

## “Favors” for Sale: Strategic Analysis of a Simple Menu Auction with Adverse Selection

Didier Laussel

*Université de la Méditerranée  
Greqam, Chateau Lafarge  
Route des Milles  
13290, Les Milles (France)*

and

Michel Le Breton

*Université de Toulouse 1 Gremaq-Idei, Manufacture des Tabacs 21, Allée de  
Brienne 31000, Toulouse (France)*

We study the distribution of a fixed amount of “favors” by an incumbent politician between two pressure groups, each of them offering to the agent a campaign contribution contingent on the quantity of “favors” received. Assuming that the total amount supplied is a private information of the politician the equilibrium contribution schedules are fully characterized. It is shown that the principals net equilibrium payoffs are larger the more quickly their marginal valuations of the favors decrease with the amount received. The equilibrium allocation of the stock of “favors” is efficient if the interest group utility functions are identical. © 2005 Peking University Press

*Key Words:* Menu auction; Adverse selection.

*JEL Classification Numbers:* C70, D72, L50.

### 1. INTRODUCTION

Incumbent politicians have long been considered by mainstream economists as benevolent dictators setting policies in order to maximize social welfare. This view of the political process in a representative democracy has been challenged by economists and political scientists who have investigated the mechanics of interest group influence. Many of them (Becker (1983), Krueger (1974), Tullock (1967), Snyder (1990), Magee, Brock and

Young (1989) ) have focused on the ability of interest groups to elicit favors from an incumbent politician or to influence the outcome of an election. A second group has insisted on interest groups as a source of information for imperfectly informed politicians (Ainsworth (1993), Austen-Smith and Wright (1992),(1994)), Ball (1993), Banks and Weingast (1992), Lohmann (1995)).

The idea that politicians pursue their own personal objectives and are influenced by interest group activities has been largely popularized by Grossman and Helpman ((1994), (1995)) who applied the economic influence model of Bernheim and Whinston (1996) to the case of trade policy. They modeled the competition among interests groups as a common agency game under complete information. By common agency one means a situation in which an individual, called the agent (the incumbent politician in Grossman and Helpman papers), decides upon an action (trade policy) which affects his well-being as well as the well being of several other individuals, called the principals (the various interest groups in Grossman and Helpman) . The interest groups, representing each one of the sector-specific factors in an open economy, offer simultaneously to the government monetary payments contingent on the trade policy chosen (a vector of import tax and/or subsidy rates), taking as given the contribution schedules of the other groups. The government sets a policy vector in order to maximize a weighted sum of total political contributions and of social welfare. An equilibrium is a set of contribution schedules such that the contribution schedule of each interest group is a best reply to the contribution schedules of the others. While there are in general many equilibria it has been shown by Bernheim and Whinston (1986) that only *truthful equilibria*, in which the principals set contribution schedules reflecting up to a constant the utility which they derive from the action chosen, are coalition-proof (CPNE). In addition, as noted by Grossman and Helpman, non-differentiable contribution schedules could entail large losses for the interest groups if they made only small miscalculations.

In Grossman and Helpman models the interest groups have conflicting preferences on the trade policy, each of them having a most preferred policy. However the conflict is highly circumscribed because the interests of the members of one interest group are in opposition to the interests of the others only because they have to protect themselves as ordinary consumers of the products of the other industries. In addition the trade policy is perfectly observable and the politicians must care about reelection: it follows that there is little room for interest groups influence. Indeed Golberg and Maggi (1999) show empirically that the government officials give much more weight to social welfare than to contributions received from the pressure groups. The conflict between the interest groups (which in this case may often be identified to large firms) is likely to be much more intense when

they compete for “ favors “ distributed by incumbent politicians. Moreover the distribution of favors and the link with the contributions paid by the pressure groups are generally neither directly nor easily observable by the voters: it is very likely that politicians care less in this case about social welfare. Several papers have considered lobbying as an “ all-pay auction “ where several bidders submit simultaneously nonnegative bids and a “ political prize “ is awarded to the highest bidder (Hillman and Riley (1989), Baye et al. ((1993),(1996)), Che and Gale (1998)). These papers however deal only, in a complete information setting<sup>1</sup>, with the case where there is only one “ political prize “ for sale. The authors argue that all-pay auctions are better suited than ordinary auctions for studying the case of lobbying since bidders cannot here credibly commit themselves to pay a monetary contribution *after* having received the prize or, alternatively, that the politician cannot commit herself to return contributions made by unsuccessful lobbyists). We don’t think the argument very convincing both on empirical and theoretical grounds. First with all-pay auctions we can symmetrically ask what is the incentive for the politician to give the prize to the highest bidder since contributions are made in advance: obviously when analyzing lobbying games where no explicit enforceable contracts are possible one must refer to a reputation argument in an implicit repeated game framework. Second there are some well-known examples of contributions made ex-post by the winner (as, for instance, in the Dassault case where the Flemish and Walloon Socialist Parties received the funds *after* the Belgian government has decided to buy the Dassault planes). In addition we think that politicians have generally more than a single favor to distribute and that this feature may be of importance: lobbying should be analyzed as a simultaneous *multi-unit auction* and there is no available model of multi-unit all-pay auctions. In our model the interest groups compete for “ favors “ in limited supply which are distributed by an incumbent politician<sup>2</sup>. For instance in many countries various groups, associations, industries or unions compete for subsidies or positions from the government or from jurisdictions like towns or regional authorities<sup>3</sup>. These favors can take different forms and may be very difficult to monitor precisely.

<sup>1</sup>The only to consider all-pay auctions with incomplete information are Amann and Leininger (1996).

<sup>2</sup>In the US case the role of lobbying groups in providing campaign contributions (nearly half of the total contributions in the 1992 campaign for the House of Representatives) and the lobbyists influence on legislation is well documented in Levitt (1985).

<sup>3</sup>The politician may for instance choose to buy planes for the Airforce from a given firm which contributes to the party to which he belongs (as in the Agusta-Dassault case in Belgium) or, at a local level, to choose a firm rather than its competitors for supplying water to the residents of a town. In France two major companies (*Compagnie Générale des Eaux*, now *Vivendi*, and *Lyonnaise des eaux*) compete for the supplying of cities and have been involved in several cases of illegal monetary transfers to local incumbent politicians (including the previous french minister of Environment Alain Carrignon, now

Hence each group is generally not able to know either the global amount of favors distributed by the government or the amount of favors received by any of its competitors. This is the reason why the model of common agency under complete information pioneered by Bernheim and Whinston (1986) and adapted by Grossman and Helpman to trade policy determination cannot be used for analyzing the distribution of “favors” : the total supply of such “favors” is likely to be private information of the politician. Moreover each interest group can condition its transfers to the politician only on the favors it receives and not on the favors received by the other ones<sup>4</sup>. In these two respects our model is close to the models of common agency with adverse selection which are the subject of Stole (1990) and Martimort ((1992),(1996)).

