, so that the equilibrium price is given by

$$p_n = \sum_{t=1}^{n-1} s_t + \bar{l}_{n-1} + (\lambda_{n1}+\lambda_{n2})y_n. \tag{5}$$

The informed trader's submission (3) shows how she determines her order strategically using private information. She makes market order in proportion to the differences between her own values of s_n and l and their expectations in the market. The intensity of informed trader's trading based on these private information is denoted each by β_{1n} and β_{2n}.

The equilibrium price (5) describes how market maker sets price using publicly known information and market orders submitted to him. The slopes λ_{1n} and λ_{2n} are market maker's prospect on short- and long-lived information. The reciprocal of $\lambda_{1n}+\lambda_{2n}$ is the measure of market depth; higher values of λ's implying that orders have larger impact on the trading price and the market is less liquid.

The conditional variance

$$\bar{\tau}_n^2 = \mathrm{Var}(l|y_1,y_2,\ldots,y_n) = \mathrm{Var}(l-\bar{l}_n)$$

represents the amount of long-lived information that is remaining exclusively to the informed trader after the n-th trade. The difference between $\bar{\tau}_n^2$ and $\bar{\tau}_{n-1}^2$ is the amount of information released to the market through the market order x_n. We may also interpret $\bar{\sigma}_n^2 = \mathrm{Var}(s_n|y_n)$ as the amount of short-lived information remaining to the informed trader, but it is meaningless since s_n becomes public information after the price is set.

The necessary and sufficient condition for a linear equilibrium is given by a recursive relationship among the parameters in the whole periods.

THEOREM 1. *With given variances σ_t^2, $t = 1, \ldots, N$, τ^2, and η^2, the necessary and sufficient condition for linear equilibrium, subject to the terminal values $\alpha_N = \xi_N = \mu_N = \psi_N = 0$, is as follows: For $n = 1, \ldots, N$,*

$$\beta_{1n} = \frac{1}{2\gamma_n}, \qquad \beta_{2n} = \frac{1 - 2\alpha_n\lambda_{2n}}{2\gamma_n} \tag{6}$$

$$\lambda_{1n} = \frac{\beta_{1n}\sigma_n^2}{\beta_{1n}^2\sigma_n^2 + \beta_{2n}^2\bar{\tau}_{n-1}^2 + \eta^2}, \quad \lambda_{2n} = \frac{\beta_{2n}\bar{\tau}_{n-1}^2}{\beta_{1n}^2\sigma_n^2 + \beta_{2n}^2\bar{\tau}_{n-1}^2 + \eta^2} \tag{7}$$

and

$$\bar{\tau}_n^2 = \beta_{2n}^{-1}\lambda_{2n}(\beta_{1n}^2\sigma_n^2 + \eta^2) \tag{8}$$

with the second order condition

$$\gamma_n = \lambda_{1n} + \lambda_{2n}(1 - \alpha_n\lambda_{2n}) > 0.$$

Also, the recursive equations for the parameters of value function are

$$\alpha_{n-1} = \alpha_n + \beta_{2n}(1 - 2\alpha_n\lambda_{2n}) - \beta_{2n}^2\gamma_n \tag{9}$$

$$\xi_{n-1} = \beta_{1n}\{1 - \lambda_{1n}\beta_{1n} - \beta_{1n}\lambda_{2n}(1 - \alpha_n\lambda_{2n})\} \tag{10}$$

$$\mu_{n-1} = \beta_{2n} + \beta_{1n}(1 - 2\alpha_n\lambda_{2n}) - 2\beta_{1n}\beta_{2n}\gamma_n \tag{11}$$

$$\psi_{n-1} = \psi_n + \alpha_n\lambda_{2n}^2\eta^2 + \xi_n\sigma_{n+1}^2. \tag{12}$$

Finally, the conditional variances are given by

$$\bar{\sigma}_n^2 = (1 - \beta_{1n}\lambda_{1n})\sigma_n^2 \tag{13}$$

$$\bar{\tau}_n^2 = (1 - \beta_{2n}\lambda_{2n})\bar{\tau}_{n-1}^2, \quad \bar{\tau}_0^2 = \tau^2. \tag{14}$$

Proof the theorem and a method of finding the solution of the system equation are given in Appendix.

