

Credit Market Imperfections and Long-Run Macroeconomic Consequences*

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This paper develops a dynamic general-equilibrium model with production to examine the inter-relationships between the real and the financial sectors with and without credit market imperfections. Due to the moral hazard problem, borrowers may take the money and run while lenders may ration credit, resulting in a widened financial spread and low effective bank loans, compared to the unconstrained equilibrium. Credit rationing causes both the loan and the deposit rates to rise. In either unconstrained or constrained equilibrium, the long-run effects of a productivity improvement on real and financial activities depends crucially on where it is originated.

Key Words: Moral hazard; Credit constraints; Real and financial activities.

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1. INTRODUCTION

It is well-documented empirically and theoretically that the financial and real activities are inter-related.¹ While the empirical evidence concerning the relationship between credit market imperfections and output growth is mixed [e.g., see a summary in Becsi and Wang 1997)], it seems more apparent that credit rationing usually causes the loan rate to rise and the financial spread to widen [e.g., see Tsiang (1980) for the case of Taiwan and Diaz-Alejandro (1985) for the Latin American economies during the 1950s and 1960s].² Are credit market imperfections really harmful to long-run growth? What are the determinants of the financial spread and how is the financial spread related to the rationing on investment loans? This paper attempts to address these important but controversial issues via some plausible dynamic general-equilibrium channels through which credit constraints on firm borrowing can influence long-run macroeconomic performance.³

To facilitate a study of the long-run effects of credit market imperfections on financial returns and economic growth, we design a stylized dynamic general-equilibrium model with two essential features. On the one hand, we fully specify the consumer behavior, the producer behavior and the financial sector so as to understand the determinants of the deposit and loan rates separately. On the other hand, for the purposes of studying the long-run interactions between the real and financial sectors, we differentiate technical progress originated in goods production from that in financial activity. In so doing, we allow for independent sources of financial and economic development through which the consequences of credit market imperfections can be examined.

More specifically, we delineate the environment of the economy with three types of optimizing agents: households, banks and firms. In the basic model, the rate of economic growth is exogenous and individual human capital is a fixed proportion of the society's stock of knowledge. The

¹For example, see a recent survey of a long list of previous studies by Levine (2005).

²More specifically, in the post-WWII period, Taiwan suffered a severe stagnation and its financial intermediation ratio was extremely low. Accompanied by a government policy toward reducing the financial market imperfection, a "Preferential Interest Rate" (at 125% per annum) was established for time deposits and credit ration was loosened. In the next decade, the Taiwan economy stabilized and grew rapidly by the 1960s. In the 1950s, Latin American countries had public development banks granting essentially negative real interest rates to favored borrowers (such as profitable non-traditional industries), leaving non-favored borrowers financing in expensive and unstable informal credit markets. This credit control was associated with negative real interest rates for depositors. Since 1960, several countries have undertaken financial reform, leading to higher savings and economic growth.

³The presence of credit constraints on U.S. entrepreneurs and firms is empirically documented by Evans and Jovanovic (1989) and Gertler and Gilchrist (1994), respectively.

young households work to receive wages and save for consumption during their retirement period. Banks employ labor and undertake financial productivity improvement to convert deposits into loan services. Firms hire workers and obtain loan services from the banking sector to invest in goods-production projects. In the presence of the moral hazard problem in that firms may borrow and abscond without repaying bank loans, it is optimal for banks to ration the credit. We fully characterize the unconstrained and credit-constrained equilibrium and then compare the respective equilibrium outcomes to understand the macroeconomic consequences of credit market imperfections in the steady state.

We find that credit rationing causes both the loan and the deposit rates to rise, and results in a widened financial spread and lowers effective bank loans. In general, credit rationing need not harm the real sector. Moreover, changes in productivity of real and financial sectors can yield different comparative statics in an unconstrained equilibrium, while such changes are qualitatively similar in credit-constrained equilibrium.

Literature Review

There are some remotely related papers to the present work, studying credit rationing on education loans or consumption loans. In modeling education loans, Tsiddon (1992) finds that, in a moral-hazard low-growth trap with credit rationing, economic growth is low and the interest rate is high, whereas Fender and Wang (2003) and Fender (2005) use an occupational choice framework to establish that credit rationing is associated with low education and low interest rates. In modeling consumption loans to motivate their empirical study, Jappelli and Pagano (1994) argue that credit constraints encourage young consumers to save and can thus spur economic growth.

Focusing on (physical capital) investment loans, one may motivate credit market imperfections in two distinct ways: incomplete markets and asymmetric information, where the latter contains adverse selection and moral hazard models. With regard to incomplete markets, Aghion et al. (2005) conclude that credit rationing lowers the interest rate and relocates investments from long term to short term, thereby reducing the mean growth rate. Using a pure exchange model with adverse selection, Azariadis and Smith (1993) find that credit rationing raises individual savings and thus reduces the interest rate. By allowing for physical capital accumulation, Ben-civenga and Smith (1993) and Hung (2005) argue that adverse selection-induced credit rationing is growth-retarding.

Perhaps the most closely related study is Aghion and Bolton (1997), in which credit rationing on investment loans occurs as a result of a moral hazard problem, as in our paper. The focus of their paper is however on how wealth distribution and evolution may influence individuals' occupational choice to become borrowers or lenders and hence the equilibrium

credit rationing outcomes. An interesting finding of their work is that both the rich and the poor become lenders whereas the middle class become borrowers.

In contrast with the aforementioned literature, our paper specifies completely an active financial sector, determining endogenously the deposit as well as the loan rates and differentiating technical progress originated in goods production from that in financial services. In so doing, we can study how the financial spread between the loan and the deposit rates responds to the credit market conditions. Moreover, we are able to differentiate changes in productivity originated from the real vs. the financial sectors, which enables us to evaluate their long-run effects on real outputs and interest rates.

2. THE BASIC MODEL

Time (indexed by t) is discrete. There are three separate theaters of economic activities: (i) each 2-period lived overlapping household (consumer/worker) is endowed with a unit of labor when young, who deposits wage incomes for future consumption, (ii) each infinitely lived producer is endowed with a production technology to manufacture the single final good using physical capital and credit facilitated by financial intermediation, and (iii) the financial sector simply converts banking deposits into loans.⁴ There is a continuum of each type of economic agents (households, firms and banks) with unit mass.

Chart 1: The Sequence of Events

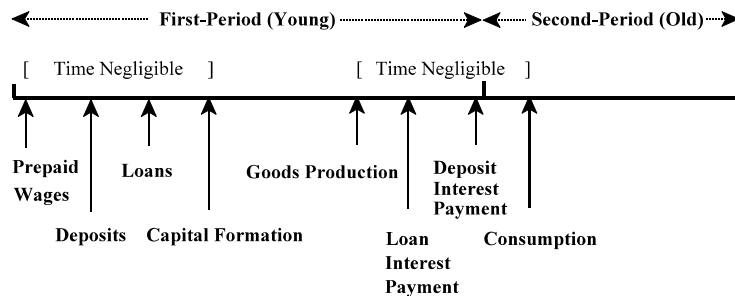


Chart 1 displays the sequence of events. When young, a household works, receives pre-paid wages (apple tree) and deposits it to the financial sector. A bank then provides a loan (apple tree) to a goods producer, which subsequently manufactures the final good (apple) and pays back the loan with

⁴We ignore, for the sake of simplicity, firm deposits and consumer loans. Indeed, one may reinterpret our bank loans as net loans (investment loans net of deposits) and bank deposits as net deposits (consumer deposits net of loans).

interests (in apples). Finally, the banks pay back the deposits with interests (in apples) to households at the end of the first period and the latter consume at the beginning of the second period (time is negligible between the end of the first and the beginning of the second periods).