In Martimort (1992) the agent chooses two “activity levels” which, together with the value of a private information parameter, affect her “well-being” measured in monetary units, each principal being interested only by one “activity” of the agent. Martimort is then able to show that the equilibrium activity levels are lower (larger) than under cooperation (i.e. when the two principals maximize their joint payoff) when the two activities are complements<sup>5</sup> (substitutes). However he does not give an explicit solution neither for the contribution schedules nor for the activity level functions. Our problem is different from Martimort’s: we study an example of “*menu auctions*” (according to Bernheim and Whinston (1986) definition) *with incomplete information*<sup>6</sup>, which is the best model for analyzing the “sale of favors” to interest groups by an incumbent politician<sup>7</sup>. As in Martimort (1992) the action of the seller has two components affecting separately the payoffs of the buyers. However here this action does not affect the utility of the seller: instead the set of possible actions is such that the sum of the two “activity levels”, say the total “activity level”, cannot be larger than a given level which is here private information of the seller. Technically we are concerned with the division of a stock between two buyers when only the seller knows the dimension of this stock and each buyer offers monetary payments which are contingent on the part of the stock which will be

---

in jail). In the US case Boylan (1998) gives a thorough account of *private bills* and the major scandals associated with them.

<sup>4</sup>Whereas in Bernheim and Whinston (1986) the payments are conditional on all the components of the action chosen.

<sup>5</sup>They are complements (substitutes) when the second cross-derivative of the agent’s utility function is negative (positive).

<sup>6</sup>The purpose of this paper is *not* to extend generally Bernheim and Whinston (1986a) in order to include incomplete information: it is rather to analyze an example of a menu auction with adverse selection.

<sup>7</sup>In the following, for the sake of simplicity, we will speak of “buyers” and “seller” instead of “interest groups” (or “principals”) and “politician” (or “agent”).

allocated to her<sup>8</sup>. Besides the problem of the sale of “ favors” on which we focus here note that there are many other examples of such situations, though they may not perfectly square with all the features of the model. The object for sale may be the future crop of a monopolized agricultural product or a stock of raw materials to be sold to several large buyers for transformation, distribution or even consumption purposes. It may be an amount of capital to be invested by a single large owner who has to choose what will be her respective investments in two jurisdictions. One may also think of the total amount of services which can be offered by a merchandise seller or a broker. Some mundane examples are given by Bernheim and Whinston (1986) such as waiters allocating their services among regular dinners or academic secretaries among professors. Another example deals with a firm supplying a resource (e.g. electricity) to several customers but whose production capacity may be subject to random shocks: in order to allocate its random supply (especially in the case of output falls) i.e. to proceed to some ex post rationing, it may ask the users to submit in advance monetary offers to allocate the priority on the rationed capacity. As noted by Noussair and Porter (1992), one could of course wait for the demand and supply conditions to be revealed and then run a spot market to allocate use but under this type market organization, demanders desiring to plan in advance must correctly anticipate spot prices and supplies. Alternatively, the use of complete contingent contracts can assist in providing information during production planning but the verification costs and the number of such contracts can be prohibitive.

In this paper the equilibrium contribution schedules of the buyers can be fully characterized. The intuition for the results is simple. Each buyer knows that the other one will not pay more for an extra unit of the good than its marginal utility for the good and accordingly is not willing herself to pay more than this value. However the overall supply of the good being unknown the value of this marginal utility is itself unknown: it follows that the marginal contribution of each buyer is a weighted mean of the marginal utilities of the other buyer. In the symmetrical case of identical buyers utility functions the differentiable equilibrium is symmetrical and, as a consequence, the allocation of the good is Pareto-efficient. We will show however that *this efficiency result does not extend to the asymmetric case*. Roughly speaking some inefficiency occurs in the latter case because, on average, whereas in equilibrium both buyers understate their marginal benefit from the consumption of the “good”, the buyer who values the more the good understates it more than the other. Another important result appears to be the following: the rents of the two buyers (i.e. their

---

<sup>8</sup>We follow here Martimort (1992). By contrast in Bernheim and Whinston, principals are allowed to condition their payments on the entire action taken by the agent.

net utilities measured in monetary units), for each value of the private information parameter, appear to be positively related to the concavity of the utility function. *The more concave are the buyers valuation functions the larger their rents and the lower the seller's one.* On the contrary linear valuation functions result in zero buyers' rents. In other words the distribution of the surplus between the buyers and the seller is determined in a very intuitive way by the relative intensity of the competition between the buyers. The latter can be approximately measured by the inverse of the absolute value of the second order derivatives of the buyers valuation functions. Intuitively the seller needs all the more a second buyer (and more generally additional buyers) as the marginal utility for the good of one buyer decreases quickly with the quantity which she consumes.

Note that the menu auctions raise a fundamental question: why to use them rather than "unitized auctions" whenever the revenue which the seller gets by using the former is less than the revenue he would get by using the latter? This phenomenon which occurs in our model in the perfect information case was pointed out by Wilson (1979) in his analysis of share auctions under a variety of assumptions. True, as noted by Bernheim and Whinston (1986), menu auctions differ from share auctions in the type of bids permitted but we will see the same phenomenon happens for menu auctions. Therefore why does not the seller precommit to use a "unitized" auction? We can think of many reasons why he would do so<sup>9</sup>. As noted by Wilson (1979) "the proposal is designed to enable smaller and more risk-averse firms to participate in the auctions of highly risky leases by allowing them to bid for fractional working-interest shares, thereby reducing their capital requirements for payment of the sale price, and also reducing their exposure to risk". It must be note that in a menu auction or in a share auction, the bidders net utilities are in general larger than their utilities if they don't participate: everybody wins something. Therefore if the transaction costs for participating are not too important all the potential bidders will come. In the case of a "unitized auction", only the seller and the highest bidder derive a net utility larger than the utilities if they don't participate. Consequently it may be the case that some bidders do not show up. This would not be a problem for the seller as long as the second highest bidder shows up but strictly speaking why should he do so if he expects to be second? Anyway as long as there is a positive probability in a unitized auction that the second bidder does not show up, there is a risk for the seller of an insufficient competition that could balance and

---

<sup>9</sup>Apart the rather simple answer that it can be impossible for a politician to precommit to "sell" a stock of "favors" in unitized auction when selling favors is itself illegal, though not uncommon, in most countries.

eventually reverse the superiority of the “unitized auction”<sup>10</sup>. We confess that the above argument is a very heuristic defense of what Wilson refers to for suggesting the use of share auctions and for sure a more rigorous game theoretical approach should be conducted to appreciate its scope of validity.

In Section 2 we present a simple discrete example in which there are two units for sale. In Section 3 we present the model in the perfectly divisible case. In Section 4 we analyze the complete information case. In Section 5 we study the differentiable equilibria in the private information case. Concluding remarks are offered in Section 6.

