Nonlinear Trend Stationarity in Real Exchange Rates: Evidence from Nonlinear ADF tests

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In this article we test for unit root in real exchange rates during the recent floating exchange rate period. In doing so, we use the unit root tests proposed by Bierens (1997a) and Bierens (1997b) that have drift hypothesis against a very general trend stationarity hypothesis, namely the alternative that the time series is stationarity about an almost arbitrary deterministic function of time. Bierens approach employs the fact that any function of time can be approximately arbitrary close by a linear function of Chebishev polynomials. The application of the tests to real exchange rate series indicate that these series are nonlinear trend stationary, and therefore, we conclude that the real exchange rate behavior may not be so different after all but simply perceived to be so because of the use of previously different unit root tests.

Key Words: Real exchange rates; PPP; Nonlinear ADF; Nonlinear trend stationary.
JEL Classification Numbers: C45, G15.

1. INTRODUCTION

Purchasing power parity (PPP) is an important theoretical concept in economics because most macroeconomics models of open economy are built upon the long-run PPP hypothesis. The relationship is important not only because it has been a cornerstone of exchange rate models, but also because of its policy implications — it provides a benchmark exchange rate and hence has some practical appeal for policymakers and exchange rate arbitragers. For example, in the case that real exchange rates are non-stationary, the principle of purchasing power parity (PPP) is no longer valid as a representation of the long run equilibrium relation between the exchange rate and relative prices. Further, the extent to which real ex-
change rate movements are random or not provides an indication of whether countries are financially integrated or autonomous. This is particularly relevant in an area of high or perfect capital mobility.\footnote{Froot and Rogoff (1995) and Rogoff (1996) provide excellent discussions.}

Although purchasing power parity has been studied extensively, empirical results have been mixed. For example, Dueker and Serletis (1997) and Serletis and Zimonopulos (1997) find that PPP does not hold over the recent floating exchange rate period, while studies by Phylaktis and Kassimatis (1994), Lothian and Taylor (1996), and Cheung and Lai (1993) report significant evidence favorable to long-run PPP. Recently, a view that real exchange rates are stationary, but highly persistent is emerging in the literature (see Rogoff (1996): Lothian and Taylor (1996) and Olekalns and Wilkins (1998)). Engle (1998) challenges this view, arguing that the power of the unit root tests in such studies is very low. Similarly, Caner and Kilian (1998) argue that tests of the stationary null hypothesis may suffer from severe size distortions. In fact, the random walk behavior of the real exchange rate was contrasted with chaotic dynamics. This is motivated the notion that the real exchange rate follows a deterministic nonlinear process which generates output that mimics the output of stochastic systems. In other words, it is possible for the real exchange rate to appear random but not to be really random. For example, Gogas and Serletis (2000) study the random walk behavior of the real exchange rate and conclude that real exchange rate movements might not be really random. They find evidence of nonlinear chaotic dynamics in 7 out of 15 real exchange rate series. Gil-Alana (2000) applies fractionally-based tests to real exchange rate data between US and five industrialized countries and his results indicate that the series are fractionally integrated with mean reversion. Gil-Alana (2002) studies the monthly real exchange rates (relative to the US dollar) from black markets of eight Asian developing countries and concludes with mean reversion in the long run. Henry and Olekalns (2002) examine the post-Woods experience of the Australian real exchange rate, and find no evidence of the long run equilibrium relation between the exchange rate and relative prices.

