

## Equilibrium Selling Mechanisms\*

Yongmin Chen

*Economics Department, University of Colorado at Boulder*

and

Ruqu Wang

*Economics Department, Queen's University and University of Colorado at Boulder*  
E-mail: ruqu.wang@colorado.edu

We consider the equilibrium choice of selling mechanisms by competing firms. For a model where a number of sellers choose *sequentially* between any two selling mechanisms, there is a unique (subgame perfect) equilibrium under fairly natural assumptions about the monotonicity and differences of the two mechanisms. All sellers choose the mechanism that has the higher per-seller surplus at a critical mass number of sellers. If a mechanism is efficient or is favored by the buyer in some “strong” sense, it will be selected as the equilibrium mechanism. Otherwise, the less efficient mechanism can emerge in equilibrium, even when the number of sellers is arbitrarily large. An increase in the number of sellers need not increase the buyer’s surplus, and can sometimes lead to a less efficient equilibrium mechanism. When more than two selling mechanisms are available, however, the equilibrium may no longer be unique; and there are usually multiple equilibria when sellers choose selling mechanisms simultaneously. © 2004 Peking University Press

*Key Words:* Selling formats; Competing mechanisms.

*JEL Classification Numbers:* C72, D43.

### 1. INTRODUCTION

In the past two decades, the study of price formation and market performance has moved beyond the framework of a given trading mechanism. Significant attention has been paid to the comparison of different trading formats. Given a set of alternative trading formats, researchers have inves-

\* We thank Stefan Dodds for his helpful comments. Chen acknowledges financial support from the US National Science Foundation under grant #SES 9911229. Wang’s research is supported by the Social Sciences and Humanities Research Council of Canada.

tigated issues such as which trading format is more attractive to buyers, which one is more profitable to sellers, or which one is more efficient socially. (See, e.g., Riley and Zeckhauser, 1983; Wang, 1993, 1995; Wernerfelt, 1994; Bulow and Klemperer, 1996; and Miller, Piankov and Zeckhauser, 2001.) Furthermore, the mechanism design literature has addressed the question of optimal selling mechanisms in certain market environments, such as Myerson (1981), Riley and Samuelson (1981), and Chen and Rosenthal (1996).

When sellers compete in a marketplace where many trading formats are available, they must decide what trading mechanism to use. This has led to the examination of endogenously determined mechanisms in competitive settings. Bester (1993) considers the endogenous selection between bargaining and posted-price selling using a model of competition between sellers who can control their product quality. McAfee (1993) studies a dynamic model where each seller announces his trading mechanism to attract buyers, and finds that auctions with efficient reserve prices are the equilibrium choice of the sellers. When the number of buyers and sellers becomes large, Peters (1994) finds that posting fixed prices by all sellers is an equilibrium if each seller announces a trading mechanism after a buyer arrives. More recently, Neeman and Vulkan (2002) explore a model of trading through bargaining versus a centralized market, and find that all serious buyers and sellers will eventually opt for trading through the centralized market. Ellison, Fudenberg and Mobius (forthcoming) study the competition between two auction markets, where buyers and sellers simultaneously choose the markets to participate, and multiple equilibria always exist. Their focus is to find conditions under which two competing markets (selling institutions) can co-exist.<sup>1</sup>

In this paper, we extend and further study a model initially proposed in Chen and Wang (forthcoming; henceforth CR). CR's model departs from most of the existing studies of competing selling mechanisms in two important ways. First, instead of being concerned with some specific selling formats, CR allows sellers to choose from *any* two arbitrary selling mechanisms. Second, instead of assuming that sellers choose their mechanisms *and* compete for buyers at the same time, CR considers a two-stage model. At the first stage, sellers *sequentially* adopt their selling mechanisms, and, observing their choices, homogeneous buyers choose the mechanism they will participate in. At the second stage, those sellers (whose particular selling mechanism has been chosen by the buyers) compete according to the

---

<sup>1</sup>Other recent contributions include Lu and McAfee (1996)'s model of choosing between auctions and bargaining; Burguet and Sakovics (1999)'s model of competition in choosing reserve prices in auctions; and Camera and Delacroix (2001)'s investigation of the endogenous determination of bargaining versus posted prices in a search economy.

rules of the mechanism.<sup>2</sup> The main result stated there is that, if certain monotonicity conditions are satisfied and if the two mechanisms have at least some minimum differences, the (subgame perfect) equilibrium mechanism is unique and it maximizes the per-seller surplus at a critical mass. The present paper will provide a complete proof for this result (which was only sketched in CR), provide additional results about the properties of this equilibrium, and further consider equilibrium selling mechanisms when there are more than two selling mechanisms and when sellers choose their selling mechanisms simultaneously. Importantly, with more than two selling formats available, the equilibrium mechanism is no longer unique even when sellers choose their format sequentially (same as in CR); and there are usually multiple equilibria when sellers choose selling formats simultaneously.

Our results suggest that some of the familiar intuition about competition under a given selling format may no longer be valid when sellers compete in selling mechanisms. For instance, under a given selling method, increased competition that reduces sellers' profits tends to benefit the buyers. But here, this competition can result in the sellers choosing a different mechanism that in turn makes the buyers worse off. Extending the analysis of competition to a setting where selling formats are determined endogenously can thus be important in understanding how a market functions. Nevertheless, competition in selling mechanisms also has similarities to competition under a given selling mechanism. For instance, as under a given selling method, there is no guarantee that competition will lead to efficiency, no matter how large is the number of the sellers. (Conditions ensuring efficiency will be provide in the analysis, however.)

In the rest of the paper, we describe our basic model in Section 2, characterize the equilibrium and analyze its properties in Section 3, extend the basic model in Section 4, and conclude in Section 5.

## 2. THE BASIC MODEL

$N > 1$  ex-ante identical sellers can supply a homogeneous product to a buyer.<sup>3</sup> There are two possible selling mechanisms,  $m_1$  and  $m_2$ , that seller

---

<sup>2</sup>The assumption that buyers choose a mechanism before knowing the price (if any) is meant to capture a realistic aspect of buyer behavior. For example, some buyers may decide to shop over the internet instead of visiting a local market, while others do the opposite; or, some buyers may prefer to buy rugs in auctions, while others prefer going to a store. Of course, these behaviors are driven by expectations of the final transaction prices. Buyers will choose a mechanism that maximizes their expected surplus, and the expected surplus can be calculated after knowing the number of sellers who adopt that mechanism.