## 2. A SIMPLE DISCRETE EXAMPLE

Suppose a politician has one or two positions to allot (equivalently a seller has one or two bottles of wine to sell) and there are two groups which compete for positions, each of them deriving a strictly positive utility  $\alpha$  from one position but no additional utility from a second position, who submit bids for “buying” one or two positions. We suppose that these bids cannot be conditional on the number of positions allotted to the other interest group nor on the total number of positions distributed<sup>11</sup>. Let us assume that the politician derives no utility from vacant positions. Let  $R_i(x)$  the contribution offered by group  $i$  for a number  $x$  of positions. Under our assumptions we will always observe  $R_i(1) = R_i(2) = r_i$ : an interest group will never be ready to pay something to get a useless second position. This position will always be allotted to the other group<sup>12</sup>. In the case of equal bids for a unique position it is allotted to each group with probability 0.5

If the number of positions to sell is known by the groups before submitting their bids the equilibrium of the complete information game is a couple  $(r_1, r_2)$  such that  $r_1 = r_2 = \alpha$  if there is only one position for sale and  $r_1 = r_2 = 0$  if there are two positions for sale because in the latter case all competition between the groups disappears.

If now the pressure groups do not know if the politician has one or two positions for sale *each of them has to balance two forces*: if there were only one position she would be ready to pay up to  $\alpha$  in order to

<sup>10</sup>By the same argument consumers in an oligopolistic market where firms decide first whether or not they enter the market and incur a sunk cost before competing in prices or quantities in a second stage may prefer Cournot competition to Bertrand competition.

<sup>11</sup>Here we differ from Bernheim and Whinston (1986a) where the principals are allowed to condition their payments on the entire action taken by the agent. This assumption does not make any difference in the complete information case but becomes important in the incomplete information one.

<sup>12</sup>We suppose that the politician will never keep a vacant position.

overbid her competitor but in the opposite case she could get a position for nothing. Obviously what is the optimal strategy depends on the other group's proposed contribution: when it is high the best is to offer nothing and hope that there will be two positions whereas when it is low the best is to offer slightly more and get one position whatever will happen.

Let us now indeed search for interest group  $i$ 's best reply to a bid  $r_j$  by the other interest group. Suppose that both pressure groups have the same priors:  $p$  is the (strictly positive) probability that the politician has two positions to distribute. Bidding 0 yields to  $i$  an expected utility level of  $p\alpha$  (she gets a position only if the politician happens to hold two positions), obviously larger than what she could obtain by bidding  $r_i \in (0, r_j)$ . Bidding  $r_j + \epsilon, \epsilon > 0$ , would yield to  $i$  an expected utility level of  $(\alpha - r_j - \epsilon)$ , tending toward  $\alpha - r_j$  as  $\epsilon \rightarrow 0$ , since here she is going get one position with probability 1. Interest group  $i$ 's best reply is now clear: bid  $r_i = 0$  if  $r_j \geq (1 - p)\alpha$  and bid "just above"  $r_j$  otherwise. It is now straightforward that this game has no pure strategy equilibrium. However there is mixed-strategy equilibrium in which each interest group's strategy is a probability distribution over the support  $[0, (1 - p)\alpha]$ . The equilibrium cumulative distribution function of bids is given by<sup>13</sup>

$$\Psi(r) = \frac{pr}{(1 - p)(\alpha - r)}$$

The support of the distribution of bids shrinks when the probability  $p$  increases: if it is more and more likely that there are indeed two positions for sale the pressure groups are less and less ready to pay for getting one position. This simple example is easily generalized to include the case where the marginal utility  $\beta$  of the second position is positive (while remaining lower than  $\alpha$ ): what is important for our results is the assumption of a decreasing marginal utility and the impossibility of bids conditional on the number of positions allotted to the other interest group<sup>14</sup>. Note that in this case the equilibrium allocation is always Pareto-efficient: when there happens to be two positions each interest group gets one.

We assumed above that when there are two positions they are sold simultaneously. However the corresponding sequential auction game has exactly the same equilibrium bidding strategies for the sale of the first position which is sold at the same prices in both cases. The only difference is that a second position would be sold at a zero price in a sequential auction: the seller strictly prefers a simultaneous auction which allows him to take advantage of the uncertainty on the number of favors for sale.

<sup>13</sup>The proof is straightforward: we leave it to the reader.

<sup>14</sup>Allowing such conditional bids would eliminate any difference with the complete information case.

### 3. THE BASIC MODEL

Consider two risk-neutral buyers and an infinitely risk-averse seller who requires a non-negative payoff in every state of nature<sup>15</sup>. The seller has to allocate a fixed amount  $\Theta$  of the perfectly divisible “commodity” between the two buyers. The two buyers have the same priors on  $\Theta$  (which is private information of the seller) represented by a cumulative distribution function  $F(\cdot)$  on  $[0, +\infty)$  with a strictly positive density function  $f(\cdot)$ . Although the supply of the good is limited the buyers are not exactly sure what is the upper bound in the support of  $\theta$  so, from their perspective,  $\theta$  is a random variable with unbounded support<sup>16</sup>. By the same argument though the supply is positive the players don’t know with certainty where the lower bound in the support of  $\theta$  is so, from their perspective, 0 is the lower bound. We only need to suppose that  $f(\Theta)$  is strictly positive over  $\mathfrak{R}^+$  but it may be very small.

Let  $q_i \in \mathfrak{R}^+$  be the amount allocated to principal  $i$ .  $q_1$  and  $q_2$  should respect the constraint

$$q_1 + q_2 \leq \Theta \tag{1}$$

Principal  $i$  ( $i = 1, 2$ ) offers to the seller non-negative monetary payments  $R_i(q_i)$  contingent on the value of  $q_i$  chosen. The monetary payment offered by a principal  $i$  to the seller cannot be conditional on  $q_j$ . We differ here from Bernheim and Whinston (1986) by assuming either that  $q_j$  cannot be observed by principal  $i$  or that contracts conditioning payments by  $i$  on  $q_j$  are not legally enforceable (either because they are prohibited or because  $q_j$  is not observable by the courts). In our interpretation of this model it is indeed reasonable to suppose that the amount of “favors” distributed to competitors is not observable by a given pressure group or, more generally, by anybody else than the politician and the members of the group. This is a key assumption in this model: if the buyers were allowed to condition their payments on the quantities sold to the other buyers we would be back to the complete information model.

We suppose that the seller does not derive any direct utility or disutility from  $q_1$  and  $q_2$ . Her utility is equal to the payments received from the

---

<sup>15</sup>This constraint may be institutional or follow from limited liability.