2.1. Households

Each household of generation t possesses a fraction of the society's knowledge stock and a unit time endowment when young, while consuming only during the second period. The latter assumption creates forced savings, which simplifies the analysis greatly. At the end of Section 4, we discuss the consequences of relaxing this assumption by allowing savings in kind via intergenerational human capital accumulation. In the concluding section, we also elaborate on a potentially positive savings effect of credit rationing with endogenous intertemporal consumption tradeoffs.

Each household allocates one unit of time endowment to goods production (ℓ) and bank operation ($1-\ell$). Assuming perfect substitution, workers are always indifferent between the two activities. In the benchmark setup, we consider exogenous growth of a labor-augmenting technology (denoted by h), which can be thought of as an embodied knowledge stock growing at a constant rate $g > 0$ across generations, that is, $h_{t+1} = (1+g)h_t$.

Since the focus of the paper is not on the formation of financial intermediation, we simply assume that all savings are channeled through the banking sector. Each household has an identical preference that is monotone increasing in consumption (c_{t+1}). In the absence of bequests, the representative household born in period t will consume all saved from the first period plus the interests (at a real deposit rate r_{t+1}).

2.2. Producers

Each producer utilizes current capital stock (k_t) and effective labor input ($\ell_t h_t$) to produce a single final good (y_t) which can be allocated to investment demand (i_t) and consumption goods supply (z_t). The production technology takes the Cobb-Douglas form: $y_t = Ak_t^\alpha (\ell_t h_t)^{1-\alpha}$ where $A > 0$ and $\alpha \in (0, 1/2)$.⁵ Next, we assume that producers are capable of converting bank loans (x_t) into fixed capital formation in such an efficient fashion that $i_t = (1+\theta)x_t$ where $\theta > 0$. Implicitly, this more-than-proportional conversion captures the potential effect of external financing on real investment decisions as a result of bank's effective monitoring.⁶ This setup is also consistent with a fractional loan-in-advance model as one unit invest-

⁵The constant-returns assumption is made so that the model accepts balanced growth in the endogenous growth case, whereas the restriction that the capital share is less than half is consistent with empirical observations.

⁶The use of financial instruments for a firm's capital formation has been emphasized since the classic work of Robinson (1969, ch. 4).

ment requires less-than-one unit bank loan. This highly stylized structure may also be viewed to capture the potential liquidity management role of financial intermediation in the sense that financial loans enable capital deepening, leading to a higher rate of returns.

We also assume that the production transformation schedule is linear so the technology applies to both capital formation and consumption good production. Moreover, we follow Diamond and Yellin (1990) assuming that the goods producer is a residual claimer, i.e., it ingests the unsold consumption goods in a fashion consistent with lifetime value maximization. This ownership assumption avoids the unnecessary Arrow-Debreu redistribution from firms to consumers while maintaining the general equilibrium nature. Moreover, as we can see below this setup is consistent with conventional profit maximization where firms rent capital from external sources.

Denote δ as the loan rate of interest and w as the effective wage rate. Then, the representative producer, at any given time t , will choose consumption goods supply, loan demand and labor demand to maximize its value (sum of present-discounted gross profit flows) subject to the capital evolution equation:

$$V(k_t) = \max_{\{i_\tau, x_\tau, \ell_\tau\}} \sum_{\tau=t}^{\infty} \left(\frac{1}{1+\sigma} \right)^\tau [y_\tau - (1+\delta_\tau)x_\tau - w_\tau \ell_\tau h_\tau]$$

$$s.t. \quad k_{\tau+1} = i_\tau + (1-d)k_\tau \tag{1}$$

$$i_\tau = (1+\theta)x_\tau \tag{2}$$

where $\tau > 0$ denotes firm owner's (constant) rate of subjective discount and $d > 0$ the (constant) rate of capital depreciation, and recall that $y_\tau = Ak_\tau^\alpha (\ell_\tau h_\tau)^{1-\alpha}$. The gross value $V(k_t)$ is crucial in determining the incentive constraint in the presence of moral hazard behavior. Under the specification of the technology and the assumption of the ownership structure, the factor demand functions are greatly simplified and the gross value of firm can be shown linear in k (and hence in x).

2.3. Banks

The reader may be reminded that this banking sector is designed mainly to specify the financial flows and to differentiate deposit from loan rates. Thus, we consider an extremely simple structure where each bank provides loan-deposit services to maximize periodic profits.⁷ The bank's operation (including, for example, monitoring firm's investment project and managing deposits and loans) is assumed to require only labor inputs, taking a

⁷The considerations of a infinitely lived bank would just complicate the analysis without altering the main findings.

fixed coefficient form.⁸ Specifically, we assume that $x_t \leq a_t$ units of loans requires $1/\phi > 0$ units of effective labor, i.e., $(1 - \ell_t)h_t = (1/\phi)x_t$, where ϕ can be regarded as the cost-saving banking innovation. Physical capital is excluded here for analytical convenience; by considering physical capital as an input in banking operation, the main results remain qualitatively unchanged as long as the banking sector is more labor intensive than the goods sector. Additionally, we assume that all consumer savings are financially intermediated and thus the bank deposit (a_t) becomes: $a_t = w_t h_t$.⁹

Under this setup, banks take deposits and effective labor as inputs, providing loan services to the real sector as intermediate inputs to subsequently produce the final goods. Here, deposits are transformed to loans via a costly financial intermediation process. In contrast to most of the existing literature, we allow the financial intermediation costs to depend on effective labor. The reallocation of labor (between goods production and bank operation) plays a crucial role in determining the effects of credit rationing and changes in the productivity and cost parameters on the real and financial activities as well as the loan and deposit rates.

At any given period t , each bank earns profit flow from loan interest receipts ($\delta_t x_t$), net of the interest payments to its depositors ($r_t a_t$) and the labor cost ($w_t(1 - \ell_t)h_t$):

$$\max_{\{x_t \leq a_t\}} \delta_t x_t - r_t a_t - w_t(1 - \ell_t)h_t = \left(\delta_t - \frac{w_t}{\phi} \right) x_t - r_t a_t \quad (3)$$

where in its optimization that determines loan supply, each bank takes the amount of deposits as parametrically given. Obviously, by examining the flow of funds, the total amount of loans must not be greater than the total amount of deposits available from household savings.

⁸For simplicity, we consider only labor input in financial services. With capital entering financial production, our main results still hold as long as the financial sector is labor intensive compared to the goods sector.

⁹The current framework ignores the conventionally emphasized roles of financial intermediation in liquidity management (cf. Bencivenga and Smith 1991) and risk-pooling (cf. Greenwood and Jovanovic 1990). This is because the primary goal of this paper is to study the consequences of credit market imperfections rather than the emergence of financial intermediation. Nevertheless, the introduction of liquidity management may indeed reinforce the negative effect of credit rationing on real activities if long-term high-return investment projects are associated with higher degree of riskiness. The incorporation of risk-pooling, on the other hand, has no direct long-run consequence on the moral hazard induced detrimental effect of credit rationing. Moreover, none of these additional roles are qualitatively critical to our results concerning interest rates.