<sup>3</sup>As it will become clear later, with a minor qualification our analysis is valid for a continuum of identical buyers of mass 1, in which case the single buyer in our analysis

$j$  can choose from, where  $j = 1, \dots, N$ . Sellers choose  $m1$  or  $m2$  sequentially. Without loss of generality, assume that seller 1 chooses first, followed by seller 2, 3,  $\dots$ ,  $N$ . Denote seller  $j$ 's choice by  $s_j \in \{m1, m2\}$ . After observing  $(s_1, \dots, s_N)$ , the buyer decides the mechanism to participate in. Denote the buyer's choice by  $b(n) \in (m1, m2)$ , where  $n$  is the number of sellers who choose  $m1$ . A possible transaction then occurs accordingly.

We allow  $m1$  and  $m2$  to be any type of trading mechanisms. For instance,  $m1$  could be each seller posting a fixed price and  $m2$  could be a second-price, sealed-bid procurement auction with the buyer setting a reserve price. When  $k$  sellers choose  $m_i$  and the buyer also participates in  $m_i$ ,  $0 \leq k \leq N$  and  $i = 1, 2$ , the expected surplus (payoff) of the buyer and the total expected surplus (profits) of the  $k$  sellers are denoted as  $B_i(k)$  and  $S_i(k)$ , respectively. A seller's profit is zero if he chooses a mechanism in which the buyer does not participate. We maintain the following two assumptions throughout the paper:

**A1.**  $B_i(0) = 0$ ,  $B_i(1) < B_l(N)$  for  $i \neq l$ , and  $B_i(k) \leq B_i(k+1)$  for  $k = 0, \dots, N-1$ .

**A2.**  $\frac{S_i(k)}{k} > \frac{S_i(k+1)}{k+1} > 0$  for  $k = 1, \dots, N-1$  and  $\frac{S_i(k)}{k} \rightarrow 0$  when  $k \rightarrow \infty$ .

Assumption A1 states that the buyer will receive zero payoff if she chooses to participate in a mechanism that is not offered by any seller; that a mechanism chosen by all  $N$  sellers will give higher surplus to the buyer than the other mechanism with only one seller; and that the buyer's surplus from a mechanism does not decrease in the number of sellers participating in the mechanism. Assumption A2 states that the average payoff to the sellers in a mechanism in which the buyer participates decreases in the number of the sellers choosing the mechanism, and this average payoff goes to zero when the number of sellers choosing the mechanism goes to infinity. These two assumptions are satisfied by most commonly observed mechanisms.

Because  $B_i(N) > B_l(1)$  and  $B_i(0) < B_l(N)$ , for  $l \neq i$ , and  $B_i(k)$  weakly increases in  $k$ , we immediately have:

LEMMA 1. For  $i, l = 1, 2$  and  $l \neq i$ , there is a unique number  $n_i \in \{1, \dots, N\}$  such that

$$\begin{aligned} B_i(n_i) &> B_l(N - n_i), \\ B_i(n_i - 1) &\leq B_l(N - n_i + 1). \end{aligned}$$

---

should be considered as a representative of the continuous buyers. We shall point out this qualification in Section 3.

Thus,  $n_i$  is the critical mass for the number of sellers in mechanism  $m_i$  to attract the buyer: the buyer will choose to participate  $m_i$  if there are  $n_i$  sellers who have chosen  $m_i$ .

We shall in addition assume the following, which rules out the buyer's and sellers' indifference around the critical number of  $n_i$ :

**A3.** For  $i, l = 1, 2$  and  $l \neq i$ ,  $B_i(n_i - 1) \neq B_l(N - n_i + 1)$ , and

$$\frac{S_2(N - n_1 + 1)}{N - n_1 + 1} \neq \frac{S_1(n_1)}{n_1}.$$

Notice that assumption A3 changes the weak inequality in Lemma 1 to a strict inequality. Then, by inverting the two inequalities in Lemma 1, we have  $n_2 = N - n_1 + 1$ . Implied from A3 is that  $m_1$  and  $m_2$  have to be different. If  $m_1$  and  $m_2$  were identical, assuming  $B_i(k)$  is strictly increasing, we would have: if  $N$  is even,  $n_1 = n_2 = \frac{N}{2} + 1$  and  $B_i(n_i - 1) = B_l(N - n_i + 1)$ ; if  $N$  is odd,  $n_1 = n_2 = \frac{N+1}{2}$  and  $\frac{S_2(N - n_1 + 1)}{N - n_1 + 1} = \frac{S_1(n_1)}{n_1}$ . In either of these cases, A3 would be violated. Obviously, the equilibrium cannot be unique when the two mechanisms are identical. A3 above provides the minimum differences between the two mechanisms that are required to guarantee that the equilibrium is unique.

### 3. EQUILIBRIUM ANALYSIS

We now characterize the equilibrium outcome of the game and study its properties.

**THEOREM 1.** *Under the assumptions of A1, A2, and A3, we have:*

(i) *If*

$$\frac{S_2(N - n_1 + 1)}{N - n_1 + 1} > \frac{S_1(n_1)}{n_1}, \quad (1)$$

*all sellers choose  $m_2$  (i.e.,  $s_j = m_2$  for all  $j$ ) in the unique subgame-perfect equilibrium outcome of the game;*

(ii) *If*

$$\frac{S_2(N - n_1 + 1)}{N - n_1 + 1} < \frac{S_1(n_1)}{n_1}, \quad (2)$$

*all sellers choose  $m_1$  (i.e.,  $s_j = m_1$  for all  $j$ ) in the unique subgame-perfect equilibrium outcome of the game.*

*Proof.* We shall prove using backward induction.

Suppose first that condition (1) holds. Consider the choice by the last seller, i.e., seller  $N$ . If at least  $n_1$  sellers have already chosen  $m_1$ , the buyer

will choose  $m1$  no matter what  $N$  chooses. Thus, it is optimal for  $N$  to choose  $m1$  as well. If fewer than  $n_1 - 1$  sellers have chosen  $m1$ , fewer than  $n_1$  sellers will choose  $m1$  regardless of seller  $N$ 's choice, which implies that the buyer will choose  $m2$  and thus seller  $N$  should also choose  $m2$ . The more complicated case is when exactly  $n_1 - 1$  sellers have chosen  $m1$  and thus exactly  $N - n_1$  sellers have chosen  $m2$ . In this case, the buyer will choose  $m1$  if seller  $N$  chooses  $m1$ , and choose  $m2$  if seller  $N$  chooses  $m2$ . Seller  $N$ 's profit is  $S_1(n_1)/n_1$  in the former and  $S_2(N - n_1 + 1)/(N - n_1 + 1)$  in the latter. Given condition (1), he will choose  $m2$ . Thus, seller  $N$  will choose  $m2$  if at most  $n_1 - 1$  sellers have chosen  $m1$ , and will choose  $m1$  otherwise.