3. STRATEGIC TRADING OF INFORMED TRADER

In this section, we investigate how informed trader trades strategically, and how information is released to market by solving the difference equation system for equilibrium in Theorem 1 for various amounts of short-lived and long-lived information. We are also interested in informed trader's contribution to market efficiency, compared to the case when informed trader has only one of short- and long-lived information.

We consider the case where there are four trading periods and the variance of market orders of liquidity traders is fixed at $\eta^2 = 1$. Inspecting the difference equation system, it can be easily seen that if η is multiplied by some $a > 0$, then λ_{1n} and λ_{2n} are divided by a and β_{1n} and β_{2n} are multiplied by a, while $\bar{\tau}_n^2$ is unchanged. Thus, as the variance of uninformed traders' market order increase, the informed trader's intensity on her private information increases proportionally. We also assume for simplicity that σ_n^2 are the same for all periods.

In order to investigate how equilibrium changes with initial amounts of information, the difference equation system is solved with various combinations of σ_n^2 and τ^2. We fix $\sigma_n^2 = 1$ and set τ^2 to 0.5, 1 and 2, and vice versa. Note that, had there been no long-lived information ($\tau^2 = 0$), it is the same as N-times replication of one-period Kyle model, and hence these parameters should be held constant at $\beta_{1n} = 1$ and $\lambda_{1n} = 0.5$ for all trading periods. And, with no short-lived information ($\sigma_n^2 = 0$), it is the same as N-period Kyle model.

Figure 1 shows the strategic behavior of informed trader. The intensity level β_{1n} on short-lived information (Figure 1(A)) is almost constant but the intensity β_{2n} on long-lived information increases pronouncedly with n (Figure 1(B)), which has been observed by Kyle as well. Informed trader would want to save long-lived information in early periods for more aggressive trading later. We also note that she trades more intensively on short-lived information than long-lived information for all trading periods, except for the last, at which point of time she has no reason to save any information.

Figure 1(A) shows also that the intensity on short-lived information decreases slightly with n, in contrast to Kyle's one-period model where the intensity is held constant (dotted line). As the informed trader has more long-lived information, she tends to trade less intensively on short-lived information. Figure 1(B) reveals that the less short-lived information the trader has, she trades more intensively on her long-lived information, and her tendency of increasing the intensity in latter periods gets more pronounced.

We are also interested in how private information is released to the market by repeated trading. Figures 1(C) and 1(D) show the proportion of information remaining to informed trader, $\bar{\sigma}_n^2/\sigma_n^2$ and $\bar{\tau}_n^2/\tau^2$, for varying levels of short- and long-lived information. The proportion of short-lived information released to the market decreases slightly as trade processes (Figure 1(C)). As the informed trader has more long-lived information, she reveals less short-lived information, and this tendency of withholding