2.4. Optimization

We begin with the optimizing behavior of the representative household, which is trivial:

$$c_t = \frac{(1+r_t)w_{t-1}h_t}{1+g} \quad (4)$$

where in deriving this we have used the evolution equation of exogenous knowledge stock $h_t = (1+g)h_{t-1}$. That is, a household when young saves its entire working income for consumption when old, in which the total amount of consumption equals the sum of the principal and the interest.

To solve a firm's optimization problem, we apply the production function, (1) and (2) and utilize the now-standard dynamic programming technique transforming the infinite-horizon problem into the recursive Bellman equation:

$$V(k_t) = \max_{\{i_t, \ell_t\}} \left[Ak_t^\alpha (\ell_t h_t)^{1-\alpha} - \left(\frac{1+\delta_t}{1+\theta} \right) i_t - w_t \ell_t h_t \right] + \frac{1}{1+\sigma} V(i_t + (1-d)k_t) \quad (5)$$

The first-order conditions for the firms optimization imply (see Appendix A):

$$\frac{k_t}{\ell_t h_t} = \left[\frac{\sigma+d}{\alpha A} \left(\frac{1+\delta_t}{1+\theta} \right) \right]^{-\frac{1}{1-\alpha}} \quad (6)$$

$$(1-\alpha)A^{\frac{1}{1-\alpha}} \left[\frac{\sigma+d}{\alpha} \left(\frac{1+\delta_t}{1+\theta} \right) \right]^{\frac{\alpha}{1-\alpha}} = w_t \quad (7)$$

In effect, (6) equates the marginal benefit of capital with its marginal cost, while (7) equates the marginal benefit of labor employment with its wage cost. Moreover, as we show in the Appendix A the gross value of firm can be expressed as:

$$V(k_t) = (1+\sigma) \left(\frac{1+\delta_t}{1+\theta} \right) k_t \quad (8)$$

Thus, the firm's value is linear in k and the firm value per effective unit (V/h) is bounded along a balanced growth path (with k and h growing at a common rate) as long as the loan rate is bounded.

From the bank's competitive profit condition, one gets:

$$r_t = \left(\delta_t - \frac{w_t}{\phi} \right) \frac{x_t}{w_t h_t} \quad (9)$$

which can be rewritten in terms of the net financial mark-up (or financial spread), defined as the ratio of the loan rate net of unit bank operation cost to the deposit rate:

$$\frac{\delta_t - w_t/\phi}{r_t} = \frac{w_t h_t}{x_t} = \frac{1}{x_t/a_t} \quad (10)$$

This suggests that the financial mark-up and the loan-deposit ratio are inversely related. We now illustrate the bank's optimizing conditions. Since the bank's objective function is linear in x , loan supply must reach the upper bound as long as $\delta > w/\phi$ (which is true under the bank's competitive profit condition):

$$x_t = a_t = w_t h_t \quad (11)$$

Thus, this implies the financial spread be unity. Finally, substituting (11) into the unit labor requirement equation of banking operation yields:

$$\ell_t = 1 - \frac{w_t}{\phi} = 1 - \frac{1}{\phi} \frac{x_t}{h_t} \quad (12)$$

3. BALANCED GROWTH EQUILIBRIUM

We are now prepared to solve for the balanced growth equilibrium. Before proving the existence and deriving the comparative statics, we outline a number of equilibrium conditions. There is one more equilibrium condition in addition to those already imposed implicitly in Section 2, including labor, deposit and loan market equilibrium and bank's zero profit condition. Goods market equilibrium requires that the total output be divided into investment and consumption. Since firm owners are residual claimers, goods demand must be equal to goods supply. Feasibility of goods allocation thus requires the following inequality to hold: $y_t - i_t = z_t \geq c_t$. With regard to feasible labor allocation, it requires that $\ell \in (0, 1)$. From (7) and (12), $\ell < 1$ is always satisfied. In order to guarantee $\ell > 0$, from (7) and (12), we impose:

Condition L: (Feasible Labor Allocation)

$$\delta \geq \delta^{\min} \equiv (1 + \theta) \left[A^{\frac{1}{\alpha}} \left(\frac{\alpha}{\sigma + d} \right) \left(\frac{1 - \alpha}{\phi} \right)^{\frac{1 - \alpha}{\alpha}} \right] - 1.$$

This condition sets a lower bound for the equilibrium value of the loan rate.

3.1. Existence

We begin by defining the concepts of equilibrium and balanced growth equilibrium.

DEFINITION 3.1. A perfect foresight equilibrium (PFE) is a tuple $\{c_t, i_t, k_t, a_t, x_t, V_t, \ell_t, \delta_t, r_t, w_t\}_{t=1}^\infty$ such that (i) each of the representative agents (household, firm and bank) optimizes, (ii) each bank reaches zero profits, and (iii) labor, deposit, loans and goods markets all clear; that is, conditions (1), (2), (4), (6)-(9), (11) and (12) are met.

DEFINITION 3.2. A perfect foresight balanced growth equilibrium (PFBGE) is a PFE such that all quantity variables, c, k, x and V grow at the same rate g as the public knowledge stock h , i.e., $m_{t+1} = gm_t, \forall t \geq 1$ and $m = c, k, a, x, V$, all price variables and time allocation ℓ are constant.

Using (1) and (2) with the definition of balanced growth, we obtain:

$$\frac{k}{h} = \left(\frac{1 + \theta}{g + d} \right) \frac{x}{h} \tag{13}$$

which can be used together with the production function, (1), (4), (6) and (11) to express the goods feasibility condition as: $1 + \delta_t \geq \alpha[(g + d)/(\sigma + d)][1 + \theta + (1 + r_t)/(1 + g)]$. Using (7), (9) and (11), we show in the Appendix B that the above inequality is guaranteed by,¹⁰

Condition F: (Feasibility) $A \left[(1 + g) \frac{\sigma + d}{g + d} - \alpha \right]^\alpha \geq (1 + g)[\alpha(\sigma + d)]^\alpha [\phi(1 + \theta)]^{1 - \alpha}$.

To obtain a PFBGE, we proceed in a recursive manner. First, substituting (7) into (11), we obtain a “labor efficiency” (LE) schedule given deposit and loan market equilibrium:

$$\frac{x}{h} = (1 - \alpha)A^{\frac{1}{1 - \alpha}} \left[\frac{\sigma + d}{\alpha} \left(\frac{1 + \delta}{1 + \theta} \right) \right]^{\frac{-\alpha}{1 - \alpha}} \tag{14}$$

which is obviously downward-sloping in $(x/h, \delta)$ space. Intuitively, when the loan rate is higher, capital accumulation slows down and by Pareto complementarity, the marginal product of labor is lower. As a consequence, the wage rate decreases and, given the fixed time endowment, labor income also reduces. Thus deposits and loan in effective unit are both lower, justifying the negative slope of the LE locus.