Now suppose that sellers  $k + 1$  through  $N$  will choose  $m2$  if at most  $n_1 - 1$  sellers have chosen  $m1$ , and will choose  $m1$  otherwise. We show that the same strategy is optimal for seller  $k$  as well.

If more than  $n_1 - 1$  sellers have already chosen  $m1$ , clearly seller  $k$  should choose  $m1$  as well, since the buyer will choose  $m1$ .

If at most  $n_1 - 1$  sellers have chosen  $m1$ , and seller  $k$  chooses  $m2$ , then the number of sellers choosing  $m1$  is still at most  $n_1 - 1$ , and therefore, all subsequent sellers will choose  $m2$  according to the specified strategy. Thus, the buyer will choose  $m2$  and seller  $k$ 's payoff is equal to  $S_2(N - n)/(N - n)$ , where  $n \leq n_1 - 1$  is the total number of sellers choosing  $m1$ .

If at most  $n_1 - 1$  sellers have chosen  $m1$  and seller  $k$  chooses  $m1$ , then the number of sellers choosing  $m1$  becomes  $n + 1 \leq n_1$ . If  $n + 1 < n_1$ , all subsequent sellers will choose  $m2$  according to the specified strategy. Thus, the buyer will choose  $m2$  and seller  $k$ 's payoff is zero. If  $n + 1 = n_1$ , then all subsequent sellers will choose  $m1$ , the buyer will choose  $m1$ , and seller  $k$ 's payoff is equal to  $S_1(n_1 + N - k)/(n_1 + N - k)$ , which is at most  $S_1(n_1)/n_1$ , since  $S_1(x)/x$  is non-increasing in  $x$ . But from our earlier analysis, for  $n = n_1 - 1$ , by choosing  $m2$  seller  $k$  earns  $S_2(N - n_1 + 1)/(N - n_1 + 1)$ , which is larger than  $S_1(n_1)/n_1$ . Therefore, he should choose  $m2$ .

Hence, the specified strategy is optimal for seller  $k$  as well. By mathematical induction, we can conclude that any seller will choose  $m2$  if at most  $n_1 - 1$  sellers have already chosen  $m1$ , and will choose  $m1$  otherwise.

Since at the beginning no seller has chosen  $m1$ , seller 1 will choose  $m2$ , and so will all subsequent sellers.

Suppose next that condition (2) holds. Let  $n_2 = N - n_1 + 1$ . Then the definition of  $n_1$  implies that we have

$$\begin{aligned} B_2(n_2) &> B_1(N - n_2), \\ B_2(n_2 - 1) &< B_1(N - n_1 + 1). \end{aligned}$$

But condition (2) can be written as

$$\frac{S_1(N - n_2 + 1)}{N - n_2 + 1} > \frac{S_2(n_2)}{n_2}.$$

The proof of (i) above therefore implies that all sellers choose  $m1$  in equilibrium. ■

Since we may consider  $A1$  and  $A2$  as monotonic conditions on the buyer's surplus and per-seller's profits, and  $A3$  as requiring the two mechanisms to have at least some minimum differences, Theorem 1 suggests that monotonicity and difference of the mechanisms are needed to ensure the equilibrium to be unique. In fact, these conditions seem quite intuitive and quite natural, if one hopes to obtain equilibrium uniqueness. Without these conditions, the buyer and/or a seller may face the same payoffs from choosing both mechanisms, which results in multiple equilibria. For instance, suppose it were the case that  $\frac{S_2(N-n_1+1)}{N-n_1+1} = \frac{S_1(n_1)}{n_1}$ . Consider the situation when exactly  $n_1 - 1$  sellers have chosen  $m1$  and thus exactly  $N - n_1$  sellers have chosen  $m2$ . Then, the buyer will choose  $m1$  if seller  $N$  chooses  $m1$ , and choose  $m2$  if seller  $N$  chooses  $m2$ . Seller  $N$ 's profit is  $S_1(n_1)/n_1$  in the former and  $S_2(N - n_1 + 1)/(N - n_1 + 1)$  in the latter. But if  $\frac{S_2(N-n_1+1)}{N-n_1+1} = \frac{S_1(n_1)}{n_1}$ , the seller is indifferent between choosing  $m1$  and  $m2$ . If his strategy is to choose  $m1$ , in equilibrium all sellers will choose  $m1$ ; and if his strategy is to choose  $m2$ , in equilibrium all sellers will choose  $m2$ . While the monotonicity and difference of the two mechanisms seem quite natural assumptions, it is interesting to see how they relate to the critical numbers of sellers that play a key role in our analysis.<sup>4</sup>

We also find it intriguing that the equilibrium mechanism is determined by the comparison of the two mechanisms' per-seller profits at the critical mass numbers  $n_1$  and  $N - n_1 + 1$ . In a sense, if  $n_1 < N - n_1 + 1$ ,  $m1$  is better for the buyer, but whether  $m1$  will be selected by the sellers in equilibrium depends on the sellers' preference at those critical numbers.

We next consider the question of whether and when the equilibrium mechanism will be the one that is optimal for the buyer and/or efficient. We shall say that

1.  $mi$  is a *common-interest* mechanism, if for all  $k = 1, \dots, N$ ,  $B_i(k) \geq B_j(k)$  and  $S_i(k) \geq S_j(k)$ , with strict inequality for some  $k$ , where  $j \neq i$ .

<sup>4</sup>Notice that if there were a continuum of identical buyers, each buyer would make the same decision as the representative buyer in our analysis, as long as the representative buyer always has a unique optimal move at every information set that she is assigned to. This qualification is satisfied in our model under the assumptions of  $A1$ ,  $A2$ , and  $A3$ .

2.  $mi$  is *efficient* if, for all  $k = 1, \dots, N$ ,  $B_i(k) + S_i(k) \geq B_j(k) + S_j(k)$ , with strict inequality for some  $k$ , where  $j \neq i$ .