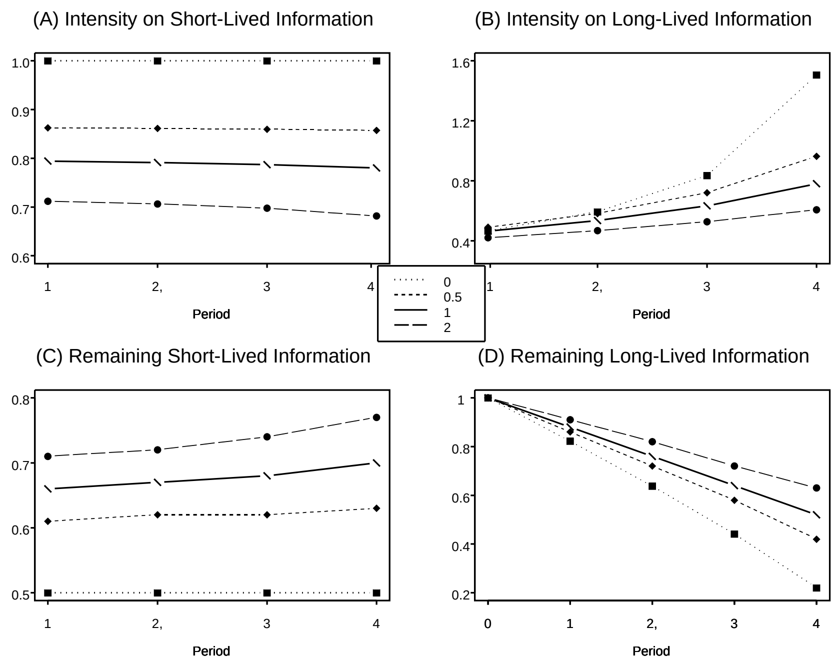

FIG. 1. Strategic trade of informed trader: (A) and (B) show trading intensities on short- and long-lived information. Proportions of remaining private information are drawn in (C) and (D). Lines are drawn with $\sigma_n^2 = 1$ and $\tau^2 = 0, 0.5, 1, 2$, for (A) and (C); $\tau^2 = 1$ and $\sigma_n^2 = 0, 0.5, 1, 2$ for (B) and (D).

short-lived information increases with n and with the amount of long-lived information. Figure 1(D) shows that the proportion of long-lived information released in each period increases with trading periods. The less short-lived information the trader has, the larger proportion of long-lived information is saved in early periods to be released later.

Next, the contribution to market efficiency made by the informed trader who knows both type of information is compared to that of informed trader who has either short- or long-lived information only by looking at the amount of long-lived information remaining to each informed trader. Shown in Figure 2 are conditional variances of long-lived information when informed trader knows only long-lived information, short-lived information and both types of information, respectively. When informed trader has only short-lived information, she only takes short-term profits and plays no role in enhancing market efficiency. When she has only long-lived information, she makes a certain contribution to market efficient by releasing the information to the market. However, when she has both type of information, this contribution is lessened because she is withholding the information. Finally, it has been found that the value of the total information is $V = 2.65$, which is much smaller than the sum of individual values, 2 and 1.46, when only one of (respectively) short- and long-lived information is given.

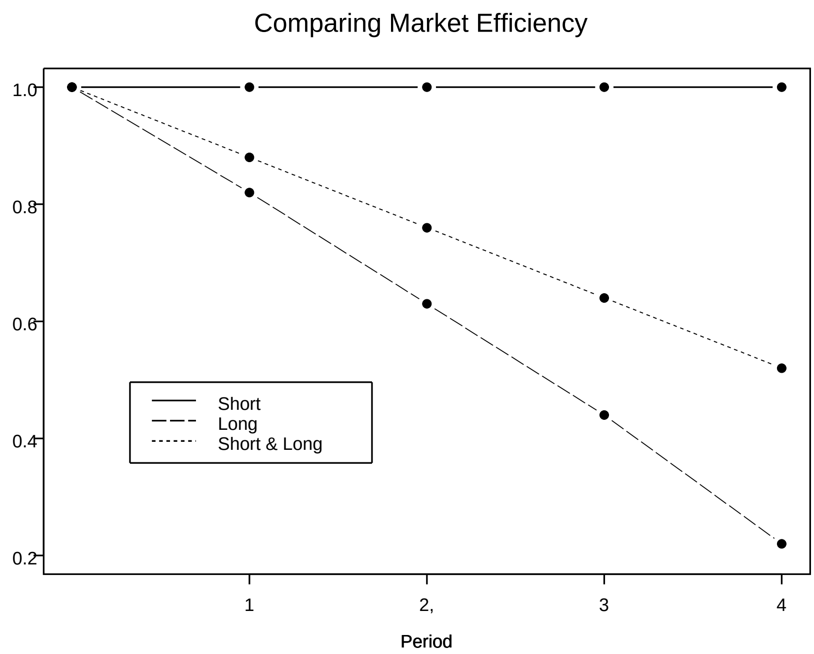

FIG. 2. Remaining Long-Lived Information to each informed trader when $\sigma_n^2 = 1$, and $\tau^2 = 1$ at each period.