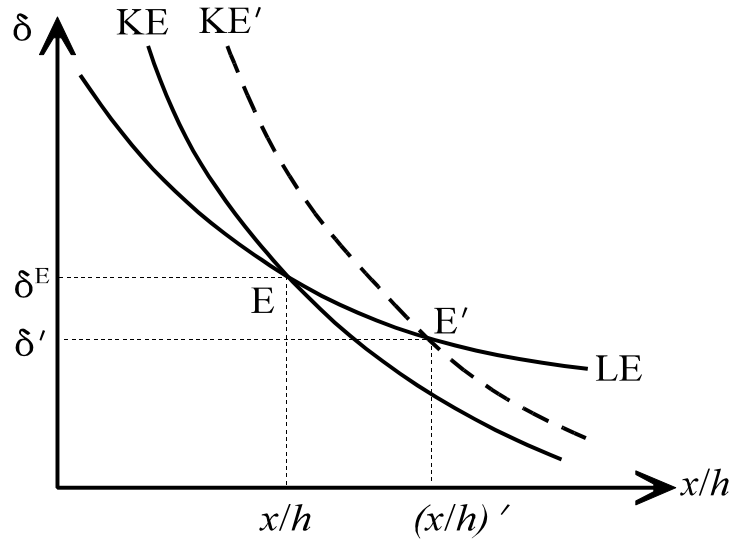
¹⁰This condition is sufficient but not necessary.

Then, utilizing (12) and (13), one can rewrite (6) to derive a “capital efficiency” (KE) schedule given loan demand as well as labor and goods market equilibrium:

$$\frac{x}{h} = \left\{ \frac{1}{\phi} + \frac{1+\theta}{g+d} \left[\frac{\sigma+d}{\alpha A} \left(\frac{1+\delta}{1+\theta} \right) \right]^{\frac{1}{1-\alpha}} \right\}^{-1} \tag{15}$$

which is also downward-sloping in $(x/h, \delta)$ space. When the loan rate increases, capital accumulation becomes more costly and hence producers undertake factor substitution leading to a lower capital-labor ratio. Along the balanced growth path, the physical capital stock is increasing in loans. Under the fixed coefficient technology, labor in the banking sector is increasing in loans, so labor devoted to goods production is decreasing in loans. Therefore, the capital-labor ratio in the goods sector is unambiguously a monotone increasing function of loans. A reduction in the capital-labor ratio must be accompanied by a decrease in loans (per effective unit), which implies a downward-sloping KE locus.

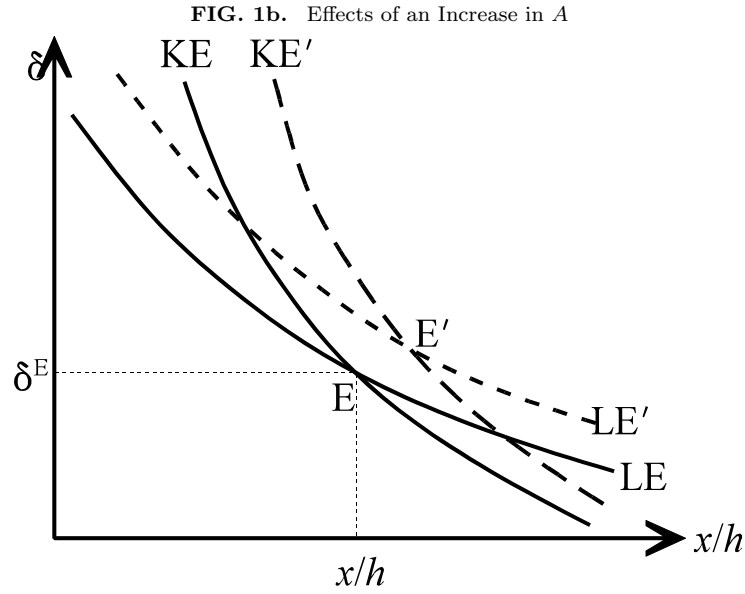
FIG. 1a. Effects of an Increase in ϕ and g



We now plot the LE and KE loci in Figure 1 for the case where KE is steeper than LE, i.e.,

Condition S: (Slope Condition)

$$\delta \leq \delta^{\max} \equiv (1+\theta) \left[\left(\frac{g+d}{1+\theta} \frac{1-\alpha}{\phi} \right)^{1-\alpha} \frac{\alpha A}{\sigma+d} \right] - 1.$$



Condition S essentially implies that when financial loan-driven capital formation becomes more efficient, the (gross) rate of returns on capital increases.¹¹ This is in the spirit of the Samuelson Correspondence Principle, ensuring that the direct effect dominates. Such a condition is imposed particularly for obtaining sensible comparative statics.

These two loci jointly determine the balanced growth equilibrium values of δ (denoted δ^E) and x/h (see the equilibrium point E in Figure 1 and detailed proof in the Appendix B), provided that:

Condition E: (Existence) $A^{\frac{1}{\alpha}} < \phi^{\frac{1-\alpha}{\alpha}} \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{g+d}{1+\theta}\right)$.

Then the equilibrium values of δ and x/h can be substituted into (2), (4), (6)-(9) and (11)-(13) to solve for other equilibrium quantities and prices. In particular, the deposit rate of interest can be derived as:

$$r = \delta - \frac{1-\alpha}{\phi} A^{\frac{1}{1-\alpha}} \left[\frac{\sigma+d}{\alpha} \frac{1+\delta}{1+\theta} \right]^{-\frac{\alpha}{1-\alpha}} \tag{16}$$

which is strictly increasing in δ . In summary, we have:¹²

¹¹To see this, define the gross rate of returns on capital as: $\Delta \equiv (1+\delta)/(1+\theta)$. Both the KE and LE loci are downward-sloping in $(x/h, \Delta)$ space and it is easily seen that $d\Delta/d\theta > 0$ iff Condition S is met. For detailed derivation, see the Appendix B.

¹²One may wonder if plausible sets of parameters would satisfy all the conditions required for existence. The answer is certainly positive. For example, we provide below

PROPOSITION 1. (*Existence*) Under Conditions E , F , L and S , there is a perfect foresight balanced growth equilibrium.

3.2. Comparative Statics

Straightforward comparative-static analysis enables us to examine how changes in A , g and ϕ affect the endogenous variables of our particular interest, including x/h , δ and r . Figures 1a and 1b display diagrammatically the effects of an increase in ϕ and A , respectively. The focus of the paper is to establish an unconstrained equilibrium and to illustrate the different effects of sectoral productivity changes. Thus, for brevity we omit other comparative static exercises.

When banking production becomes more efficient (i.e., ϕ increases), labor saving induces a reallocation from banking to goods sector, leaving the labor efficiency locus unchanged. Since labor and capital are Pareto-complements and capital formation is based on a fixed coefficient technology in terms of bank loans, the demand for loans increases for a given level of the loan rate. As a consequence, the KE locus shifts rightwards (see the new KE' locus and the new equilibrium point E' in Figure 1a). These imply higher loans per effective unit (x/h) and lower loan and deposit rates (δ and r). Intuitively, by Pareto complementarity, the wage rate responds positively to bank loans; the resultant increase in the wage rate leads to a decrease in the loan rate due to the standard downward-sloping factor price frontier (i.e., factor substitution). The decrease in the deposit rate is a result of bank's zero profit condition. Thus, a cost-reducing bank innovation enables more bank loans and higher capital formation. These findings are consistent with the Schumpeterian view of financial development.

In response to a higher labor-augmenting technical progress rate (i.e., g increases), the marginal product of labor in the goods sector is higher. This also causes labor reallocation from banking to goods sector, thereby leading to similar comparative statics to the case of more efficient banking production.