Despite the voluminous literature, studies on PPP remain inconclusive. A number of possible reasons can be put forward for the failure to find evidence for PPP. These include traditional forms of price stickiness (Dornbusch, 1976) as well as explanations based on trade costs (e.g., Dumas, 1992) and price discrimination (e.g., Chari, Kehoe and McGrattam, 2000). The empirical literature on the time-series properties of the real exchange rate has primarily focused on real factors and cross-country differences in productivity growth. For example, Cegłowski (1996) attempts to capture the effects of relative productivity differentials on the yen real exchange
rates using the procedures suggested by Perron (1990) and Perron and Vogelsang (1992). Those tests allow consideration of breaks in the mean as well as in the trend. The use and application of panel methods have typically allowed the production of more evidence in favor of real exchange rate mean reversion (e.g. MacDonald, 1996; Wau, 1996; Papell, 1997; Oh, 1996). Cheung and Lai (2001) focus on the possibility of long-memory dynamics. They consider eight bilateral exchange rates looking for evidence of fractional integration and find that the order of integration of all series considered is between zero and one. The use of fractionally integrated processes allows long cycles and long-term memory and provides a flexible enough framework simultaneously to describe large swings and mean-reverting dynamics that may characterize real exchange rate behavior. There is also a growing literature on modeling exchange rates using non-linear models such as the TAR and STAR (e.g., Sarantis, 1999; Baum, Barkoulas and Caglayan, 2001; Taylor, Peel and Sarno, 2001).

Another possible reason for the non-stationarity in the real exchange rates could be the presence of structural breaks in the series. Perron (1989, 1990) and Perron and Vogelsang (1992) have shown that when a time series has structural breaks in the mean, the unit root hypothesis is often accepted before structural breaks are taken into account, while it is rejected after structural breaks are considered. The fact that our sample period includes some economic and financial events that took place in the sample size (i.e., European Monetary events) makes it very likely to have some structural breaks. In this paper, we examine data for real exchange rate series for evidence of nonstationarity in the presence of level shifts. We use the Bierens (1997a,1997b) nonlinear augmented Dickey-Fuller (NLADF) test here since it allows the trend to be an almost arbitrary deterministic function of time. These tests differ from others in that they use Chebishev time polynomials rather than regular time polynomials, a parametric specification of the dynamics rather than using a Newey-West (1987) type long-run variance estimator, and the null hypothesis is the unit root with constant drift hypothesis rather than the unit root with nonlinear trended drift hypothesis. The application of the tests to real exchange rate series indicate that, even after allowing for nonlinear trend breaks, the unit root hypothesis still could not be rejected for all countries. Overall, we conclude that there is no evidence of mean-reversion in the level of real exchange rates and the conflicting results obtained from the tests might be that the real exchange rate series are nonlinear trend stationary, with a more complicated trend than a linear trend.

The rest of the paper is organized as follows. Section 2 presents the nonlinear augmented Dickey-Fuller tests proposed by (1997a,1997b). Section 3 presents the empirical results. And section 4 concludes.
2. NONLINEAR AUGMENTED DICKEY FULLER TESTS

Various methodological approaches have been used including cointegration tests for exchange rates and prices, variance ratio test, and unit root tests on real exchange rate series. A sufficient condition for a violation of absolute PPP is that the real exchange rate is characterized by a unit root. A number of approaches have been developed to test for unit roots. Nelson and Plosser (1982), using augmented Dickey-Fuller (ADF) type regressions argue that most macroeconomic time series (including real exchange rates) have a unit root. Perron (1989) however, has shown that conventional unit root tests are biased against rejecting a unit root where there is a break in a trend stationary process. Motivated by these considerations, Serletis and Zimonopoulous (1997), using the methodology suggested by Perron and Vogelsang (1992) and quarterly dollar-based and Deutchmark-based real exchange rates (over the period from 1957:1 to 1995:4) for 17 OECD countries, show that the unit root hypothesis cannot be rejected even if allowance is made for the possibility of a one-time change in the mean of the series at an unknown point in time.

A few unit root tests have been developed for time series with structural breaks. We use the Bierens (1997a,1997b) nonlinear augmented Dickey-Fuller (NLADF) test here since it allows the trend to be an almost arbitrary deterministic function of time. The test is based on an ADF type auxiliary regression model where the deterministic trend is approximated by a linear function of Chebishev polynomials. These tests differ from others in that they use Chebishev time polynomials rather than regular time polynomials, a parametric specification of the dynamics rather than using a Newey-West (1987) type long-run variance estimator, and the null hypothesis is the unit root with constant drift hypothesis rather than the unit root with nonlinear trended drift hypothesis. As stated by Bierens (1997a), the Chebishev polynomials have substantial advantages over regular time polynomials because they are orthogonal and bounded.