<sup>16</sup>This assumption will avoid the “corner problems” which appear when the support of  $f(\cdot)$  is a closed bounded interval: in this case the buyers equilibrium strategies are defined only when they have a strictly positive probability of being implemented, that is for values of  $\Theta$  belonging to the interval. However, if we want to be sure that an equilibrium has been reached, we need to consider deviations from this equilibrium, deviations which may lead to activity levels which are outside the equilibrium set of activities. Martimort’s (1992) solution is to extend the strategies outside the set of activities which can be observed in equilibrium by using first-order conditions. With our solution (extending the support of  $\Theta$  to  $\mathfrak{R}^+$ ) the reaction of a principal to a deviation by the seller or the other principal is unambiguously defined by her equilibrium strategy.

buyers, i.e. to  $R_1 + R_2$ . The buyers have quasi linear utility (valuation) functions:

$$V_i(q_i) - R_i$$

In the following we will restrict our attention to the symmetric case where the two buyers have the same utility function  $V$ , except when we will explicitly suppose the contrary.

We now introduce three assumptions. The first one requires that the maximum expected utility level (surplus) is finite: this condition, besides being a sensible one, is needed in order to ensure that the buyers problems are bounded.

$$\text{ASSUMPTION 1. } \int_0^{+\infty} V\left(\frac{\Theta}{2}\right) f(\Theta) d\Theta \ll +\infty$$

The second assumption is now usual: the hazard rate  $h(\Theta) = \frac{f(\Theta)}{1-F(\Theta)}$  should not be decreasing in  $\Theta$ .

$$\text{ASSUMPTION 2. } \frac{dh(\Theta)}{d\Theta} \geq 0.$$

We finally introduce a third, non-standard assumption.

$$\text{ASSUMPTION 3. } \frac{dM(x)}{dx} \leq 0 \text{ where } M(x) = h(2x)V'(x).$$

Note that this assumption is obviously satisfied when the hazard rate is constant ( $g(\Theta) = e^{-k\Theta} \rightarrow h(\Theta) = k$  when  $\Theta \in [0, +\infty)$ <sup>17</sup>). Other examples where Assumption 3 is satisfied include cases where the hazard rate does not increase “too fast” relative to the decrease of the marginal valuation  $V'$  ( $F(\Theta) = 1 - e^{-\frac{\Theta^{\beta+1}}{\beta+1}}$ ,  $V(q) = q^\alpha$  with  $\alpha + \beta < 1$  is an example among others where the three above assumptions are satisfied).

The strategic interaction between the buyers and the seller is modeled as a three stage game. In a first stage the two buyers offer simultaneously their contribution schedules to the seller. In a second stage the seller accepts or refuses the offers (contrary to Martimort (1992) he can accept only one offer while rejecting the other). Finally in the third stage, the seller learns the value of  $\theta$  and chooses the values of  $q_1$  and  $q_2$ .

## 4. EQUILIBRIUM PREDICTIONS

### 4.1. The Complete Information Benchmark

<sup>17</sup>It is also easy to construct a constant hazard rate distribution when the support of  $\Theta$  is bounded.

Two cases are usually of interest: complete information (i.e. the buyers know the value of  $\Theta$ ) and cooperation (the two buyers maximize the sum of their expected utility levels, i.e. they act as if there was only one principal). The second one is here trivial: a single principal would leave no rent to the seller and allocate efficiently  $\Theta$  between  $q_1$  and  $q_2$ . We study below the complete information case.

As proved by Bernheim and Whinston (1986) the values of  $q_1$  and  $q_2$  should maximize the sum of the utilities of the seller and each principal  $i$  ( $i = 1, 2$ )

$$\begin{aligned} \underset{q_i, q_j}{Max} \quad & V(q_i) + R_j(q_j), \quad i, j = 1, 2, \quad i \neq j \\ \text{s.t.} \quad & q_i + q_j \leq \Theta \end{aligned} \quad (2)$$

On the other hand the values of  $q_1$  and  $q_2$  should also maximize the utility of the seller, i.e.  $R_i(q_i) + R_j(q_j)$  subject to the same constraint. From Bernheim and Whinston we know that a couple  $(R_1(\cdot), R_2(\cdot))$  of *truthful* contribution schedules such that  $R_i(q_i) = V(q_i) - K_i$ ,  $i = 1, 2$ , is a Nash equilibrium iff the constants  $K_i$  are the largest ones satisfying the constraints<sup>18</sup>

$$\begin{aligned} K_1 &\leq W(\{1, 2\}) - W(\{2\}) \\ K_2 &\leq W(\{1, 2\}) - W(\{1\}) \\ K_1 + K_2 &\leq W(\{1, 2\}) \end{aligned} \quad (3)$$

where  $W(\{1, 2\}) = \underset{q_1, q_2}{Max} \{ V(q_1) + V(q_2) \text{ s.t. } q_i + q_j \leq \Theta \} = 2V\left(\frac{\Theta}{2}\right)$ ,  $W(\{2\}) = V(\Theta) = W(\{1\})$ . Basically the constraints mean that the seller cannot be better by accepting the offer of only one principal or by rejecting both offers. Given our assumptions on the function  $V$  the only solution for the constants is given by:

$$K_i = 2V\left(\frac{\Theta}{2}\right) - V(\Theta), \quad i = 1, 2 \quad (4)$$

Note that  $K_i$  is simply principal  $i$ 's equilibrium rent (net utility level). The seller's rent is then equal itself to  $2[V(\Theta) - V(\frac{\Theta}{2})] > 0$ . Even under complete information the competition between two buyers in a menu auction is enough for a strictly positive seller's rent. Clearly *the buyers equilibrium rents* (net utility levels) are zero whenever the utility function

---

<sup>18</sup>They can be derived from the participation constraints though Bernheim and Whinston do not explicitly consider such constraints. The general structure is explained in Laussel and Le Breton (2001).

$V$  is linear (i.e.  $2V(\frac{\Theta}{2}) = V(\Theta)$ ) and are all the more important the more concave is  $V$ . On the contrary the rent of the seller, which equals  $2[V(\theta) - V(\frac{\theta}{2})]$ , is the lower the more concave is  $V$ . It is even not necessarily always increasing in  $\theta$ . It is the case iff  $V'(\theta) \geq \frac{1}{2}V'(\frac{\theta}{2})$  for all  $\theta$ , which means that the marginal valuation does not decrease too fast. A simple counterexample is the following where  $V(q) = a - \frac{a}{1+q}$ . The seller's rent equals here  $\frac{2a}{2+\theta} - \frac{a}{1+\theta}$ : it is maximum for  $\theta = \sqrt{2}$  and decreases for  $\theta > \sqrt{2}$ .

Finally note that the revenue of the seller in a menu auction is less than the rent  $V(\theta)$  which he would obtain in a "unitized" auction since concavity of  $V$  implies that  $2V(\frac{\theta}{2}) \geq V(\theta)$ .