The measure of market depth, $1/(\lambda_{n1} + \lambda_{n2})$, is shown in Figure 3, revealing how the information release influences the market. In Admati and Pfleiderer, market is deepest in some period in which informed trader and discretionary liquidity traders concentrate their trading volume. It is shown that market gets deeper as trades proceed and informed trader has less private information. We also note that the market depth is more sensitive to short-lived information.

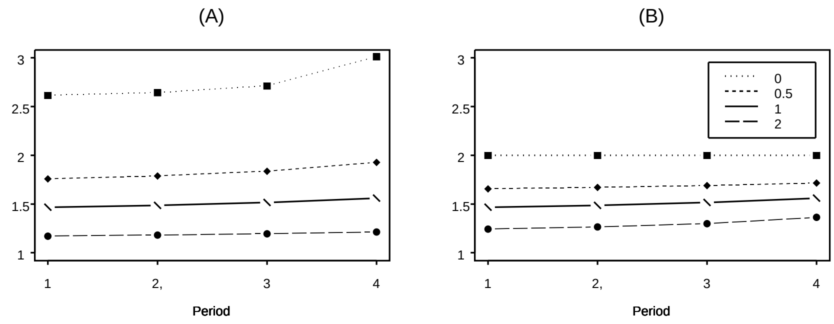

FIG. 3. Market depths (A) with $\tau^2 = 1$ and $\sigma_n^2 = 0, 0.5, 1, 2$, and (B) with $\sigma_n^2 = 1$ and $\tau^2 = 0, 0.5, 1, 2$.

4. CONCLUSION

We considered a market model where there is a single informed trader who has monopoly on both short- and long-lived information and found necessary and sufficient condition for the linear equilibrium which is the form of a difference equation system. We discovered that the amount of one type of information affects informed trader's intensity on the other type of information. She tends to trade less intensively on long(short)-lived information as she has larger amount of short(long)-lived information.

We investigated how private information is released to the market. As trades proceed, informed trader reveals the smaller proportion of short-lived information and the larger proportion of the long-lived information. The less short(long)-lived information informed trader has, the larger proportion of long(short)-lived information is released. We also found that informed trader who has both types of information plays no role in enhancing market efficiency, comparing to the informed trader who has only long-lived information.

We have analyzed in this paper the market model with a single informed trader who has monopoly both on short- and long-lived information. It would be interesting to extend to the model that includes multiple informed traders; they may share the same information or may be asymmetrically informed. Also, while we have considered only perfect information, the market model imperfect information with varying life time may be considered in future studies.

APPENDIX A

A. Proof of Theorem 1: We verify the validity of the value function (2) by induction and find the equilibrium condition. Assume (2) is true for $n+1$. Then,

$$\begin{aligned} V_n &= \max_{x_n} E[(v-p_n)x_n + V_{n+1}|\Phi_n^I] \\ &= \max_{x_n} E[(v-p_n)x_n + \xi_n s_{n+1}^2 \\ &\qquad + \alpha_n(l-\bar{l}_n)^2 + \mu_n s_{n+1}(l-\bar{l}_n) + \psi_n|\Phi_n^I] \end{aligned} \tag{A.1}$$

If we insert (4) and (5) here and compute the conditional expectations, we have

$$\begin{aligned} V_n = \max_{x_n} \big[&\{s_n + (l-\bar{l}_{n-1}) - (\lambda_{1n}+\lambda_{2n})x_n\}x_n + \\ &\alpha_n(l-\bar{l}_{n-1}-\lambda_{2n}x_n)^2 + \alpha_n\lambda_{2n}^2\eta^2 + \xi_n\sigma_{n+1}^2 + \psi_n\big] \end{aligned} \tag{A.2}$$