An increase in the goods production scaling factor A encourages sectoral reallocation toward goods production, leading to a higher demand for labor and demand for loans. The former causes a rightward shift of the LE locus (to LE') whereas the induced demand for loans enhances capital accumulation and causes an rightward shift of the KE locus (to KE'). The new equilibrium point is thus at E' (see Figure 1b) and the effects on the effective loans and the loan rate are generally ambiguous. Intuitively, a higher

a list of parameter values which are either commonly used in calibrated models or chosen for normalization purposes): $\alpha = 1/3$, $g = \sigma = 0.025$, $d = \theta = 0.05$, $A = 1$, $\phi = 3$. Under this parametrization, all conditions are met.

goods productivity induces capital and hence loan demand. By Pareto complementarity and factor substitution, the loan rate is lower (similar to what described above). Through factor reallocation between sectors, more labor is devoted to goods production and by diminishing returns the marginal product of labor decreases. By Pareto complementarity, this lowers the demand for capital and bank loans, offsetting the direct productivity effect. Moreover, the wage rate is lower in response to a lower marginal product of labor. Using the relationship of factor price frontier (factor substitution), the loan rate must be higher, also offsetting previous effect from induced demand.

PROPOSITION 2. (*Characterization of the Unconstrained Equilibrium*) Under Conditions E , F , L and S , the uniquely determined PFBGE possesses the following properties:

(i) a cost-reducing banking innovation or a labor-augmenting technical progress increases bank loans per effective unit, enhances capital formation, and lowers the loan and the deposit rates;

(ii) a more efficient goods production has ambiguous effects on bank loans, capital formation and loan and deposit rates.

Interestingly, while the rate of labor-augmenting technical progress generates unambiguous comparative-static outcomes similar to banking productivity, higher real sector productivity yields generally ambiguous long-run effects.

Remark 3.1. One may easily extend the basic framework to allow for endogenous growth. A natural way is to consider endogenous intergenerational knowledge accumulation via parental time devoted to children's education following Glomm and Ravikumar (1992) where parents are concerned with children's knowledge capital (i.e., h_{t+1} enters generation t 's utility). In response to an increase in banking productivity (ϕ), labor reallocates from banking to children education and hence raises economic growth. Similarly, higher real sector productivity (A) increases the marginal product of knowledge, thereby encouraging more children education and leading to higher growth.

4. MORAL HAZARD AND CREDIT-CONSTRAINED EQUILIBRIUM

We turn next to examine what happens if moral hazard causes banks to ration investment loans. Conventionally, moral hazard behavior is modeled

as for borrowers to take excessively risky projects after obtaining the loan from banks in that the lender can ensure that the money is invested but cannot appropriate the return (see Hart and Moore 1994 and papers cited therein). In this paper, we adopt a parsimonious form of moral hazard that is sufficient to capture Keynes' (1964) consideration of the lender's risk.¹³ Specifically, the lender cannot ensure that the money lent is indeed invested and thus fails to ensure the repayment. An individual firm, in anticipating a low rate of returns on productive investment, may have an incentive to "take the money and run" (i.e., to abscond), without repaying the loan.¹⁴ While both approaches generate the possibility of credit rationing, the latter is analytically much simpler. Moreover, we may reinterpret the absconding story as one similar to Hart and Moore in terms of ex post effort. Consider a borrower to exert an effort on an investment project upon obtaining the loan. Then absconding is equivalent to assuming such an effort is a step function, taking values of 0 and 1 only (the value of 0 implies absconding while the value of 1 means undertaking the investment).

Assume that failing to repay the loan, an individual firm owner would have part of the productive capital stock seized and this fraction is denoted by η (later referred to as the unit absconding cost). Thus, the cost of absconding is measured by ηk and the value of taking the money (the amount of loan, x) and run is $x + (1 - \eta)k$. Moreover, by absconding an individual producer would lose the value accrued from goods production and hence the value of production measures the opportunity cost of absconding.¹⁵ The incentive compatibility constraint (IC) that eliminates this moral hazard behavior is therefore given by:

$$V(k) \geq x + (1 - \eta)l \quad (17)$$

which implies that the value of undertaking production exceeds the net value of absconding.¹⁶ Using (8) and (13), we can rewrite (17) as:

$$\delta \geq D \equiv \frac{1}{1 + \sigma} [(g + d) + (1 - \eta)(1 + \theta)] - 1 \quad (18)$$

¹³Banerjee and Newman (1993) illustrate this type of moral hazard problem: "[an agent may] attempt to avoid his obligations by fleeing from his village, albeit at the cost of lost collateral" (p. 280). For further discussion, the reader is referred to Fender and Wang (2003).

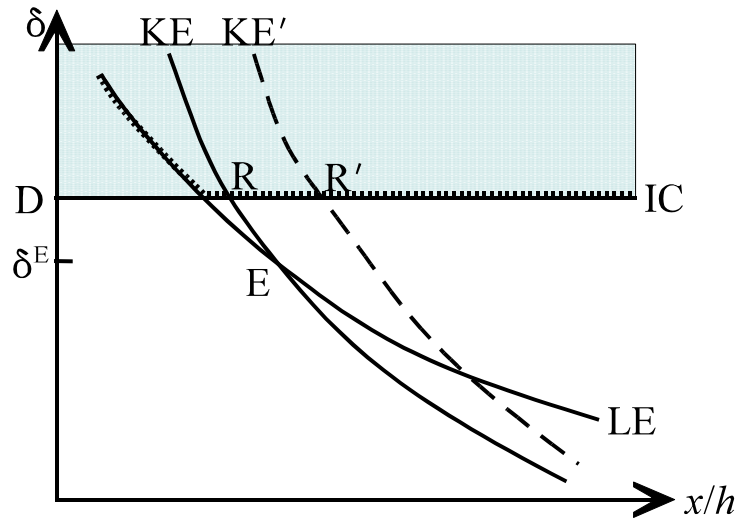
¹⁴In modeling the absconding behavior, it is not necessary to fully specify the structure of uncertainty. Of course, the necessity for the moral hazard behavior to occur is that banks cannot detect the possibility of absconding ex ante. The incentive compatibility constraint can be thus written without probabilistic measurement.

¹⁵Our setup follows Kehoe and Levine (1993) in which "creditors can seize the assets of debtors who default on their debts" (p. 869).

¹⁶The reader may find our incentive compatibility constraint is analogous to the limited liability constraint (PAii) in Sappington (1983, p. 6).