The tests are based on the following. Let $y_t$ be a univariate time series of interest, and consider the null hypothesis:

$$H_0 : y_t = y_{t-1} + \mu + u_t$$  \hspace{1cm} (1)

where $\mu$ is a constant drift parameter and $u_t$ is a stationary AR($p$) process. Bierens (1997a) proposed tests of this null hypothesis against the alternative of nonlinear trend stationary:

$$H_1 : y_t = g(t) + u_t$$  \hspace{1cm} (2)

where $g(t)$ is a possibly nonlinear trend function. Then following Dickey and Fuller (1979, 1981), these tests will be based on an Augmented Dickey-
Fuller (ADF) type auxiliary regression model:

\[ \Delta y_t = \beta y_{t-1} + \sum_{j=1}^{p} \nu_j \Delta y_{t-j} + \theta^T P^{(m)}_{t,n} + \varepsilon_t \]  

(3)

where

\[ P^{(m)}_{t,n} = (P^{*}_{0,n}(t), P^{*}_{1,n}(t), \ldots, P^{*}_{m,n}(t))^T \]  

(4)

is a vector of orthogonal Chebyshev polynomials. Under the null hypothesis of unit root, \( \beta = 0 \), and \( \theta^T = 0 \). The unit root hypothesis is tested based on the \( t \)-statistic of \( \beta \), the test statistic

\[ Am = (n - p - 1)\beta \left| 1 - \sum_{j=1}^{p} \nu_j \right|, \]

and the \( F \)-test of the joint hypothesis that \( \beta \) and the last \( m \) components of \( \theta^T \) are zero.

In the literature on unit root testing, it is well-known that the standard unit root tests such as the ADF and Phillips-Perron tests fail to reject the null hypothesis of a unit root in a near unit root economic time series. The null hypothesis is always accepted unless there is strong evidence against it. To avoid this problem, tests have been designed under the null hypothesis that the time series under test is stationary around a long-term mean, against the alternative that the time series has a unit root. One possibility is to use the Kwiatkowski, Phillips, Schmidt and Shin (1992) (KPSS hereafter). In this paper, we do not use KPSS, but an alternative stationarity test proposed by Bierens and Guo (1993). The test is designed with the null hypothesis:

\[ y_t = \mu + \varepsilon_t \]  

(5)

against the alternative

\[ \Delta y_t = y_t - y_{t-1} = \varepsilon_t \]  

(6)

where \( \varepsilon_t \) is a zero-mean stationary process and \( \mu \) is the long-term mean. Bierens and Guo (1993) design four types of Cauchy tests against unit root hypothesis, based on an auxiliary linear time trend regression. Large values of these tests would lead to the rejection of the stationarity null hypothesis.

3. EMPIRICAL RESULTS

The real exchange rate series of country \( i \) at time \( t \) is defined as:

\[ y_{it} = \log(S_t P^i_t P^{us}_t) \]  

(7)

where \( S_t \) is the nominal exchange rate in the home currency of country \( i \) per dollar, \( P^i_t \) and \( P^{us}_t \) enote the consumer price indices of country \( i \) and US,
respectively. Under long-run purchasing power parity, the long-run equilibrium real exchange rate is equal to 1 in the absolute version of PPP, which would imply that $y_{it} = 0$ — in the relative version of PPP the first logged difference of the real exchange rate would be zero, that is $\Delta y_{it} = 0$. In the short-run, however, one would expect deviations from PPP, and the question is whether these deviations are permanent or transitory. Therefore, we apply the test statistic to test for persistence in the series of $\Delta y_{it}$.