Notice that our efficiency concept is a relative one. If one mechanism is efficient, the other must be less efficient or inefficient. While two mechanisms may not always be comparable in the sense of efficiency, we are mostly interested in situations where they are comparable, as are the case in the two examples that we shall consider later. We have:

PROPOSITION 1. *If  $mi$  is a common-interest mechanism, then all sellers choose  $mi$  in the unique subgame-perfect equilibrium.*

*Proof.* See Appendix. ■

Notice that by definition a common-interest mechanism must be efficient. In fact, since in a common-interest mechanism both the buyer and the sellers prefer it to the alternative mechanism, the common-interest mechanism is efficient in a “strong” sense, and it seems intuitive that every seller would choose it. Potentially, however, it is still possible that there is failure of coordination among the sellers and the inefficient mechanism occurs in equilibrium. Our proof rules out such a possibility.

What happens when the buyer prefers a mechanism that may not be preferred by the sellers? As a corollary to Theorem 1, that mechanism will be chosen by the sellers if certain conditions are satisfied.

PROPOSITION 2. *Suppose that there exists some given  $\hat{n}$ , independent of  $N$ , such that  $B_i(\hat{n}_i) > B_l(k)$  for all  $k$ , where  $l \neq i$ . Then, all sellers choose  $mi$  if  $N$  is sufficiently large.*

*Proof.* Suppose that such a  $\hat{n}$  exists. Clearly, the critical mass number for mechanism  $mi$ ,  $n_i \leq \hat{n}$ . But it must be true that, when  $N$  is sufficiently large,

$$\frac{S_i(n_i)}{n_i} > \frac{S_l(N - n_i + 1)}{N - n_i + 1}.$$

From Theorem 1,  $s_j^* = mi$  for all  $j$ . ■

If the buyer likes  $mi$  so much more than  $ml$  that  $mi$  with just a few sellers dominates  $ml$  with any number of sellers, then  $mi$  must be chosen in equilibrium as long as there are enough sellers around. Thus, if a mechanism is favored by the buyer in a “strong” sense, it will be the equilibrium mechanism as the market becomes more competitive. Notice that for this situation to occur, the joint surplus of the buyer and the sellers, or the total surplus, under  $ml$  must be much higher than that under  $mi$ . For



example, under  $m1$  each seller simultaneously posts a price, and under  $m2$  the buyer makes a final price offer to a randomly selected seller. In this case, increasing the number of sellers in  $m2$  does not increase total surplus, and  $m1$  would be selected if  $N$  is sufficiently large.

On the other hand, even if one mechanism is both efficient and preferred by the buyer, in general there is no guarantee that it will be chosen in equilibrium (see example 1 later for an illustration). This is because the sellers' preference has to be considered as well. However, if a seller's interest is not too different from that of the buyer, then the mechanism that is both efficient and favored by the buyer will be selected in equilibrium, as in the following:

PROPOSITION 3. *Assume that  $B_1(k) + S_1(k) \geq B_2(k) + S_2(k)$  and  $B_1(k) > B_2(k)$  for all  $k$ . Then there exists some  $\delta > 0$  such that if*

$$\frac{S_1(k)}{k} > \frac{S_2(k)}{k} - \delta,$$

*for all  $k$ , every seller chooses  $m1$  in equilibrium.*

*Proof.* See Appendix. ■

We now turn our attention to the relationship between the number of sellers and consumer surplus. When sellers compete in prices, it is often the case that an increase in the number of sellers leads to lower prices and larger consumer surplus.<sup>5</sup> In our model, this is not always true. The following example illustrates that more sellers do not always increase the buyer's surplus. Furthermore, it illustrates that the equilibrium mechanism may be the less efficient one.

EXAMPLE 3.1. Suppose that in  $m1$  each seller posts a price, and, in  $m2$  the buyer randomly chooses a seller and makes a take-it-or-leave-it price offer. Thus, in contrast to  $m1$ , the buyer sets the price in  $m2$ .<sup>6</sup> Assume that each seller's cost  $c$  is distributed i.i.d. according to the c.d.f.  $F(c)$ . The buyer has a unit demand for the product, with her valuation  $v$  distributed according to the c.d.f.  $G(v)$ . After the choice of mechanisms is sequentially made by all sellers, and then by the buyer, each seller's cost realization is learned by the seller himself as well as by all other sellers, but it remains unknown to the buyer. The realization of the buyer's valuation

<sup>5</sup>There are, however, exceptions. See, for instance, Rosenthal (1981) for a model where the presence of more sellers actually reduces consumer surplus.

<sup>6</sup>This mechanism is somewhat similar to the one used by Priceline.com, an internet retailer which sells airline tickets, etc., by letting consumers name their own prices. It is not clear, however, whether Priceline.com takes each consumer's offer to just one seller or to more than one seller.

is learned by the buyer and remains her private information. Assume that  $F(c) = c/t$ , where  $0 < c < t$  with parameter  $t < 2/5$ , and  $G(v) = v$ , where  $0 < v < 1$ .

In this example,  $n_1 = 2$  regardless of the value of  $N$ . That is, the buyer prefers  $m_1$  with two sellers to  $m_2$  with any number of sellers. Also,  $m_1$  is efficient. However, in equilibrium,  $m_2$  will be chosen if  $N = 2$  or  $N = 3$ , and  $m_1$  will be chosen if  $N \geq 4$ . (See the Appendix for the detailed calculations.)

This example shows that the efficient mechanism need not be chosen in equilibrium. Furthermore, when  $N$  increases from 2 to 3 in this example, neither the buyer's surplus nor the total surplus is increased. However, in this example when the number of sellers ( $N$ ) is large enough, the efficient mechanism will be the equilibrium mechanism and the buyer's surplus will increase in the number of sellers. Intuitively, while total surplus is higher under the efficient mechanism, it is not possible to make transfer payments from the buyer to the seller. Hence, it is possible that a mechanism is both efficient and preferred to by the buyer, but the per-seller profit at the critical mass is so low that the mechanism is not selected in equilibrium. On the other hand, this example satisfies the condition in Proposition 2, since  $B_1(2) > B_2(k)$  for all  $k$ ; thus when  $N$  is large, the mechanism preferred to by the buyer, which is also efficient, will be chosen in equilibrium.

However, even if the number of sellers is large, it is *not* always true that the equilibrium mechanism will be efficient. To see this, consider the next example.

**EXAMPLE 3.2.** In  $m_1$  each seller posts a price and in  $m_2$  sellers participate in a second-price auction with a reserve price set by the buyer. We assume that the buyer has a unit demand for the product for which her valuation is  $v$ , which is always above a seller's cost. This cost is distributed i.i.d. according to the c.d.f.  $F(c)$ . After the choice of mechanism is sequentially made by all sellers and then by the buyer, each seller's cost realization is learned by the seller himself as well as by all other sellers, but remains unknown to the buyer.