The first order condition is then given by

$$2\{\lambda_{1n}+\lambda_{2n}(1-\alpha_n\lambda_{2n})\}x_n = s_n + (1-2\alpha_n\lambda_{2n})(l-\bar{l}_{n-1})$$

along with the second order condition

$$\gamma_n = \lambda_{1n} + \lambda_{2n}(1-\alpha_n\lambda_{2n}) > 0, \tag{A.3}$$

and we thus have (6). By the projection theorem for normally distributed random variables, we derive (7) for $\lambda_{1n}, \lambda_{2n}$ and (13), (14) for $\bar{\sigma}_n^2$, $\bar{\tau}_n^2$. From (14), together with (7), we have

$$\begin{aligned} \bar{\tau}_n^2 &= (1-\beta_{2n}\lambda_{2n})\bar{\tau}_{n-1}^2 \\ &= \frac{\beta_{1n}^2\sigma_n^2+\eta^2}{\beta_{1n}^2\sigma_n^2+\beta_{2n}^2\bar{\tau}_{n-1}^2+\eta^2}\,\bar{\tau}_{n-1}^2 \\ &= \frac{\lambda_{2n}}{\beta_{2n}}\,(\beta_{1n}^2\sigma_n^2+\eta^2). \end{aligned}$$

By plugging the optimal order $x_n = \beta_{1n}s_n + \beta_{2n}(l-\bar{l}_{n-1})$ in (A.2) and after comparing corresponding terms, we obtain the recursive relations (9)–(12).

Conversely, it is clear that if parameters satisfy these conditions then x_n is the market order of the informed trader and p_n is trading price set by the market maker in equilibrium. Therefore, it is a necessary and sufficient condition for a linear equilibrium. □

B. Solving the system equations in Theorem 1: For given variances σ_t^2, $t = 1,\ldots,N$, τ^2 and η^2, we solve the system equations starting from the last trading period with the terminal condition $\alpha_N = \xi_N = \mu_N = \psi_N = 0$. For an arbitrary terminal variance $\bar{\tau}_N^2$, we can find the intermediate values of β_{1N}, β_{2N}, λ_N and λ_{2N}, and then obtain the values of $\bar{\tau}_{N-1}^2$ and $\alpha_{N-1}, \xi_{N-1}, \mu_{N-1}$, ψ_{N-1}, by the recursive equations (9)–(12) and (14). We repeat this process backward until the first trading period, and come up with an induced value of $\bar{\tau}_0^2$, which is hence regarded as a function of terminal variance $\bar{\tau}_N^2$. Hence choosing a value of $\bar{\tau}_N^2$ so that $\bar{\tau}_0^2$ is the same as the given value of τ^2 results in an equilibrium.

For each trading period n, let $\bar{\tau}_n^2$, α_n, ξ_n, μ_n, and ψ_n be given. Letting $\phi_n = \beta_{1n}\lambda_{1n}$, we have $\bar{\sigma}_n^2 = (1-\phi_n)\sigma_n^2$. Clearly, ϕ_n satisfies $0 < \phi_n < 1$, and a larger value of ϕ_n implies that more information about s_n is revealed to the market maker by the market order. In the single period model in Kyle (1985) with monopoly informed trader whose information consists only of s_n, ϕ_n is exactly $1/2$. Since the market maker in our model learns less about s_n by the market order in the presence of the long-lived information, we see that $0 < \phi_n < 1/2$.

For given ϕ_n, we have from (6)

$$\beta_{1n} = \frac{\phi_n}{\lambda_{1n}}, \quad \text{and} \quad \beta_{2n} = \frac{\phi_n}{\lambda_{1n}}(1 - 2\alpha_n\lambda_{2n}). \tag{A.4}$$

From (8), we also have

$$\frac{\lambda_{2n}}{\bar{\tau}_n^2}\left(\frac{\phi_n^2}{\lambda_{1n}^2}\sigma_n^2 + \eta^2\right) = \frac{\phi_n}{\lambda_{1n}}(1 - 2\alpha_n\lambda_{2n}) \tag{A.5}$$