There are other equilibria in the complete information case than the *truthful equilibrium* exhibited above<sup>19</sup>. Note however that any differentiable equilibrium is truthful and that truthful equilibria are the only coalition-proof equilibria of the game (Bernheim and Whinston (1986)), i.e. the only to be robust to non-binding communication between the buyers.

#### 4.2. The Equilibrium with Adverse Selection

DEFINITION 4.1. An equilibrium of this model is a triple of pure strategies  $(R_1(\cdot), R_2(\cdot), \gamma(R_1(\cdot), R_2(\cdot), \Theta))$  such that :

- (i)  $\gamma(R_1(\cdot), R_2(\cdot), \Theta) \in \text{Arg Max}_{q_1, q_2} [R_1(q_1) + R_2(q_2)]$  subject to  $q_1 + q_2 \leq \Theta$
- (ii)  $R_i(\cdot) \in \text{Arg Max}_{\rho_i(\cdot)} E_{\Theta} [V(q_i(\Theta)) - \rho_i(q_i(\Theta))]$ , where  $E$  is the mathematical expectations operator, subject to  $(q_1(\Theta), q_2(\Theta)) = \gamma(\rho_i(\cdot), R_j(\cdot), \Theta)$ , and to  $\rho_i(0) \geq 0, \forall \Theta \in \mathfrak{R}^+, i, j = 1, 2, i \neq j$ .

The first part of the definition states that the seller chooses optimally the values of the "activity levels"  $q_1$  and  $q_2$  depending on the "contribution schedules" previously chosen by the buyers and the true value of the private information parameter  $\Theta$ . The second part states that the buyers non cooperatively and simultaneously choose their contribution schedules in order to maximize their expected net utility. Note that  $\gamma(\cdot)$  as defined above is single-valued: when the seller's problem does not have a unique solution, it is supposed that  $\gamma(\cdot)$  selects one solution among all the possible ones. The constraint that *the buyers should never require from the seller a strictly positive payment is enough to ensure that the participation constraints are*

<sup>19</sup>For instance  $R_i(q_i) = V(\Theta)$  for  $q_i = \Theta$  and  $R_i(q_i) = 0$  otherwise,  $i = 1, 2$ , is a Nash equilibrium.

automatically satisfied<sup>20</sup>. Suppose indeed that  $(q_1(\Theta), q_2(\Theta)) \in \text{Arg Max}_{q_1, q_2} [R_1(q_1) + R_2(q_2)]$  subject to  $q_1 + q_2 \leq \Theta$ . Then clearly

$$R_1(q_1(\Theta)) + R_2(q_2(\Theta)) \geq R_i(q'_i) + R_j(0)$$

for all  $0 \leq q'_i \leq \Theta$  and  $i, j = 1, 2, i \neq j$ . If  $R_j(0) \geq 0$  we immediately conclude that the participation constraints of the seller are satisfied since

$$R_1(q_1(\Theta)) + R_2(q_2(\Theta)) \geq R_i(q'_i)$$

for all  $0 \leq q'_i \leq \Theta$  and  $i, j = 1, 2, i \neq j$ , (the seller cannot increase her utility by contracting only with one principal) and, on the other hand,

$$R_1(q_1(\Theta)) + R_2(q_2(\Theta)) \geq 0$$

so that the seller cannot be better off by rejecting the offers of both buyers.

We will now restrict our attention to the differentiable equilibria of the game by assuming that the contribution schedules are  $C^2$ <sup>21</sup>. In the single principal-agent literature this differentiability restriction is derived from a “revealed preference” argument and a single-crossing property of the agent’s utility function whereas in *common* agency theory the agent’s utility depends on the contract offered by the other principal and any single-crossing condition should involve some assumption on the other seller’s equilibrium contribution schedule.

For the sake of simplicity<sup>22</sup> let us first assume that the equilibrium contribution schedules are such that: (i)  $R'_i(q) > 0$  for all  $q \geq 0$ <sup>23</sup> (ii)  $R''_i(q) < 0$  for all  $q \geq 0$  and (iii)  $R'_i(0) = +\infty$ <sup>24</sup>. We will check *ex post* these assumptions.

Note that the last condition ensures that any non-decreasing function  $q_i(\Theta)$  is *implementable* (on this point see for instance Fudenberg and Tirole (1991), p. 260-262), that we may apply the Implicit function Theorem to derive the functions  $q_i(\Theta)$  from the seller’s equilibrium condition and that these functions are  $C^2$ . Moreover the optimality conditions for the seller’s problem now imply:

$$\frac{d\pi(\Theta)}{d\Theta} = R'_j(\Theta - q_i(\Theta)), \quad i = 1, 2, \quad i \neq j \quad (5)$$

<sup>20</sup>It is even sufficient to require that this constraint is satisfied for zero activity levels.

<sup>21</sup>Grossman and Helpman ((1994), footnote 8) argued by using non differentiable contribution schedules the sellers could suffer large losses from small miscalculations.

<sup>22</sup>See Martimort ((1992), (1996)) for a similar approach in a common agency model.

<sup>23</sup>This implies trivially that  $q_1(\Theta) + q_2(\Theta) = \Theta$ .

<sup>24</sup>Hence the first-order conditions of the agent’s problem, namely  $R'_1(q_1) = R'_2(\Theta - q_1)$ , give the unique solution of this problem for all  $\Theta \in \mathbb{R}^+$ .

where  $\pi(\Theta) = \underset{q_1, q_2}{Max} [R_1(q_1) + R_2(q_2)]$  s.t.  $q_1 + q_2 \leq \Theta$ . Integrating this equation we obtain

$$\pi(\Theta) = \int_0^\Theta R'_j(s - q_i(s)) ds$$

From the above incentive constraint if buyer  $i$  decides to increase his share of the total supply  $s$  for values  $s < \Theta$  this results in a larger rent for the type  $\Theta$ -seller. Indeed in this case the type  $\Theta$ -seller should receive a larger transfer from buyer  $i$  since principal  $j$ 's marginal contribution rate is increased for values of  $q_j < \Theta - q_i(\Theta)$ , creating an incentive for the seller to allocate to  $j$  more than  $\Theta - q_i(\Theta)$  and hence less to  $i$  than  $q_i(\Theta)$ .