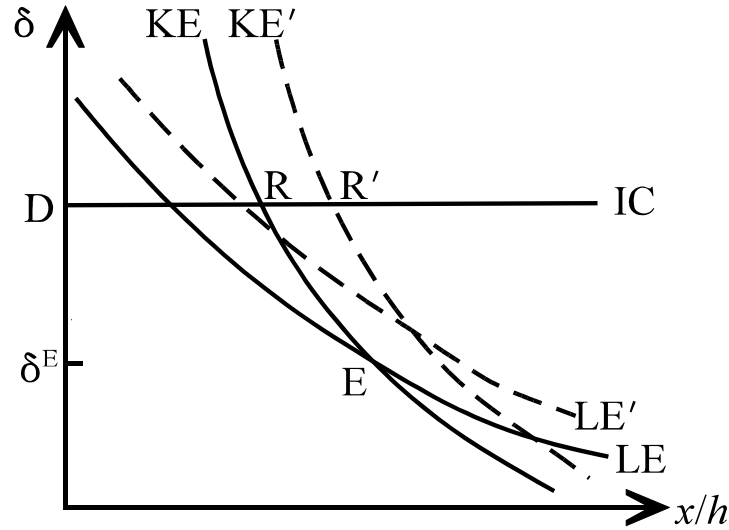
Notably, if D exceeds the unconstrained loan rate δ^E , credit rationing is present. In this case, equation (11) is no longer applicable, as is the LE schedule (14). Thus, the “effective” labor efficiency locus is now represented by the kinked dash line in Figure 2a where the shaded area represents the effective region in which the incentive compatibility constraint is met. Our framework is obviously different from the static, partial-equilibrium loanable funds model of Stiglitz and Weiss (1981) in which an increase in the loan rate encourages firms to undertake riskier projects. In our equilibrium credit rationing model in the absence of an explicit specification of riskiness, banks’ profit maximization implies a desire to have the maximum amount of loans subject to the incentive compatibility condition and other equilibrium conditions. Since an equilibrium must be along the downward-sloping capital efficiency locus, the maximum amount of incentive-compatible loans is attained at point R (see Figure 2) where $\delta = D$. Thus, in equilibrium, the incentive compatibility constraint (17) is binding, while equilibrium credit ration occurs in the sense that the amount of loans is below the unconstrained level (indicated by point E). The moral hazard behavior is not observed in equilibrium and the credit-constrained equilibrium is associated with an loan rate higher than that in the unconstrained equilibrium.¹⁷

FIG. 2a. Effects of an Increase in ϕ and g with Credit Rationing



¹⁷Similar results are obtained in the moral hazard trap model of Tsiddon (1992) under a very different setting.

FIG. 2b. Effects of an increase in A with Credit Rationing



The reader may ask if the loan contract specified above is optimal under our competitive setting. On the one hand, no individual bank would undercut in the loan rate, since it would obviously result in a moral hazard problem, leading to a failure of loan repayments. On the other hand, if an individual bank would offer a higher loan rate, it would end up with no customers. Thus, the credit-constrained loan rate must be equal to the exogenous value D with no individual banks deviating in equilibrium.

From the above discussion, we learn that the necessary and sufficient condition for the presence of credit rationing is to have the unconstrained equilibrium loan rate δ^E below D . From Proposition 2 and the definition of D in (18), it is easily seen that in order for credit rationing to occur, we need the unit absconding cost (η) and the unit labor requirement of bank operation ($1/\phi$) to be sufficiently low. More precisely, the necessary and sufficient condition to guarantee the presence of credit rationing is (see the derivation in the Appendix C),

Condition R: (Credit Rationing) $Q < \left[\frac{\sigma+d}{\alpha A} \left(\frac{1+D}{1+\theta} \right) \right]^{\frac{\alpha}{1-\alpha}}$ where Q solves $Q = (1 - \alpha)A \left(\frac{1}{\phi} + \frac{1+\theta}{g+d} Q^{\frac{1}{\alpha}} \right)$.

Notice that the fixed point mapping of Q corresponds to the intersection of the KE and LE loci. Furthermore, to ensure the feasibility of loanable funds (i.e., $x < a$), we impose:

Condition N: (Loanable Funds Feasibility)

$$D > (1 + \theta) \left[\frac{A^{1/\alpha}}{\phi^{(1-\alpha)/\alpha}} \left(\frac{\alpha}{\sigma + d} \right) \right] - 1.$$

The wage rate under the constrained equilibrium is:

$$w^R = (1 - \alpha)A \left[\frac{\sigma + d}{\alpha A} \left(\frac{1 + D}{1 + \theta} \right) \right]^{\frac{-\alpha}{1-\alpha}} \tag{19}$$

Since the presence of credit rationing is associated with a higher loan rate, it is clear that the wage rate is lower than that in unconstrained equilibrium. From (6), (13), (19) and the first equality of (12) (recalling that the second inequality of (12) involves the use of (11) which is invalid in the case of credit rationing), the deposit rate under the constrained equilibrium can be written as:

$$r^R = [(1 - \alpha)A]^{\frac{1}{\alpha}} \left(\frac{g + d}{1 + \theta} \right) \left(D - \frac{w^R}{\phi} \right) \left(1 - \frac{w^R}{\phi} \right) (w^R)^{\frac{1-\alpha}{\alpha}} \tag{20}$$

which yields the financial spread as:

$$\frac{D - w^R/\phi}{r^R} = \left\{ [(1 - \alpha)A]^{\frac{1}{\alpha}} \left(\frac{g + d}{1 + \theta} \right) \left(1 - \frac{w^R}{\phi} \right) (w^R)^{\frac{1-\alpha}{\alpha}} \right\}^{-1}.$$

Comparing the credit-constrained equilibrium with the unconstrained equilibrium (point *E*) and utilizing (9), (10) and (13), we can establish:

PROPOSITION 3. (*Credit-Constrained Equilibrium*) Under Conditions *E*, *F*, *L*, *S*, *R* and *N*, there is a perfect foresight balanced growth equilibrium with credit rationing. The presence of credit constraints causes the loan and the deposit rates and the financial spread to increase, and effective bank loans and the effective capital formation to decrease.

While most of the results are straightforward, the finding concerning the deposit rate and financial spread deserves further comments. We show in the Appendix C that the deposit rate is unambiguously higher under credit rationing than in unconstrained equilibrium. This is because the positive banking marginal revenue effect of a higher loan rate dominates the negative loan reduction effect, leading to a higher bank profit and hence requiring a higher deposit rate to restore the zero profit condition. This contrasts with the pure exchange model of Azariadis and Smith (1993) in which credit rationing reduces intergenerational borrowing and the enlarged forced savings cause interest rates to fall. Moreover, our model suggests that a higher loan rate and a lower wage rate tends to increase the

financial spread, whereas a higher deposit rate lowers the financial spread. Under Condition N which ensures that the loan-deposit ratio is lower under credit rationing, equation (10) then implies that the former effect must dominate the latter such that the financial spread is widened under credit rationing.

Our results regarding the size of loans and the loan rate can be compared with those in the existing literature, particularly the overlapping generations models by Tsiddon (1992) and Bencivenga and Smith (1993). The moral-hazard trap in Tsiddon and the adverse-selection induced credit-constrained equilibrium in Bencivenga and Smith are both associated with a higher loan rate of interest. In our investment-loan production economy, credit rationing leads to lower bank loans and thus less capital accumulation. By diminishing returns, the marginal product of capital is higher, as is the constrained equilibrium rate of loan.

The diagrammatic analysis of the comparative statics with respect to changes in ϕ and A in the presence of credit rationing is displayed in Figures 2a and 2b. A cost-reducing banking innovation (a higher ϕ) shifts the KE locus rightward to KE' which causes the effective bank loans to increase, but it has no effect on the loan rate (in the sense of a local analysis). Thus, the wage rate is not affected because banking innovation has no effect on the goods production technology (and the factor price frontier in the goods sector). However, the deposit rate increases due to banks' zero profit condition in response to an increase in the marginal profit. As a result, the financial spread is narrowed in response to a cost-reducing banking innovation. Since an increase in bank loans raises capital accumulation and goods production, banking innovation causes a negative correlation between real output and the financial spread, consistent with the empirical evidence in Lehr and Wang (2000).¹⁸

An improvement in goods production efficiency (a higher A) shifts both the KE and LE loci rightward to KE' and LE' respectively. As long as the horizontal rightward shift of LE is less than that of KE , the new constrained equilibrium is still determined by the intersection of KE' and the horizontal line D . In this case, the loan rate is not affected, whereas the bank loans in effective units increase. Moreover, an improvement in production efficiency raises the marginal productivity of labor which increases the equilibrium wage rate. Although the direct effect of an improvement in production efficiency is to increase the deposit rate, the indirect effect through the wage rate is ambiguous. As a consequence, the change in financial spread is also ambiguous.