We examine the US dollar-based real exchange rate series for the following countries: Canada, France, Germany, Japan, United Kingdom, Sweden, Switzerland and Netherlands. The data is quarterly series covering the period 1973:Q1 — 2001:Q2 and obtained from Datastream. To construct the real exchange rate series, we calculate the real exchange rate as the ratio of actual spot exchange rate relative to the PPP spot rate. The PPP spot exchange rate for period $t + n$, is defined as:

$$S_{PPP,t+n} = S_{$/i,0} = \frac{P_{us,t+n}}{P_{i,t+n}}$$

where 0 represents the base period, and $i$ denotes the foreign country. By definition, the PPP spot rate is the spot rate that reestablishes PPP relative to some base period. Ideally, a base period is one where the exchange rate takes a value that allows both countries to achieve their domestic and international policy objectives (i.e., internal and external balance). Some select a period immediately after a major exchange rate change. Therefore, we select 1973:Q1 as the base period.

Fig 1 plots the quarterly observations of three real exchange rate series, namely, the Canadian dollar, the French franc, and the Deutsche mark. It is observed that the French franc and Deutsche mark behaved very similarly and all exhibit long and large deviations from the sample means in the past three decades. The Canadian real exchange rate also shows some fluctuations over time, but less than those of the other two series. Before conducting the unit root tests with structural breaks, we first conduct standard ADF tests for a unit root for each individual series. Table 1 presents the results. The regressions are run with and without a trend and the lag length is set to 4 for our data.²

Tables 2 and 3 present the results of the nonlinear augmented Dicky-Fuller tests and the associated critical values. The results show that, even after allowing for nonlinear trend breaks, the unit root hypothesis still could not be rejected for all countries, since the t-statistics are all below the critical values. The results of the stationarity test of Bierens and Guo (1993) are also reported in Table 4. The null hypothesis of stationarity in

²Other lag lengths are used and the results are qualitatively the same and are not reported.
FIG. 1. Real Exchange Rate for Canada, France, and Germany

The first logged difference of the real exchange rate would be zero, that is \( \Delta y_{it} = 0 \). In the short-run, however, one would expect deviations from PPP, and the question is whether these deviations are permanent or transitory. Therefore, we apply the test statistic to test for persistence in the series of \( \Delta y_{it} \).

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\[
\frac{P_{t+n}^{PP}}{P_{t}^{PPP}}
\]

where \( 0 \) represents the base period, and \( i \) denotes the foreign country. By definition, the PPP spot rate is the spot rate that reestablishes PPP relative to some base period. Ideally, a base period is one where the exchange rate takes a value that allows both countries to achieve their domestic and international policy objectives (i.e., internal and external balance). Some select a period immediately after a major exchange rate change. Therefore, we select 1973:Q1 as the base period.

Figure 3.1: Real Exchange Rate for Canada, France, and Germany

<table>
<thead>
<tr>
<th>Country</th>
<th>ADF-trend</th>
<th>ADF-without trend</th>
<th>Country</th>
<th>ADF-trend</th>
<th>ADF-without trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>-1.656</td>
<td>-0.889</td>
<td>UK</td>
<td>-2.349</td>
<td>-2.326</td>
</tr>
<tr>
<td>France</td>
<td>-2.042</td>
<td>-1.836</td>
<td>Norway</td>
<td>-2.058</td>
<td>-1.795</td>
</tr>
<tr>
<td>Germany</td>
<td>-1.821</td>
<td>-1.827</td>
<td>Sweden</td>
<td>-1.812</td>
<td>-1.363</td>
</tr>
<tr>
<td>Japan</td>
<td>-2.121</td>
<td>-1.793</td>
<td>Switzerland</td>
<td>-2.082</td>
<td>-2.447</td>
</tr>
</tbody>
</table>

Notes: the table reports the ADF tests for unit roots applied to real exchange rate series. The critical values are computed using Mackinnon’s (1990) method. The level of real exchange rates is rejected for Canada, France, Netherlands, and Sweden, but not for those of Germany, Japan, Switzerland and UK. Actually, for Sweden, there is evidence of stationarity from the type 3 and type 4 Cauchy tests at the 90% significance level. Overall, we conclude that there is no evidence of mean-reversion in the level of real exchange rates and the conflicting results obtained from Tables 2 and 4 might be that the real exchange rate series are nonlinear trend stationary, with a more complicated trend than a linear trend.