Using the implementability property and substituting in principal  $i$ 's objective function  $\pi(\Theta) - R_j(\Theta - q_i(\Theta))$  for  $R_i(q_i(\Theta))$  (the so-called "Mirrlees trick") we can simplify principal  $i$ 's problem and rewrite it as follows:

$$\underset{q_i(\Theta)}{Max} \int_0^\infty [V(q_i(\Theta)) + R_j(\Theta - q_i(\Theta)) - \pi(\Theta)] f(\Theta) d\Theta$$

subject to (5) and

$$\pi(\Theta) \geq \underset{R_j(\Theta)}{Max} [0, R_j(\Theta)]$$

Note that the last, participation, constraint is automatically satisfied when, as assumed,  $R_i(0)$  is constrained to be non negative for  $i = 1, 2$ . It is clearly optimal for each principal  $i$  to set  $R_i(0) = 0$ . Integrating by parts the principal  $i$ 's maximand we obtain the following expression to be maximized with respect to  $q_i(\Theta)$ :

$$\int_0^\infty [(V(q_i(\Theta)) + R_j(\Theta - q_i(\Theta))) f(\Theta) + (F(\Theta) - 1)R'_j(\Theta - q_i(\Theta))] d\Theta$$

since

$$\int_0^\infty \pi(\Theta) f(\Theta) d\Theta = \lim_{\Theta \rightarrow +\infty} \pi(\Theta)(F(\Theta) - 1) + \int_0^\infty (1 - F(\Theta)) R'_j(\Theta - q_i(\Theta)) d\Theta$$

and, on the other hand,  $\int_0^\infty \pi(\Theta) f(\Theta) d\Theta \leq \int_0^\infty 2V(\frac{\Theta}{2}) f(\Theta) d\Theta \ll +\infty$  (from Assumption 1 and the non negativity of the buyers expected incomes), implying that

$$\lim_{\Theta \rightarrow +\infty} [\pi(\Theta) f(\Theta)] = 0 = \lim_{\Theta \rightarrow +\infty} [\pi(\Theta)(1 - F(\Theta)) h(\Theta)]$$

and hence, since from Assumption 2  $h(\cdot)$  is a non decreasing function of  $\Theta$

$$\lim_{\Theta \rightarrow +\infty} [\pi(\Theta)(F(\Theta) - 1)] = 0.$$

The first-order condition of principal  $i$ 's problem is now:

$$(V'(q_i(\Theta)) - R'_j(\Theta - q_i(\Theta))) f(\Theta) + (1 - F(\Theta))R''_j(\Theta - q_i(\Theta)) = 0 \quad (6)$$

for  $i, j = 1, 2, i \neq j$ .

On the other hand, from the Implicit Function Theorem and the seller's first-order condition

$$\frac{dq_i}{d\Theta} = \frac{-R''_j(q_j)}{R''_j(q_j) + R''_i(q_i)}$$

and using equations (6) we finally obtain for  $i, j = 1, 2, i \neq j$ :

$$\frac{dq_i}{d\Theta} = \frac{V'(q_i(\Theta)) - R'_i(q_i(\Theta))}{V'(q_i(\Theta)) - R'_i(q_i(\Theta)) + V'(q_j(\Theta)) - R'_j(q_j(\Theta))} \quad (7)$$

Let us now suppose, without loss of generality, that  $R'_2(q) > R'_1(q)$  for all  $q \in [\alpha, a]$  where  $\alpha$  is the supremum of the set  $\{u < \Theta, q_1(u) = q_2(u)\}$ <sup>25</sup>. From the seller optimality conditions and the assumption  $R''_i(q) < 0$ ,  $i = 1, 2$ , we deduce that  $q_1(\Theta) < q_2(\Theta)$  for all  $\Theta \in (0, 2a)$ . But equation (7) implies that  $q_1(\Theta) > q_2(\Theta)$  for all  $\Theta \in (0, 2a)$ , a contradiction. We conclude that *there is no asymmetric equilibrium satisfying the conditions initially assumed*.

We now have to search for a symmetric equilibrium. It should satisfy the following differential equation which is derived from equation (6) assuming that the two buyers choose the same contribution schedules and setting  $q = \frac{\Theta}{2}$ <sup>26</sup>:

$$R''(q) = -h(2q)(V'(q) - R'(q)) \quad (8)$$

The solutions of this equation are given by

$$R'(q) = -e^{\int_0^q h(2t)dt} \left[ C + \int_0^q h(2t)V'(t)e^{-\int_0^t h(2u)du} dt \right] \quad (9)$$

where  $C$  is an integration constant. We are searching for an equilibrium where  $R'(q) > 0$ , and  $R''(q) < 0, \forall q \in \mathfrak{R}^+$ . Then, from(8), we deduce that  $R'(q) < V'(q)$ . The integration constant must then be such that:

$$C = - \int_0^\infty h(2t)V'(t)e^{-\int_0^t h(2u)du} dt \quad (10)$$

<sup>25</sup>This supremum is well defined since  $q_1(0) = q_2(0)$ .

<sup>26</sup>If the contribution schedules satisfy the conditions initially postulated then  $q_1(\Theta) = q_2(\Theta) = \frac{\Theta}{2}$ .

Substituting this value for  $C$  in equation (9) we finally obtain

$$R'(q) = \int_q^\infty h(2t)V'(t)e^{-\int_q^t h(2u)du} dt \quad (11)$$

We have now obtained a candidate equilibrium such that the contribution schedules satisfy equation (11) and  $R(0) = 0$ . One should note that  $R'(q)$  is a weighted mean of marginal utilities  $V'(t)$  for values of  $t$  larger than  $q$  and, consequently, that  $V''(q) < 0 \implies R'(q) < V'(q)$ ,  $\forall q \in \mathfrak{R}^+$ . When the buyers have strictly concave valuation functions their *contribution schedules are not truthful*: they voluntarily understate the marginal benefit to the buyers from a marginal increase in the “activity levels”. Each principal  $i$  by doing so wants to reduce the seller’s rents: by lowering the activity levels  $q_i(s)$  for  $s < \Theta$  he increases  $q_j(s) = s - q_i(s)$  and hence reduces the marginal contribution paid by principal  $j$ . He can then reduce the contribution paid to the type  $\Theta$ -seller without inducing him to choose values of  $q_i < q_i(\Theta)$ .

Proposition 1, proved in the appendix asserts that this is indeed an equilibrium.

PROPOSITION 1. *Given Assumptions 1, 2 and 3 the couple  $(R(\cdot), R(\cdot))$  such that  $R(\cdot)$  is defined by equation (11) and by  $R(0) = 0$  is an equilibrium.*

The intuition of this result is the same as in the simple example of Section 2. Principal  $i$  knows that principal  $j$  will not pay a contribution for buying a marginal increase in her stock of favors larger than her utility for this increase. However since she does know in advance the total stock of favors for sale and hence the amount which  $j$  will buy, she offers a marginal contribution which is a weighted mean of  $j$ ’s marginal utilities for larger values of  $q$  (see equation (11)).

From equation (11), integrating by parts we obtain an equivalent expression for the marginal contribution:

$$R'(q) = V'(q) + \int_q^\infty V''(t)e^{-\int_q^t h(2u)du} dt$$

Thus the wedge between the marginal contribution and the marginal utility of any principal for a given amount of “favors” is the larger the more quickly the marginal utility of the principal decreases when the amount of favors acquired increases.