In this example,  $n_1 = N/2 + 1$  if  $N$  is even and  $n_1 = (N + 1)/2$  if  $N$  is odd. Whenever  $N$  is even,  $S_2(N/2)/(N/2) > S_1(N/2 + 1)/(N/2 + 1)$ , and thus  $m_2$  will be chosen in equilibrium; while when  $N$  is odd the efficient mechanism will be chosen in equilibrium. (See the Appendix for the detailed calculations.) However,  $m_2$  is less efficient compared to  $m_1$ , since  $m_2$  has an optimal reserve price that is below the buyer's valuation, which sometimes prevents mutually beneficial trade from occurring.

This example demonstrates that it is possible for the less efficient mechanism to be chosen in equilibrium no matter how large  $N$  is (as long as  $N$  is an even number). Moreover, it is possible that an increase in the number of sellers (from an odd number to an even number) changes the equilibrium mechanism from the efficient one to the inefficient one. Unlike in Example 1, where the critical mass  $n_1$  is constant no matter how large  $N$  is, here  $n_1$  increases propositionally in  $N$ , which enables  $\frac{S_2(N-n_1+1)}{N-n_1+1} > \frac{S_1(n_1)}{n_1}$  to hold for any even number of  $N$  and thus  $m_2$  as the equilibrium mechanism.

Summarizing our finding from the two examples, we have the following:

**Observation 1** In our model, it is possible that an increase in the number of sellers increases neither buyer's surplus nor total surplus. It is also possible that an increase in the number of sellers changes the equilibrium from the efficient mechanism to the inefficient one. Furthermore, the efficient mechanism may not be chosen in equilibrium no matter how large the number of competing sellers is.

#### 4. EXTENSIONS

In this section, we relax our previous assumptions by considering two possible extensions of the basic model: allowing more than two selling mechanisms, and allowing sellers to choose their selling mechanisms simultaneously.

##### 4.1. More than two selling mechanisms

When there are only two selling mechanisms available, we have shown that there is a unique subgame perfect equilibrium in a model of sellers choosing mechanisms sequentially. Interestingly, the uniqueness property of the equilibrium need not hold when there are three or more selling mechanisms available. We demonstrate this result in the following example.

EXAMPLE 4.1. Suppose that there are three mechanisms, four sellers and one buyer. The buyer's preference is as follows:

$$B_1(n) < B_2(n) < B_3(n), n = 1, 2, 3, 4.$$

And

$$B_i(n+1) > B_j(n), n = 1, 2, 3, \forall i, j.$$

That is, the buyer prefers  $m_3$  to  $m_2$  to  $m_1$ , given the same number of sellers. Meanwhile, the buyer prefers a mechanism with more sellers to one with fewer sellers: for example,  $B_1(4) > B_3(3)$ . Hence, the buyer will choose to participate in the mechanism with the largest number of sellers,

and if two mechanisms have the same number of sellers, she will choose the one with the higher index.

Assume that the sellers' preference is in the opposite order to the buyer's:

$$\frac{S_1(n)}{n} > \frac{S_2(n)}{n} > \frac{S_3(n)}{n}, n = 1, 2, 3, 4.$$

And

$$\frac{S_i(n+1)}{n+1} < \frac{S_j(n)}{n}, n = 1, 2, 3, \forall i, j.$$

That is, with the same number of sellers in a mechanism, a seller's surplus is highest in  $m_1$ , followed by  $m_2$ , and then  $m_3$ . Meanwhile, a seller has a higher surplus in a mechanism with fewer sellers.

Given these preferences, we can analyze the sellers' choices by using backward induction in the game where sellers choose mechanisms sequentially. We find that there are two equilibrium outcomes. (See Appendix for a detailed analysis.)

**Equilibrium Outcome 1:** The first seller chooses  $m_2$ , with the second seller choosing  $m_1$  and the rest of sellers choosing  $m_2$ . The buyer participates in  $m_2$ , and the second seller earns zero profit.

**Equilibrium Outcome 2:** All sellers choose  $m_3$ , and so does the buyer. In this equilibrium, the sellers choose their least preferred mechanism.

Outcome 2 is easier to understand. Since  $m_3$  is most preferred by the buyer, the sellers have a tendency to choose that mechanism. Outcome 1 is more surprising. Seller 2 chooses a mechanism in which the buyer does not participate. The intuition is as follows. If seller 2 chooses  $m_2$  as well, then sellers 3 and 4 would both choose  $m_3$ , and seller 2 ends up with zero profit. If seller 2 chooses  $m_3$ , then sellers 3 and 4 both choose  $m_1$  to maximize their own profit. Therefore, seller 2 earns zero profit either way.

Hence, we have the following observation:

**Observation 2** With more than two selling mechanisms, there can be multiple equilibria. At some of these equilibria, two or more mechanisms may be chosen by different sellers, implying that some sellers earn zero profit.

#### 4.2. Sellers Moving Simultaneously

We next investigate the situation where sellers choose their selling mechanisms simultaneously. Suppose that there are  $k$  selling mechanisms,  $m_1, m_2, \dots, m_k$ , that the sellers can choose from. Following the earlier notations, let  $B_i(n)$  denote the buyer's surplus from buying from  $m_i$  when there are a total of  $n$  sellers choosing  $m_i$ , and let  $S_i(n)$  be the total surplus of these

$n$  sellers choosing  $mi$  when the buyer participates in  $mi$ . Obviously, since  $S_i(k)/k > 0$  for all  $k$ , and since a seller receives 0 if he chooses a mechanism in which the buyer does not participate, all sellers will choose the same mechanism in any pure-strategy equilibrium. We have the following result.