Hence, we may eliminate β_{1n} and β_{2n} to have a simpler system equation for λ_{1n} and λ_{2n} only:

$$\begin{aligned} (1-2\phi_n)\lambda_{1n} &= 2\phi_n\lambda_{2n}(1-\alpha_n\lambda_{2n}) \\ \lambda_{2n}\phi_n^2\sigma_n^2 + \lambda_{2n}\lambda_{1n}^2\eta^2 &= \bar{\tau}_n^2\phi_n\lambda_{1n}(1-2\alpha_n\lambda_{2n}) \end{aligned} \tag{A.6}$$

Further eliminating λ_{1n}, we have a fourth-order equation $f(\lambda_{2n}) = 0$, where

$$f(\lambda_{2n}) = \tilde{\phi}_n^2\eta^2\lambda_{2n}^2(1-\alpha_n\lambda_{2n})^2 - \bar{\tau}_n^2\phi_n\tilde{\phi}_n(1-\alpha_n\lambda_{2n})(1-2\alpha_n\lambda_{2n}) + \phi_n^2\sigma_n^2, \tag{A.7}$$

and $\tilde{\phi}_n = 2\phi_n/(1-2\phi_n)$. Since $f(1/2\alpha_n) > 0$ and $f(0) < 0$ if ϕ_n satisfies the condition

$$\underline{\phi}_n < \phi_n < \frac{1}{2}, \quad \text{where} \quad \underline{\phi}_n = \max\left(\frac{1}{2} - \frac{\bar{\tau}_n^2}{\sigma_n^2}, 0\right) \tag{A.8}$$

we see that the equation (A.7) has a single root in $(0, 1/2\alpha_n)$ for any ϕ_n satisfying (A.8), and that it is the only root satisfying the second order condition (A.3). With a such ϕ_n and a root λ_{2n}, we can then compute the values of λ_{1n} from (A.6), β_{1n}, β_{2n} and $\bar{\tau}^2_{n-1}$ from (6) and (14).

The equation (7) provides another way of obtaining λ_{1n} from β_{1n}, β_{2n} and $\bar{\tau}^2_{n-1}$. If the ϕ_n is the value in equilibrium, then the two values of λ_{1n} should be identical. We adopt a search algorithm to find an equilibrium value of ϕ_n. (The existence of an equilibrium value ϕ_n in $(\underline{\phi}_n, 1/2)$ is shown in **C** below.)

Since $\bar{\tau}^2_{n-1} > \bar{\tau}^2_n$, the induced value of $\bar{\tau}^2_0$ increases strictly as $\bar{\tau}^2_N$ increases when other parameters are fixed. Therefore, there exists a value of $\bar{\tau}^2_N$ in $(0, \tau^2)$ that makes $\bar{\tau}^2_0$ computed from $\bar{\tau}^2_N$ the same as τ^2.

C. Existence of Equilibrium: We will show the existence of an equilibrium value of ϕ_n in $(\underline{\phi}_n, 1/2)$ for every trading period n. Let $\lambda_{2n}(\phi_n)$ be a solution of the equation, $f(\lambda_{2n}) = 0$ in $(0, 1/2\alpha_n)$, for given $\phi_n \in (\underline{\phi}_n, 1/2)$, and define

$$g_{1n}(\phi_n) = \frac{2\phi_n}{1-2\phi_n}\lambda_{2n}(\phi_n)(1-\alpha_n\lambda_{2n}(\phi_n)), \tag{A.9}$$

$$g_{2n}(\phi_n) = \frac{\beta_{1n}(\phi_n)\sigma_n^2}{\beta_{1n}^2(\phi_n)\sigma_n^2 + \beta_{2n}^2(\phi_n)\bar{\tau}^2_{n-1}(\phi_n) + \eta^2}, \tag{A.10}$$

where

$$\begin{aligned}
\beta_{1n}(\phi_n) &= \frac{1-2\phi_n}{2\lambda_{2n}(\phi_n)(1-\alpha_n\lambda_{2n}(\phi_n))} \\
\beta_{2n}(\phi_n) &= \beta_{1n}(\phi_n)(1-2\alpha_n\lambda_{2n}(\phi_n)) \\
\bar{\tau}^2_{n-1}(\phi_n) &= \frac{\bar{\tau}^2_n}{1-\beta_{2n}(\phi_n)\lambda_{2n}(\phi_n)}.
\end{aligned}$$