The critical values are computed using Mackinnon’s (1990) method. It is found that the null hypothesis that real exchange rate series contain unit
TABLE 2.
Bierens (1997) Nonlinear ADF test applied to levels of real exchange rates

<table>
<thead>
<tr>
<th>RER</th>
<th>Canada</th>
<th>France</th>
<th>Germany</th>
<th>Japan</th>
<th>Netherlands</th>
<th>Sweden</th>
<th>Switzerland</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>$-0.0701$</td>
<td>$-0.092$</td>
<td>$-0.0915$</td>
<td>$-0.105$</td>
<td>$-0.114$</td>
<td>$-0.0842$</td>
<td>$-0.1046$</td>
<td>$-0.101$</td>
</tr>
<tr>
<td>$F$-test</td>
<td>1.730</td>
<td>3.345</td>
<td>3.384</td>
<td>2.344</td>
<td>3.058</td>
<td>2.728</td>
<td>3.251</td>
<td>1.923</td>
</tr>
</tbody>
</table>

Notes: The table reports the results of Bierens (1997) non-linear Augmented Dickey-Fuller tests for unit root. Critical values of $t$-stat (5%) = $-3.97$; (10%) = $-3.64$; Critical values of Am (5%) = $-27.2$; (10%) = $-23$; Critical values of $F$-test (5%) = 4.88; (10%) = 5.68.

TABLE 3.

<table>
<thead>
<tr>
<th>RER</th>
<th>Canada</th>
<th>France</th>
<th>Germany</th>
<th>Japan</th>
<th>Netherlands</th>
<th>Sweden</th>
<th>Switzerland</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>$-0.625$</td>
<td>$-0.913$</td>
<td>$-1.018$</td>
<td>$-0.760$</td>
<td>$-0.975$</td>
<td>$-0.849$</td>
<td>$-0.939$</td>
<td>$-0.939$</td>
</tr>
<tr>
<td>t-stat</td>
<td>$-4.140$</td>
<td>$-4.614$</td>
<td>$-4.673$</td>
<td>$-4.085$</td>
<td>$-4.255$</td>
<td>$-5.121$</td>
<td>$-4.463$</td>
<td>$-5.010$</td>
</tr>
<tr>
<td>Am</td>
<td>$-43.69$</td>
<td>$-77.96$</td>
<td>$-114.33$</td>
<td>$-53.57$</td>
<td>$-59.68$</td>
<td>$-74.63$</td>
<td>$-70.79$</td>
<td>$-105.79$</td>
</tr>
<tr>
<td>$F$-test</td>
<td>5.768</td>
<td>7.142</td>
<td>7.645</td>
<td>5.625</td>
<td>6.037</td>
<td>8.743</td>
<td>6.672</td>
<td>8.379</td>
</tr>
</tbody>
</table>

Notes: The table reports the results of Bierens (1997) non-linear Augmented Dickey-Fuller tests for unit root. Critical values of $t$-stat (5%) = $-3.97$; (10%) = $-3.64$; Critical values of Am (5%) = $-27.2$; (10%) = $-23$; Critical values of $F$-test (5%) = 4.88; (10%) = 5.68.

TABLE 4.
Bierens-Guo (1993) stationarity tests applied to levels of real exchange rates.

<table>
<thead>
<tr>
<th>RER</th>
<th>Canada</th>
<th>France</th>
<th>Germany</th>
<th>Japan</th>
<th>Netherlands</th>
<th>Sweden</th>
<th>Switzerland</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1</td>
<td>192.44</td>
<td>27.75</td>
<td>4.848</td>
<td>1.434</td>
<td>39.35</td>
<td>63.79</td>
<td>0.072</td>
<td>0.184</td>
</tr>
<tr>
<td>Type 2</td>
<td>113.37</td>
<td>33.94</td>
<td>5.049</td>
<td>1.581</td>
<td>43.20</td>
<td>44.99</td>
<td>0.072</td>
<td>0.185</td>
</tr>
<tr>
<td>Type 3</td>
<td>10.98</td>
<td>9.32</td>
<td>2.35</td>
<td>$-53.5$</td>
<td>29.66</td>
<td>5.54</td>
<td>0.150</td>
<td>0.220</td>
</tr>
<tr>
<td>Type 4</td>
<td>7.17</td>
<td>9.27</td>
<td>2.25</td>
<td>5.625</td>
<td>29.66</td>
<td>0.144</td>
<td>0.144</td>
<td>0.203</td>
</tr>
</tbody>
</table>