**THEOREM 2.** *All sellers choosing  $mi$ ,  $i = 1, \dots, k$ , is a subgame-perfect equilibrium in the game where all sellers move simultaneously if and only if*

- (i)  $B_i(N - 1) > B_j(1)$ , for every  $j \neq i$ ; or*  
*(ii) For every  $j$  where  $B_j(1) > B_i(N - 1)$ , the following holds:*

$$S_j(1) < \frac{S_i(N)}{N}.$$

*Proof.* Suppose a seller deviates from the proposed strategy and chooses  $mj$  instead of  $mi$ . This deviation is profitable if and only if the buyer will choose  $mj$  (where there is only one seller) instead of  $mi$  (where there are  $N - 1$  sellers), and the single seller of  $mj$  receives more surplus compared to the equilibrium surplus of  $B_i(N)/N$ . The deviation is not profitable if and only if either of these conditions is not satisfied, i.e. if either (i) or (ii) holds. ■

It is interesting to note that, similar to the model of sequential moves, the most efficient mechanism ( $B_i(N) + S_i(N) > B_j(N) + S_j(N)$ ,  $\forall j \neq i$ ) is not always among the equilibrium mechanisms. Imagine that a mechanism (say,  $m1$ ) is most efficient when there are  $N$  sellers choosing it, but that it gives the buyer all of the surplus from trade. If there is another mechanism (say,  $m2$ ) with one seller, which gives the buyer more surplus than  $m1$  with  $N - 1$  sellers, then  $m1$  is not an equilibrium mechanism. For example, let  $m1$  be the mechanism of posting a price. When there are two sellers whose costs are the same but are private information, it is an efficient mechanism. Let  $m2$  be the mechanism where the buyer makes the final offer to one of the sellers. It is not efficient, but the buyer gets more from  $m2$  (with one seller) than from  $m1$  (with  $N - 1$  sellers). Therefore, both sellers choosing  $m1$  simultaneously is not an equilibrium.

We now return to the mechanisms discussed in Example 3.1. That is, under  $m1$  each seller posts a price and under  $m2$  the buyer randomly chooses a seller and makes a take-it-or-leave-it price offer. Under the assumptions about the seller's cost distribution and the buyer's valuation distribution, we have  $n_1 = 2$ .

If  $N > n_1$ , then,  $B_1(N - 1) > B_2(1)$  since  $B_1(N - 1) > B_1(2)$  and

$$B_1(2) - B_2(1) = \frac{1}{4}t^2 + \frac{1}{2} - \frac{2}{3}t - \left( \frac{2}{3}t^2 + \frac{1}{2} - t \right) = \frac{1}{12}t(4 - 5t) > 0;$$

and  $B_2(N - 1) > B_1(1)$  since  $B_2(N - 1) = B_2(1)$  and

$$B_2(1) - B_1(1) = \frac{2}{3}t^2 + \frac{1}{2} - t - \left( -\frac{1}{8}t + \frac{1}{24}t^2 + \frac{1}{8} \right) = \frac{5}{8}t^2 + \frac{3}{8} - \frac{7}{8}t > 0.$$

Thus, from Theorem 2, it is an equilibrium for all sellers to choose  $m1$  and another equilibrium for all sellers to choose  $m2$ . Notice that for large  $N$ ,  $m1$  is clearly both the efficient mechanism and the mechanism which favors the buyer, yet it need not be the equilibrium mechanism.

On the other hand, if  $N \leq n_1$  (in this case  $N = 2$ , since  $n_1 = 2$ ), we have:  $B_2(N - 1) = B_2(1) > B_1(N - 1) = B_1(1)$  but

$$S_2(1) = \frac{1}{6}t(3 - 4t) > \frac{S_1(2)}{2} = \frac{1}{24}t(4 - 3t);$$

and thus the only (pure-strategy) subgame perfect equilibrium of the game is for all sellers to choose  $m2$ . We thus have the following observation:

**Observation 3** If sellers move simultaneously in choosing selling mechanisms, it is possible that an equilibrium mechanism is inefficient and/or is not favored by the buyer, no matter how many competing sellers there are. It is also possible that adding more sellers results in a mechanism that is less efficient or is less favorable to the buyer.

## 5. CONCLUDING REMARKS

In this paper, we have considered a model of competition in selling mechanisms. When a given number of sellers sequentially choose among any two possible selling mechanisms, there exists a unique (subgame perfect) equilibrium under fairly natural assumptions about monotonicity and differences of the two mechanisms. The unique equilibrium mechanism is the one that gives higher average seller surplus at some critical number of sellers. The equilibrium mechanism will be efficient and/or preferred to by the buyer under some strong sense of efficiency and/or the buyer's preference. Otherwise it need be neither efficient nor preferred to by the buyer, even if the number of sellers is arbitrarily large. An increase in the number of sellers need not increase the buyer's surplus, and can sometimes changes the equilibrium from the efficient mechanism to the less efficient one. It appears that the assumptions of two selling formats and sequential choices

are both crucial for the unique equilibrium result. When there are more than two selling formats or when sellers choose simultaneously, more than one selling format can emerge in equilibrium.<sup>7</sup>

In a model of endogenous trading mechanisms, more competition under a particular selling mechanism can be a mixed blessing to a buyer, since it may result in an equilibrium mechanism that is less desirable to the buyer. This raises interesting issues about how a market may evolve as transaction and information costs become lower, resulting in more competition under a certain trading mechanism. While it is well known that a market need not maximize consumer surplus or be efficient under a certain trading mechanism – even when the number of sellers becomes arbitrarily large – the same result in our model is perhaps more intriguing, since the competition in selling mechanisms here is conducted under complete information.

### APPENDIX

This appendix contains the proofs for Proposition 1, Proposition 3 and the analysis for Examples 3.1, 3.2 and 4.1.

**Proof of Proposition 1** Without loss of generality, suppose that  $m_1$  is the common-interest mechanism. Since  $B_1(k) \geq B_2(k)$  for all  $k$ , we must have

$$B_1(n_1) > B_2(N - n_1)$$

and

$$B_1(n_1 - 1) < B_2(N - n_1 + 1)$$

for some  $n_1 \leq \frac{N}{2}$ . Since  $S_1(k) \geq S_2(k)$  for all  $k$ , we have

$$\frac{S_1(n_1)}{n_1} \geq \frac{S_2(n_1)}{n_1} > \frac{S_2(N - n_1 + 1)}{N - n_1 + 1},$$

since  $N - n_1 + 1 > n_1$  and  $\frac{S_2(k)}{k}$  decreases in  $k$ . Therefore,  $s_j^* = m_1, \forall j$ , in the unique subgame-perfect equilibrium. That is, all sellers choose  $m_1$  in the equilibrium. ■

**Proof of Proposition 3** Since  $B_1(k) > B_2(k)$ , we have  $n_1 \leq \frac{N}{2}$ . Thus

$$\frac{S_2(n_1)}{n_1} > \frac{S_2(N - n_1 + 1)}{N - n_1 + 1}.$$

<sup>7</sup>We have assumed that there is a single buyer, or a continuum of identical buyers. If consumers have heterogeneous preferences for different selling mechanisms, one can easily imagine that there could be different selling mechanisms being offered in equilibrium, similar to the arise of product differentiation.