¹⁸Lehr and Wang (2000) find that in post-World War II U.S., U.K. and West Germany, financial innovations, measured by the structural disturbances to the inverse of the loan-deposit interest rate differential, are positively correlated with long-run movements in real output.

PROPOSITION 4. (*Characterization of the Constrained Equilibrium*) *The comparative statics of changes in any productivity parameters in the presence of credit rationing are:*

(i) *an advancement in banking or goods productivity raises the effective bank loans;*

(ii) *while an advancement in banking productivity results in a higher deposit rate and a lower financial spread, an improvement in goods productivity has ambiguous effects on them.*

It may be of interest to compare our findings with those in the static, pure-exchange framework of Williamson (1986) in which credit rationing emerges as a result of costly bank operation. A common outcome is that the presence of credit rationing leads to a higher loan rate. When credit rationing is present, Williamson (1986) finds that an increase in the bank operation (monitoring) cost reduces the deposit rate (from maintaining zero profit), while a higher unit labor requirement (lower ϕ) in our model also results in a downward change in the deposit rate. In another static, pure-exchange model by Holmström and Tirole (1997), the presence of the moral hazard problem causes credit rationing. They, in a partial equilibrium setting, find that credit rationing implies a higher loan rate and a lower deposit rate. While a higher loan rate in their study is similar to our result, a lower deposit rate is different from ours. Different from both Williamson (1986) and Holmström and Tirole (1997), our model incorporates potential effects via intertemporal substitution and production factor reallocation.

Remark 4.1. Under an endogenous growth setup discussed in Remark 3.1 (Section 3.2), one can see that the size of loans in effective units is lower and the loan rate is higher, both suppressing the capital-labor ratio and reducing the marginal product of knowledge capital. On the one hand, the reduction in the returns to knowledge discourages parents' incentive to educate their children, thus decreasing economic growth. We refer to this as the parental incentive effect. On the other hand, a reduced loan size requires less labor inputs, leading to labor reallocation from banking to children's education and causing economic growth to rise. This latter channel can be referred to as the labor reallocation effect. As a result of these two opposing effects, the net effect of credit constraints on economic growth is ambiguous, which may serve to explain why the empirical relationship between credit rationing and economic growth is mixed. Should the parental incentive effect dominates, credit rationing retards growth, which corroborates with findings established in Aghion and Bolton (1997) and Bencivenga and Smith (1993). However, if the labor reallocation effect is sufficiently strong, credit constraints may raise growth.

5. CONCLUDING REMARKS

This paper develops a dynamic general-equilibrium model with production to study the long-run consequences of credit market imperfections. Our results suggest that while changes in productivity of real and financial sectors can generate very different comparative-static outcomes in a steady-state equilibrium in the absence of credit constraints, such changes under credit-market imperfections are qualitatively similar. Moreover, credit rationing need not harm the real sector, though it unambiguously causes the loan rate to rise and the financial spread to widen.

A natural extension of our model is to allow for intertemporal consumption-saving choice through which the effect of credit rationing may alter. In particular, credit rationing results in two opposing effects. On the one hand, it reduces the wage rate which decreases savings in goods. On the other, it increases the deposit rate, and thus increases savings in goods, provided that the substitution effect of a higher deposit rate on savings dominates the associated wealth effect. In the absence of direct lending from households to firms, there would be no first-order long-run real effects. By allowing a direct link between household savings and firm capital formation, the reduction in wages retards capital accumulation whereas the rise in savings enhances it. Should the former effect be dominated by the later, the net effect of credit rationing is to spur the economy. This therefore provides an empirically testable hypothesis concerning the long-run effects of credit rationing on household savings and long-run macroeconomic performance.

APPENDIX

A. Derivation of the Firm's Optimization Conditions:

From (5) we can derive the first-order conditions for i and ℓ , respectively, as:

$$\left(\frac{1}{1+\sigma}\right)V'_{t+1} = \frac{1+\delta_t}{1+\theta} \quad (\text{A.1})$$

$$A(1-\alpha)\left(\frac{k_t}{\ell_t h_t}\right)^\alpha = w_t \quad (\text{A.2})$$

The Benveniste-Scheinkman condition is:

$$V'_t = A\alpha k_t^{\alpha-1}(\ell_t h_t)^{1-\alpha} + \frac{1-d}{1+\sigma}V'_{t+1} \quad (\text{A.3})$$

Guess $V(k)$ is linear in k : $V(k_t) = V_0 + V_1 k_t$. Then (A.1) and (A.3) imply

$$V_1 = \alpha A \frac{1 + \sigma}{\sigma + d} \left(\frac{k_t}{\ell_t h_t} \right)^{-(1-\alpha)} = (1 + \sigma) \left(\frac{1 + \delta_t}{1 + \theta} \right) \tag{A.4}$$

From (A.2) and (A.4) we get (6) and (7). Substituting (A.4) into the above $V(k)$ expression and using (5) and (A.2), we can prove $V_0 = 0$ and derive the linear value function (8).

B. Conditions for the Balanced Growth Equilibrium:

Condition F: (Feasibility)

$$A \left[(1 + g) \frac{\sigma + d}{g + d} - \alpha \right]^\alpha \geq (1 + g) [\alpha(\sigma + d)]^\alpha [\phi(1 + \theta)]^{1-\alpha}.$$

Feasibility requires $y - i \geq c$. From the production function, (2), (11) and (13), the feasibility condition can be rewritten as $A \left(\frac{k}{h\ell} \right)^{-(1-\alpha)} \geq (g+d) + \frac{c}{k}$. Substituting (4), (6), (9) and (13) into this inequality, one can see that the feasibility holds if and only if $\frac{1+\delta}{1+\theta} \geq \alpha \left(\frac{g+d}{\sigma+d} \right) \left(1 + \frac{1}{1+g} \frac{1+r}{1+\theta} \right)$, and by substituting into (9) and (7), this inequality can be re-written as

$$\left[\frac{1 + g}{\alpha} \frac{\sigma + d}{g + d} - 1 \right] \Delta \geq (1 + g) - \frac{(1 - \alpha) A^{1/(1-\alpha)}}{\phi(1 + \theta)} \left(\frac{\alpha}{\sigma + d} \right)^{\alpha/(1-\alpha)} \Delta^{-\alpha/(1-\alpha)} \tag{A.5}$$

where $\Delta = \frac{1+\delta}{1+\theta}$. As one can see, the *LHS* of (A.5) is linear in Δ whereas the *RHS* is increasing and concave in Δ with a positive horizontal intercept and an asymptote $1 + g$ as Δ approaches infinity. Define a critical value $\Delta_c \equiv \alpha A \left\{ (\sigma + d)^\alpha \left[\phi(1 + \theta) \left(\frac{1+g}{\alpha} \frac{\sigma+d}{g+d} - 1 \right) \right]^{(1-\alpha)} \right\}^{-1}$ such that the slope of *LHS* is equal to the slope of *RHS* at which $(LHS - RHS)$ is minimized. Thus, if $(LHS - RHS)$ is nonnegative at Δ_c , the above inequality (A.5) must always hold for any value of Δ . This yields Condition F.