Notes: The table reports the four types of Gauchy tests of Bierens-Guo (1993) stationarity tests applied to levels of real exchange rates. Critical values are (5%) = $12.706$ and (10%) = $6.314$. The tests are based on $m = 16 = [c.n^r]$, where $c = 5$, $r = .25$, $n = 114$.

roots cannot be rejected at conventional significance levels, and therefore conclude that all series are nonstationary.

Hsieh (1991), for example, asserts that nonstationarity is synonymous with structural change (changes in the data generating process (DGP)). There may be many reasons for structural changes: technological and financial innovations, policy changes, etc. It would be difficult to argue that the structure of the economic and financial system has remained constant from 1973 to 2001. Thus, for example, during the period under analysis we run into the incorporation into the EMS of France (13-3-79), Spain
(19-6-89), Italy (13-3-79) and the United Kingdom (8-10-90), and the later abandonment in 1992 (21-9-92) of Italy and UK because of the EMS crisis. The Maastricht Treaty was agreed upon in December of 1991, and in February of 1992, it is signed by the EU finance and foreign ministries. At the beginning of 1993 the EU single market begins and by the summer of 1993, the fluctuation bands within the ERM were widened considerably. By the end of 1996, the Dublin summit agrees the main outlines of the Stability and Growth Pact, while in 1997 the Amsterdam summit adopts the Stability and Growth Pact. Also, in September of 1992, the French public votes for the Maastricht Treaty, while in August of 1993, the fluctuation band for the French Franc is widened to ±15%. In fact, during the same year (i.e., 1993) the German constitutional court rules in favor of the Maastricht Treaty too. In Japan, during March of 1995, the Japanese yen appreciated dramatically, in part due to the huge current account surplus. This was despite the aggressive intervention by the Federal Reserve Bank. On March 31, 1995 the Bank of Japan allowed its overnight call rate to fall to an historic low value of 1.75, but still the dollar plunged to an all time during April 1995.

The fact that these events might have an impact on our results, is not far fetched. For example, Del Rio and Santamaria (2000) applied an empirical density test to check whether different sample periods of the exchange rate series have the same empirical density as the whole series of the French Franc, British pound, Italian Lira and Spanish peseta for periods of before and after incorporating exchange rate mechanism of EMS and their results show that the distribution of daily variations is different for each one of the two periods. Further, Kruger and Kugler (1993) argue that exchange rates might show regime-switching behavior, in particular under a system of managed floating such as occurred in the 1980s when it was attempted to stabilize the exchange rate of the US dollar. Intuitively, monetary authorities may intervene in the foreign exchange market as a reaction to large depreciations or appreciations of a currency, which lead to different behavior for moderate and large changes of the exchange rate. Similar behavior may be observed for an exchange rate which is constrained to lie within a prescribed band or target zone, as was the case in the Exchange Rate Mechanism (ERM) in Europe (see Chappell et al., 1996). In this case, the level of the exchange rate rather than the change in the exchange rate determines the regimes.

4. CONCLUSION

The empirical literature that tests for purchasing power parity (PPP) by focusing on the stationarity of real exchange rates has so far provided, at best, mixed results. This paper contributes to this discussion by providing
new evidence on the stationarity of bilateral real exchange rates, after allowing nonlinear behavior in the unit root tests. We test for a unit root in real exchange rates by using a nonlinear augmented Dickey-Fuller test proposed by Bierens (1997a) and Bierens (1997b). Our results confirm overall that, there is no evidence of mean-reversion in the level of real exchange rates and the conflicting results might be that the real exchange rate series are nonlinear trend stationary, with a more complicated trend than a linear trend. Thus, the real exchange rate behavior may not be so different after all but simply perceived to be so because of the use of previously restrictive unit root tests.

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