Let

$$\delta = \frac{S_2(n_1)}{n_1} - \frac{S_2(N - n_1 + 1)}{N - n_1 + 1}.$$

If

$$\frac{S_1(n_1)}{n_1} > \frac{S_2(n_1)}{n_1} - \delta,$$

we have

$$\frac{S_1(n_1)}{n_1} > \frac{S_2(n_1)}{n_1} - \delta = \frac{S_2(N - n_1 + 1)}{N - n_1 + 1}.$$

Thus, from Theorem 1,  $s_j^* = m1$  for all  $j$ . ■

**Analysis for Example 3.1** Since under  $m2$  the buyer will choose price

$$a(v) = \begin{cases} t & \text{if } v > 2t \\ \frac{v}{2} & \text{if } v \leq 2t \end{cases},$$

we have

$$\begin{aligned} B_2(n) &= \int_0^1 ((v - a(v))F(a(v))) dv = \int_0^1 \left( \left( v - \min\left\{t, \frac{v}{2}\right\} \right) \frac{\min\left\{t, \frac{v}{2}\right\}}{t} \right) dv \\ &= \int_0^{2t} \left( \left( v - \frac{v}{2} \right) \frac{\frac{v}{2}}{t} \right) dv + \int_{2t}^1 (v - t) dv = \frac{2}{3}t^2 + \frac{1}{2} - t, \end{aligned}$$

for any  $n = 1, \dots, N$

$$\begin{aligned} B_1(1) &= \int_{\underline{c}}^{\bar{c}} \int_{p_m(c)}^{\bar{v}} (v - p_m(c)) dG(v) dF(c) \\ &= \int_0^t \left( \int_{\frac{1+c}{2}}^1 \left( v - \frac{1+c}{2} \right) dv \right) \frac{1}{t} dc = -\frac{1}{8}t + \frac{1}{24}t^2 + \frac{1}{8}. \end{aligned}$$

For  $n \geq 2$ ,

$$\begin{aligned} B_1(n) &= \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{c_2} \int_{\min\{p^m(c_1), c_2\}}^{\bar{v}} (v - \min\{p^m(c_1), c_2\}) dG(v) \frac{dF(c_1)}{F(c_2)} dF_2(c_2, n) \\ &= \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{c_2} \left( \int_{c_2}^{\bar{v}} (v - c_2) dv \right) \frac{1/t dc_1}{c_2/t} \left( n(n-1) \left[ 1 - \frac{c_2}{t} \right]^{n-2} \frac{c_2}{t^2} \right) dc_2 \\ &= \int_0^t \left( \left( \int_0^{c_2} \left( \int_{c_2}^1 (v - c_2) dv \right) dc_1 \right) n(n-1) \left( 1 - \frac{c_2}{t} \right)^{n-2} \frac{1}{t^2} \right) dc_2 \\ &= \int_0^t \left( \frac{1}{2} (1 - c_2)^2 c_2 n(n-1) \left( 1 - \frac{c_2}{t} \right)^{n-2} \frac{1}{t^2} \right) dc_2. \end{aligned}$$



Therefore,

$$B_1(2) = \int_0^t \left( \frac{1}{2} (1 - c_2)^2 c_2 2(2 - 1) \left( 1 - \frac{c_2}{t} \right)^{2-2} \frac{1}{t^2} \right) dc_2 = \frac{1}{4}t^2 + \frac{1}{2} - \frac{2}{3}t.$$

We have

$$\begin{aligned} & B_1(2) - B_2(N - 2) \\ &= \begin{cases} \frac{1}{4}t^2 + \frac{1}{2} - \frac{2}{3}t - \left( \frac{2}{3}t^2 + \frac{1}{2} - t \right) = \frac{1}{12}t(4 - 5t) > 0 & \text{if } N > 2 \\ \frac{1}{4}t^2 + \frac{1}{2} - \frac{2}{3}t > 0 & \text{if } N = 2 \end{cases}. \end{aligned}$$

$$B_1(1) - B_2(N - 1) = -\frac{1}{8}t + \frac{1}{24}t^2 + \frac{1}{8} - \left( \frac{2}{3}t^2 + \frac{1}{2} - t \right) = \frac{7}{8}t - \frac{5}{8}t^2 - \frac{3}{8} < 0,$$

Thus  $n_1 = 2$ . To determine which mechanism is the equilibrium mechanism, we have

$$\begin{aligned} S_2(n) &= \int_{\underline{c}}^{\min\{\bar{c}, a(1)\}} \left( \int_{a^{-1}(c)}^1 (a(v) - c) dv \right) f(c) dc \\ &= \int_0^t \left( \int_{a^{-1}(c)}^1 (a(v) - c) dv \right) f(c) dc \\ &= \int_0^t \left( \int_{2c}^{2t} \left( \frac{v}{2} - c \right) dv + \int_{2t}^1 (t - c) dv \right) \frac{1}{t} dc = \frac{1}{6}t(3 - 4t) \end{aligned}$$

for any  $n$ , and

$$\begin{aligned} & S_1(n)/n \\ &= \int_{\underline{c}}^{\bar{c}} \int_{c_1}^{\bar{c}} [\min\{p^m(c_1), c\} - c_1][1 - G(\min\{p^m(c_1), c\})] dF_1(c, n - 1) dF(c_1) \\ &= \int_0^t \left( \int_{c_1}^t (c - c_1)(1 - c)(n - 1) \left[ 1 - \frac{c}{t} \right]^{n-2} \frac{1}{t} dc \right) \frac{1}{t} dc_1 \\ &= \frac{t(n + 2 - 3t)}{n(n + 1)(n + 2)}. \end{aligned}$$

So

$$\frac{S_1(n_1)}{n_1} = \frac{1}{24}t(4 - 3t).$$

Comparing it with

$$\frac{S_2(N - n_1 + 1)}{N - n_1 + 1} = \frac{\frac{1}{6}t(3 - 4t)}{N - 1},$$

we conclude that

$$\frac{S_1(n_1)}{n_1} > \frac{S_2(N - n_1 + 1)}{N - n_1 + 1}$$

if and only if  $N \geq 4$ . Therefore,  $m_1$  is the equilibrium mechanism if and only if  $N \geq 4$ . Since  $m_1$  is the efficient mechanism, it is chosen when the number of sellers is sufficiently large in this example. It is easy to verify that the buyer's surplus increases in the number of sellers.