For given ϕ_n, $g_{1n}(\phi_n)$ is the value of λ_{1n} obtained from (A.6), and $g_{2n}(\phi_n)$ is another value of λ_{1n} obtained from (7) using the values, $\beta_{1n}(\phi_n)$, $\beta_{2n}(\phi_n)$, and $\bar{\tau}^2_{n-1}(\phi_n)$. The value of ϕ_n in equilibrium is thus the value ϕ_n^* at which $g_{1n}(\phi_n^*) = g_{2n}(\phi_n^*)$, or the ratio of the two is one. Inserting $\beta_{1n}(\phi)$, $\beta_{2n}(\phi)$ and $\bar{\tau}_{n-1}(\phi)$ into (A.10), we see that the ratio is expressed by

$$r_n(\phi_n) = \frac{g_{1n}(\phi_n)}{g_{2n}(\phi_n)} = \phi_n\Gamma_n(\phi_n),$$

where

$$\Gamma_n(\phi_n) = 1 + (1-2\alpha_n\lambda_{2n})^2\frac{\bar{\tau}^2_{n-1}}{\sigma_n^2} + \frac{4\lambda_{2n}^2(1-\alpha_n\lambda_{2n})^2}{(1-2\phi_n)^2}\frac{\eta^2}{\sigma_n^2}. \tag{A.11}$$

We now investigate the behavior of $r_n(\phi_n)$ near the endpoints in the interval $(\underline{\phi}_n, 1/2)$. But, we first make a remark on $\lambda_{2n}(\phi_n)$.

Proposition 1. *When* $0 < \frac{\bar{\tau}_n^2}{\sigma_n^2} < \frac{1}{2}$, $\lim_{\phi_n \downarrow \underline{\phi}_n} \lambda_{2n}(\phi_n) = 0$. *Also,* $\lim_{\phi_n \uparrow \frac{1}{2}} \frac{\lambda_{2n}(\phi_n)}{1-2\phi_n} = \infty$.

Proof. The first statement is trivial since

$$f(0) = -\frac{2\phi_n^2 \bar{\tau}_n^2}{1-2\phi_n} + \phi_n^2\sigma_n^2 \uparrow 0 \text{ as } \phi_n \downarrow \underline{\phi}_n = \frac{1}{2} - \frac{\bar{\tau}_n^2}{\sigma_n^2},$$

and

$$f'(0) = \frac{6\alpha_n \phi_n^2 \bar{\tau}_n^2}{1-2\phi_n} \to 3\alpha_n^2\sigma_n^2\underline{\phi}_n^2 > 0.$$

For the second, note that, since $\lambda_{2n}(\phi_n)$ is the solution of $f(\lambda_{2n}) = 0$, ϕ_n and λ_{2n} satisfy

$$\eta^2\Big(\frac{2\phi_n}{1-2\phi_n}\Big)^2\lambda_{2n}^2(1-\alpha_n\lambda_{2n})^2 - \bar{\tau}_n^2\frac{2\phi_n^2}{1-2\phi_n}(1-3\alpha_n\lambda_{2n}+2\alpha_n^2\lambda_{2n}^2)+\phi_n^2\sigma_n^2 = 0.$$

Letting $T_n = \frac{\lambda_{2n}}{1-2\phi_n}$, we have

$$4\phi_n^2\{\eta^2(1-\alpha_n\lambda_{2n})^2 - \bar{\tau}_n^2(1-2\phi_n)\}T_n^2 + 6\bar{\tau}_n^2\alpha_n T_n - 2\bar{\tau}_n^2\frac{2\phi_n^2}{1-2\phi_n} + \phi_n^2\sigma_n^2 = 0.$$