Condition S: (Slope condition)

$$\delta \leq \delta^{\max} \equiv (1 + \theta) \left[\left(\frac{g + d}{1 + \theta} \frac{1}{\phi} \frac{\alpha}{1 - \alpha} \right)^{1-\alpha} \frac{\alpha A}{\sigma + d} \right] - 1.$$

Differentiating (14) and (15) gives:

$$\begin{aligned} \frac{-d(x/h)|_{LE}}{d\delta} &= \frac{x}{h} \Big|_{LE} \frac{1}{2 + \theta + \delta} \frac{\alpha}{1 - \alpha}, \\ \frac{-d(x/h)|_{KE}}{d\delta} &= \frac{x}{h} \Big|_{KE} \frac{\frac{1+\theta}{g+d} \left[\frac{\sigma+d}{\alpha A} \left(\frac{1+\delta}{1+\theta} \right) \right]^{\frac{1}{1-\alpha}} \left(\frac{1}{2+\theta+\delta} \right) \left(\frac{1}{1-\alpha} \right)}{\frac{1}{\phi} + \left(\frac{1+\theta}{g+d} \right) \left[\frac{\sigma+d}{\alpha A} \left(\frac{1+\delta}{1+\theta} \right) \right]^{\frac{1}{1-\alpha}}} \end{aligned}$$

respectively. At the intersection of both LE and KE loci, i.e., $(\frac{x}{h})_{LE} = (\frac{x}{h})_{KE}$, the KE locus is steeper than the LE locus in $(x/h, \delta)$ space if $\frac{1+\theta}{g+d} \left[\frac{(\sigma+d)}{\alpha A} \left(\frac{1+\delta}{1+\theta} \right) \right]^{\frac{1}{1-\alpha}} < \frac{1}{\phi} \frac{\alpha}{1-\alpha}$, which can be rewritten as that specified in the Condition S.

Condition E: (Existence) $A^{\frac{1}{\alpha}} < \phi^{\frac{1-\alpha}{\alpha}} \left(\frac{\alpha}{1-\alpha} \right) \left(\frac{g+d}{1+\theta} \right)$.

By equating the RHS of (14) and that of (15) and solving the resulting equation for δ , one can solve for the equilibrium δ^E . Thus, we can prove the existence of balanced growth equilibrium by showing such an equation has a solution for Q . Equating the RHS of both (14) and (15) yields

$$\frac{1}{\phi} + \frac{1+\theta}{g+d} \left[\frac{\sigma+d}{\alpha A} \left(\frac{1+\delta}{1+\theta} \right) \right]^{\frac{1}{1-\alpha}} = \frac{1}{(1-\alpha)A} \left[\frac{\sigma+d}{\alpha A} \left(\frac{1+\delta}{1+\theta} \right) \right]^{\frac{\alpha}{1-\alpha}}$$

Defining $Q \equiv \left[\frac{\sigma+d}{\alpha A} \left(\frac{1+\delta}{1+\theta} \right) \right]^{\frac{\alpha}{1-\alpha}}$, the above equation can then be rewritten as:

$$Q = H(Q) \equiv (1-\alpha)A \left[\frac{1}{\phi} + \frac{1+\theta}{g+d} Q^{1/\alpha} \right] \tag{A.6}$$

Notice that $H'(Q) = \frac{1-\alpha}{\alpha} \frac{1+\theta}{g+d} A Q^{(1-\alpha)/\alpha}$ is continuous and increasing in Q ; moreover, $H'(Q=0) = 0$. Let \bar{Q} be such that $H'(\bar{Q}) = 1$. To show the existence of an equilibrium, it is sufficient to show that $H(\bar{Q}) < \bar{Q}$. One can easily compute $\bar{Q} = \left[\frac{g+d}{1+\theta} \frac{1}{A} \frac{\alpha}{1-\alpha} \right]^{\alpha/(1-\alpha)}$. By substituting this result into $H(\bar{Q}) < \bar{Q}$, one can derive Condition E.

C. Moral Hazard, Credit Rationing and Proposition 3

Condition R: (Credit Rationing) $Q < \left[\frac{\sigma+d}{\alpha A} \left(\frac{1+D}{1+\theta} \right) \right]^{\frac{\alpha}{1-\alpha}}$ where Q solves $Q = (1-\alpha)A \left(\frac{1}{\phi} + \frac{1+\theta}{g+d} Q^{\frac{1}{\alpha}} \right)$.

Recall that $Q = \{[(\sigma+d)/\alpha A][(1+\delta)/(1+\theta)]\}^{\alpha/(1-\alpha)}$ in unconstrained equilibrium. When $D > \delta^E$ (the unconstrained equilibrium level of the loan rate), credit rationing occurs. Thus Condition R follows from setting $\delta = D$.

Condition N: (Loanable Funds Feasibility)

$$D > (1+\theta) \left[\frac{A^{1/\alpha}}{\phi^{(1-\alpha)/\alpha}} \left(\frac{\alpha}{\sigma+d} \right) \right] - 1.$$

When the credit rationing constraint (18) is binding, the wage rate (w^R) obtained from (7) is

$$w^R = (1-\alpha)A \left[\frac{\sigma+d}{\alpha A} \left(\frac{1+D}{1+\theta} \right) \right]^{\frac{-\alpha}{1-\alpha}} \tag{A.7}$$

Notice that w^R is decreasing in D . Moreover, from the bank's zero profit condition, one can derive the deposit rate in constrained equilibrium (r^R) as

$$r^R = [(1 - \alpha)A]^{\frac{1}{\alpha}} \left(\frac{g + d}{1 + \theta} \right) \left(D - \frac{w^R}{\phi} \right) \left(1 - \frac{w^R}{\phi} \right) (w^R)^{\frac{1-\alpha}{\alpha}} \quad (\text{A.8})$$

It can be easily shown that $\left. \frac{d \ln r^R}{d \delta} \right|_{\delta=D} \propto \frac{1-\alpha}{\alpha} \frac{1}{w^R} (2 + \theta + w^R/\phi) + \frac{1}{\phi} \frac{D - w^R/\phi}{1 - w^R/\phi} > 0$. Furthermore, from the bank's zero profit condition, the constrained equilibrium financial markup is derived as:

$$q \equiv \frac{\delta - \frac{w}{\phi}}{r} \left(= \frac{\alpha/h}{x/h} \right) = \left\{ [(1 - \alpha)A]^{\frac{1}{\alpha}} \left(\frac{g + d}{1 + \theta} \right) \left(1 - \frac{w^R}{\phi} \right) (w^R)^{\frac{1-\alpha}{\alpha}} \right\}^{-1}. \quad (\text{A.9})$$

One can verify that $\frac{dq}{dD} > 0$ if and only if $D > (1 + \theta) \left[\frac{A^{1/\alpha}}{\phi^{(1-\alpha)/\alpha}} \left(\frac{\alpha}{\sigma + d} \right) \right] - 1$. As one might notice that this is also the condition for $x < a$ since $q = 1$ when the loan rate is δ^E (see equation (10)) and $D > \delta^E$ (which is required for credit rationing to occur). As a result we have Condition N.

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