Since  $R(0) = 0$  we finally obtain

$$R(q) = V(q) + \int_0^q \int_s^\infty V''(t)e^{-\int_s^t h(2u)du} dt$$

We conclude that *the rents which can be earned by the buyers are the larger the more concave is their valuation function.* Of course their rents tend toward zero when we consider a sequence  $\{V^n(\cdot)\}$  of functions tending toward a function  $V$  such that  $V'(q) = c$ , where  $c$  is a constant, for all  $q \geq 0$ .

Let us now assume that the buyers utility functions are not necessarily the same and denote by  $\alpha_i = q_i^{-1}$  the inverse of the output function  $q_i$ .<sup>27</sup> Equations (6) and (7) remain valid with the appropriate addition of subscripts to distinguish the utility functions of different buyers. From (6) we easily obtain:

$$R'_i(q) = \int_q^\infty h(\alpha_i(t))V'_j(\alpha_i(t) - t)e^{-\int_q^t h(\alpha_i(u))du} dt$$

Hence the marginal contribution of buyer  $i$  for  $q_i(\theta)$  is a weighted mean of the marginal utilities of  $j$  for values of  $q_j$  larger than  $\theta - q_i(\theta)$ .

We will say that an equilibrium is *efficient* whenever the allocation of the good between the buyer is efficient, i.e. such that  $V'_1(q_1^*(\theta)) = V'_2(\theta - q_1^*(\theta))$ . Proposition 2 states that an equilibrium is efficient iff the buyers are alike.

PROPOSITION 2. *A differentiable equilibrium of the menu auction game with adverse selection is efficient iff the buyers have identical utility functions.*

What does happen in the case of buyers who value differently the good? Let us suppose for instance that buyer 1 has a larger marginal valuation of the good than buyer 2, i.e.  $V'_1(q) > V'_2(q)$ ,  $\forall q \geq 0$  (implying trivially that  $V_1(q) > V_2(q)$ ,  $\forall q > 0$ ). Let  $(q_1^*(\theta), \theta - q_1^*(\theta))$  and  $(\hat{q}_1(\theta), \theta - \hat{q}_1(\theta))$  be respectively the efficient and the equilibrium allocations of the good between the buyers. It easy to show that  $\frac{\theta}{2} < \hat{q}_1(\theta)$ , i.e. *the buyer with the larger valuation of the good gets in equilibrium a larger share of the good.* Suppose to the contrary that there exists  $(\alpha, \beta)$  such that  $\hat{q}_1(\theta) \leq \theta - \hat{q}_1(\theta)$ ,  $\forall \theta \in (\alpha, \beta)$ . Then from (7) there must exist an interval  $(\alpha, \gamma] \subset (\alpha, \beta)$  such that  $V'_1(\hat{q}_1(\theta)) - R'_1(\hat{q}_1(\theta)) \leq V'_2(\theta - \hat{q}_1(\theta)) - R'_2(\theta - \hat{q}_1(\theta))$  and, since  $R'_1(\hat{q}_1(\theta)) = R'_2(\theta - \hat{q}_1(\theta))$  from the seller's first-order condition, we obtain  $V'_1(\hat{q}_1(\theta)) \geq V'_2(\theta - \hat{q}_1(\theta)) \Leftrightarrow \hat{q}_1(\theta) > \theta - \hat{q}_1(\theta)$ ,  $\forall \theta \in (\alpha, \gamma]$ , hence a contradiction.

On the other hand it is easy to show that  $\hat{q}_1(\theta)$  cannot be equal to or larger than  $q_1^*(\theta)$  for all  $\theta$ : on average *the buyer who values the more the good gets a share of the good lower than the efficient one* Suppose to the contrary that  $\hat{q}_1(\theta) \geq q_1^*(\theta) > \frac{\theta}{2}$ ,  $\forall \theta$ . This is equivalent to  $V'_1(\hat{q}_1(\theta)) \leq V'_2(\theta - \hat{q}_1(\theta))$ ,  $\forall \theta$ . Then from (7) and the seller's first-order condition we obtain  $\frac{d\hat{q}_1(\theta)}{d\theta} \leq \frac{1}{2}$ ,  $\forall \theta$  contradicting the inequality  $\hat{q}_1(\theta) > \frac{\theta}{2}$ .

<sup>27</sup>It is strictly increasing from our assumptions on the contribution schedules.

### 5. CONCLUDING REMARKS

In this paper we have studied the distribution of favors by an incumbent politician when the total supply of favors is private information of the politician. We have shown that, whenever the two interest groups (buyers) have the same valuation function, this kind of private information does not preclude an efficient allocation of the supply between them. However we showed that this efficiency result does not hold with asymmetric buyers. We derived explicitly the equilibrium contribution schedules of the buyers which, contrary to the perfect information case, are not truthful except in the limiting case of linear valuation functions: in equilibrium the buyers always offer an amount of money which is lower than their valuation of the quantity to be acquired and, more importantly, their marginal offer is lower than their marginal valuation of the commodity. The difference between the two is the larger the more concave is their valuation function. Finally the buyers rents (net utilities) appear to be themselves the larger the more concave their utility function. Interestingly this result is qualitatively the same as in the perfect information case and seems then to be robust. Intuitively indeed if the marginal utilities of the users for the good or the service decrease sharply with the quantity used, the supplier has a strong incentive to divide his stock between the bidders and cannot credibly threaten to sell it only to one of them.

An extension of our model to encompass the case of  $N > 2$  identical interest groups (or “buyers”) is possible and leaves our qualitative results unchanged. The equilibrium contribution function becomes

$$R(q, N) = V(q) + \int_0^q \int_s^\infty V''(t) e^{-\int_s^t h(Nu) du} dt$$

and differentiating with respect to  $N$  we obtain<sup>28</sup>

$$\frac{\partial}{\partial N} R(q, N) = - \int_0^q \int_s^\infty \left( \int_s^t u h'(Nu) du \right) V''(t) e^{-\int_s^t h(Nu) du} dt \geq 0$$

Hence *the difference between the money value for a “buyer” of a given quantity of the “good” and the monetary contributions offered for it decreases as the number of buyers increases*: there is less to gain by understating one’s valuation of a good or service when the number of people who are interested in buying it increases. This for instance very clear in the discrete example of Section 2: if the number of interest groups is 3 or more the only equilibrium of the game has each interest group offering a contribution  $\alpha$  for “buying” one position since even if there two positions

<sup>28</sup>Remember that, from Assumption 2, the hazard rate is increasing in its argument.

for sale there will always be a competition between two or more buyers for the second position so nobody can expect to get the second unit at a lower “price”.