Since it is a second-order equation for T_n, we get

$$T_n = \frac{\lambda_{2n}}{1-2\phi_n} = \frac{1}{2}\left(-A \pm \sqrt{A^2 + 4\phi_n^2 B\Big(2\bar{\tau}_n^2\frac{2\phi_n^2}{1-2\phi_n} - \phi_n^2\sigma_n^2\Big)}\right), \tag{A.12}$$

where $A = 3\bar{\tau}_n^2\alpha_n$ and $B = \eta^2(1-\alpha_n\lambda_{2n})^2 - \bar{\tau}_n^2(1-2\phi_n)$. Since $\phi_n \in (0, 1/2)$ and $\lambda_{2n} \in (0, 1/2\alpha_n)$, we see that $T_n > 0$, and the result follows. ▌

We now verify the existence of a root of $r_n(\phi_n) = 1$ by showing that it crosses one in $(\underline{\phi}_n, 1/2)$.

Proposition 2.
There exist ϕ_n^L *and* ϕ_n^R *in* $(\underline{\phi}_n, 1/2)$ *such that* $r_n(\phi_n^L) < 1$ *and* $r_n(\phi_n^R) > 1$. *Thus* $r_n(\phi_n) = 1$ *must have a root in* $(\underline{\phi}_n, 1/2)$.

Proof. Fix n and let $\{\epsilon_m > 0\}$ be a sequence of real numbers monotonely decreasing to 0 where $\epsilon_1 < \min(\bar{\tau}_n^2/\sigma_n^2, 1/4)$. When $\bar{\tau}_n^2/\sigma_n^2 \geq 1/2$ and thus $\underline{\phi}_n = 0$, it is immediate that $\Gamma_n(\epsilon_m)$ is bounded since $\lambda_{2n} \in (0, \frac{1}{2\alpha_n})$,

$\alpha_n > 0$ and $\bar{\tau}^2_{n-1} < \bar{\tau}^2_0 < \infty$. Hence $r_n(\epsilon_m) = \epsilon_m \Gamma_n(\epsilon_m)$ goes to zero as ϵ_m goes to 0, and it implies the existence of ϕ_n^L such that $r_n(\phi_n^L) < 1$.

When $0 < \bar{\tau}^2_n/\sigma^2_n < 1/2$ and thus $\underline{\phi}_n = \frac{1}{2} - \frac{\bar{\tau}^2_n}{\sigma^2_n}$, let $\phi_{nm} = \underline{\phi}_n + \epsilon_m$ and write

$$\Gamma_n(\phi_{nm}) = 1 + \frac{2(1-\alpha_n\lambda_{2n})(1-2\alpha_n\lambda_{2n})^2}{1+2\phi_{nm}(1-2\alpha_n\lambda_{2n})}\frac{\bar{\tau}^2_n}{\sigma^2_n} + \frac{4\lambda^2_{2n}(1-\alpha_n\lambda_{2n})^2}{(1-2\phi_{nm})^2}\frac{\eta^2}{\sigma^2_n}$$

using the relation

$$\bar{\tau}^2_{n-1} = \frac{\bar{\tau}^2_n}{1-\beta_{2n}\lambda_{2n}} = \frac{2(1-\alpha_n\lambda_{2n})}{1+2\phi_{nm}(1-2\alpha_n\lambda_{2n})}\bar{\tau}^2_n.$$

Since λ_{2n} goes to zero as ϕ_{nm} goes to $\underline{\phi}_n$ by Remark 3.1 and $0 < \underline{\phi}_n < 1/2$, we have

$$\lim_{m\to\infty} \phi_{nm}\Gamma_n(\phi_{nm}) = \underline{\phi}_n + \frac{(1-2\underline{\phi}_n)\underline{\phi}_n}{1+2\underline{\phi}_n} < 1,$$

which implies the existence of ϕ_n^L near $\underline{\phi}_n$ such that $r_n(\phi_n^L) < 1$.

Lastly, let $\phi_{nm} = 1/2 - \epsilon_m$. Then the second statement in Proposition 1 implies that $\Gamma(\phi_{nm})$ goes to infinite, and hence there must be a ϕ_n^R with $r_n(\phi_n^R) > 1$. ∎

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