**APPENDIX: PROOFS OF PROPOSITIONS 1 AND 2**

**Proof of Proposition 1**

- From the above definition of  $R(\cdot)$  and the assumptions  $V'(q) > 0$  and  $V''(q) < 0$  we deduce that (i)  $R'(q) > 0$  and (ii)  $R'(q) < V'(q), \forall q \in \mathfrak{R}^+$ . Then (i) at any equilibrium the constraint (1) is binding and (ii) from equation (8)  $R''(q) < 0 \forall q \in \mathfrak{R}^+$ : the agent’s pay-off function is globally strictly concave. Given the couple of contribution schedules defined in Proposition 1 above, choosing  $q_1(\Theta) = q_2(\Theta) = \frac{\Theta}{2}$  maximizes the agent’s pay-off for each  $\Theta \in \mathfrak{R}^+$ .

- Let us now consider the derivative of principal 1’s expected utility with respect to  $q_1$  as above defined (the LHS of equation (3)) and substitute for  $R'(\Theta - q_1)$  its value as defined in equation (8). Use the agent’s first-order condition and the fact above demonstrated that the constraint (1) is binding in equilibrium and obtain:

$$\frac{dE(W_1)}{dq_1} = [V'(q_1) - V'(\Theta - q_1)]h(\Theta) + [h(\Theta) - h(2\Theta - q_1)] [V'(\Theta - q_1) - R'(\Theta - q_1)]$$

After substituting for  $R'(\Theta - q_1)$  its value from the definition of  $R(\cdot)$ , we finally obtain:

$$\begin{aligned} \frac{dE(W_1)}{dq_1} &= [V'(q_1) - V'(\Theta - q_1)]h(\Theta) + [h(\Theta) - h(\Theta - 2q_1)]V'(\Theta - q_1) \\ &\quad - \int_{\Theta - q_1}^{\infty} h(2t)V'(t)e^{-\int_{\Theta - q_1}^t h(2u)du} dt \end{aligned}$$

Under Assumptions 2, 3 and 4, the above expression is negative when  $q_1 > \frac{\Theta}{2}$  and positive when  $q_1 < \frac{\Theta}{2}$ : this follows from the fact that a)  $[V'(q_1) - V'(\Theta - q_1)]h(\Theta) + [h(\Theta) - h(2\Theta - 2q_1)]V'(\Theta - q_1)$  equals zero for  $\Theta = 2q_1$  and is strictly decreasing in  $q_1$  and b) the sign of  $-[h(\Theta) - h(2\Theta - 2q_1)] \int_{\Theta - q_1}^{\infty} h(2t)V'(t)e^{-\int_{\Theta - q_1}^t h(2u)du} dt$  is the sign of  $\Theta - 2q_1$ . This completes the proof. ■

**Proof of Proposition 2**

- (i) *sufficiency*: straightforward
- (ii) *necessity*:

Suppose that  $(q_1^*(\theta), \theta - q_1^*(\theta))$  is an efficient equilibrium allocation. From the definition of an efficient equilibrium  $V_1'(q_1^*(\theta)) = V_2'(\theta - q_1^*(\theta))$  and from the seller first-order condition  $R_1'(q_1^*(\theta)) = R_2'(\theta - q_1^*(\theta))$ . We obtain  $V_1'(q_1^*(\theta)) - R_1'(q_1^*(\theta)) = V_2'(\theta - q_1^*(\theta)) - R_2'(\theta - q_1^*(\theta))$  and hence, from (7)  $q_1^*(\theta) = \frac{\theta}{2}, \forall \theta \geq 0$ . It follows that, for efficiency, the utility functions must be identical. ■

## REFERENCES

- Amann, E. and Leininger, W., 1996, Asymmetric all-pay auctions with incomplete information. *Games and Economic Behaviour* **14**, 1-18.
- Austen-Smith, D. and Wright, J.R., 1992, Competitive lobbying for a legislator vote. *Social Choice and Welfare* **9**, 229-257.
- Austen-Smith, D. and Wright, J.R., 1994, Counteractive lobbying. *American Journal of Political Science* **38**, 25-44.
- Ball, R., 1991, Political lobbying as welfare improving signalling. Department of Agricultural and Resource Economics, University of Berkeley, Working Paper.
- Becker, G.S., 1983, A theory of competition among pressure groups for political influence. *Quarterly Journal of Economics* **98**, 371-400.
- Banks, J. and Weingast, B. R., 1992, The political control of bureaucracies under asymmetric information. *American Journal of Political Science* **36**, 509-524.
- Baye, M.R., Kovenock, D., and de Vries, C.G., 1993, Rigging the lobbying process: An application of all-pay auctions. *American Economic Review* **83**, 289-294.
- Baye, M.R., Kovenock, D., and de Vries, C.G., 1996, The all-pay auction with complete information. *Economic Theory* **8**, 291-305.
- Bernheim, B. D. and Whinston, M., 1986, Menu auctions, resource allocation, and economic influence. *Quarterly Journal of Economics* **101**, 1-33.
- Boylan, R.T., 1998, Private bills: A theoretical and empirical study of lobbying. *Mimeo*. University of Washington.
- Goldberg, P.K. and Maggi, G., 1999, Protection for sale: An empirical investigation. *American Economic Review* **89**, 1135-1155.
- Grossman G. and Helpman, H., 1994, Protection for sale. *American Economic Review* **85**, 833-850.
- Grossman G. and Helpman, H., 1995, Trade wars and trade talks. *Journal of Political Economy* **103**, 675-708
- Hillman, A.L. and Riley, J.G., 1989, Politically contestable rents and transfers. *Economics and Politics* **1**, 17-39.
- Krueger, A. O., 1974, The political economy of the rent seeking society. *American Economic Review* **6**, 291-303.
- Laussel, D. and Le Breton, M., 2001, Conflict and cooperation: The structure of equilibrium payoffs in common agency. *Journal of Economic Theory* **100**, 93-128.
- Levitt, S.D., 1995, Congressional campaign finance reform. *Journal of Economic Perspectives* **9**, 183-93.
- Lohmann, S., 1995, Information, access, and contributions: A signaling model of lobbying. *Public Choice* **85**, 267-284.

- Magee, S.P., Brock, W.A., and Young, L., 1989, *Lack Hole Tariffs and Endogenous Policy Theory: Political Economy in General Equilibrium*. Cambridge: Cambridge University Press.
- Martimort, D., 1992, Multiprincipaux avec anti-Sélection. *Annales d'Economie et de Statistique* **28**, 1-37.
- Martimort, D., 1996, Exclusive dealing, common Agency, and multiprincipals incentive theory. *Rand Journal of Economics* **27**, 1-31.
- Moulin, H., 1988, *Axioms of Cooperative Decision Making*. Cambridge, Cambridge University Press.
- Snyder, J.M., 1990, Campaign contributions as investments: The US house of representatives 1980-1986. *Journal of Political Economy* **98**, 1195-1227.
- Snyder, J.M., 1991, On buying Legislatures. *Economics and Politics* **3**, 93-109.
- Stole, L., 1990, Mechanism design under common agency. MIT, *Mimeo*.
- Tullock, G., 1967, The welfare cost of tariffs, monopolies, and theft. *Western Economic Journal* **5**, 224-232.