To compare a seller's profit in either mechanism, we have

$$\frac{S_2(N)}{N} = \frac{t(3 - 4t)}{6N},$$

and

$$\frac{S_1(N)}{N} = \frac{t(N + 2 - 3t)}{N(N + 1)(N + 2)}.$$

Given our assumption that  $t < 2/5$ , it is easy to verify that for  $n \geq 4$ ,  $S_2(N)/N \geq 0.23t/N$ , but  $S_1(N)/N < 0.20t/N$ . Therefore, all sellers choosing  $m_2$  induces higher profit than all sellers choosing  $m_1$ . ■

**Analysis for Example 3.2** From Bulow and Klemperer (1996), we can conclude that

$$B_1(n + 1) > B_2(n), \forall n.$$

On the other hand, since  $B_2(n) > B_1(n)$  is always true for all  $n$ , it must be true that the cut-off value  $n_2 = N/2$ , since  $B_2(N/2) > B_1(N/2)$  and  $B_2(N/2 - 1) < B_1(N/2) < B_1(N/2 + 1)$ . Therefore, the cut-off value  $n_1 = N - n_2 + 1 = N/2 + 1$ . (In the case where  $N = 2k + 1$  is an odd number,  $n_2 = k + 1$  and  $n_1 = k + 1$ .)

We know that  $S_1(n) > S_2(n)$ ,  $\forall n$ . This is because a seller has a higher probability of winning and pays a weakly lower price in  $m_1$  than in  $m_2$ , the number of competitors being the same. When  $N$  is odd,  $n_1 = n_2 = k + 1$ , and thus  $S_1(n_1)/n_1 > S_2(n_2)/n_2 = S_2(n_2)/(N - n_1 + 1)$ . So by Theorem 1,  $m_1$  will be the equilibrium mechanism.

When  $N$  is even,  $n_1 = N/2 + 1$  and  $n_2 = N/2$ ,  $S_1(n_1)/n_1 = S_1(N/2 + 1)/(N/2 + 1)$ , and  $S_2(n_2)/n_2 = S_2(N/2)/(N/2)$ . In this case, we compare  $S_1(n_1)/n_1$  and  $S_2(n_2)/n_2$ . Since mechanism  $m_2$  has a reserve price but one fewer seller, the comparison depends on whether a seller prefers an optimal reserve price (set by the buyer) or one more seller. It turns out that there are situations where a seller would prefer a reserve price to one more competing seller. Consider the following:

Suppose that a buyer's value is 1, and that a seller's cost is distributed uniformly on  $[0, x]$ , where  $x < 1$ . The buyer's optimal reserve price  $r$  is

always equal to 0.5 regardless of the value of  $x$ , since it maximizes  $(1 - r)F(r)$ , the buyer's surplus when there is just one seller.

If  $x = 0.5$ , imposing the reserve price does not affect a seller's profit at all, but adding a seller will surely reduce his expected profit. So in this case,  $S_2(N/2)/(N/2)$  is greater than  $S_1(N/2 + 1)/(N/2 + 1)$ , and mechanism  $m_2$  will be chosen in equilibrium. In this case mechanism  $m_2$  is still efficient, however, since the reserve price is equal to the highest possible cost and does not reduce beneficial trade.

However, when  $x$  is higher than but sufficiently close to 0.5, by continuity,  $S_2(N/2)/(N/2)$  is still greater than  $S_1(N/2 + 1)/(N/2 + 1)$ . Then,  $m_2$  will still be the equilibrium mechanism, but now it is not efficient compared to  $m_1$ , since  $m_1$  is equivalent to a second-price auction without a reserve price and it avoids the loss of beneficial trading opportunities under  $m_2$ . ■

**Analysis for Example 4.1** Let  $(n_1, n_2, n_3) \rightarrow (n_1 + 1, n_2, n_3)$  denote that the seller being examined chooses  $m_1$  given that  $n_i$  sellers have already chosen mechanism  $i$ ,  $i = 1, 2, 3$ ; let  $(n_1, n_2, n_3) \rightarrow (n_1, n_2 + 1, n_3)$  denote that the seller being examined chooses  $m_2$  given that  $n_i$  sellers have already chosen mechanism  $i$ ,  $i = 1, 2, 3$ ; and let  $(n_1, n_2, n_3) \rightarrow (n_1, n_2, n_3 + 1)$  denote that the seller being examined chooses  $m_3$  given that  $n_i$  sellers have already chosen mechanism  $i$ ,  $i = 1, 2, 3$ .

We first consider the last (fourth) seller's optimal choice.  $(1, 1, 1) \rightarrow (2, 1, 1)$ ;  $(0, 2, 1) \rightarrow (0, 2, 2)$ ;  $(0, 1, 2) \rightarrow (0, 1, 3)$ ;  $(1, 0, 2) \rightarrow (1, 0, 3)$ ;  $(2, 0, 1) \rightarrow (2, 0, 2)$ ;  $(1, 2, 0) \rightarrow (1, 3, 0)$ ;  $(2, 1, 0) \rightarrow (2, 2, 0)$ ;  $(3, 0, 0) \rightarrow (4, 0, 0)$ ;  $(0, 3, 0) \rightarrow (0, 4, 0)$ ;  $(0, 0, 3) \rightarrow (0, 0, 4)$ .

Given this, the third seller's optimal choice can be described as follows.  $(1, 1, 0) \rightarrow (1, 2, 0) \rightarrow (1, 3, 0)$ ;  $(1, 0, 1) \rightarrow (1, 0, 2) \rightarrow (1, 0, 3)$ ;  $(0, 1, 1) \rightarrow (1, 1, 1) \rightarrow (2, 1, 1)$ ;  $(0, 2, 0) \rightarrow (0, 2, 1) \rightarrow (0, 2, 2)$ ;  $(2, 0, 0) \rightarrow (2, 1, 0) \rightarrow (2, 2, 0)$ ;  $(0, 0, 2) \rightarrow (0, 0, 3) \rightarrow (0, 0, 4)$ .

Based on the third and the fourth sellers' reactions, the second seller's choices are as follows.

If the subgame is  $(1, 0, 0)$ ,

$$\begin{aligned} & \mathbf{(1, 0, 0)} \rightarrow \mathbf{(1, 1, 0)} \rightarrow \mathbf{(1, 3, 0)}; \quad \mathbf{or} \rightarrow \mathbf{(1, 0, 1)} \rightarrow \mathbf{(1, 0, 3)}; \\ & \mathbf{or} \rightarrow \mathbf{(2, 0, 0)} \rightarrow \mathbf{(2, 2, 0)}. \end{aligned}$$

Obviously, the second seller would choose  $m_2$  when the subgame is  $(1, 0, 0)$ , because other choices yield less preferred final outcomes.

If the subgame is  $(0, 1, 0)$ ,

$$\begin{aligned} & \mathbf{(0, 1, 0)} \rightarrow \mathbf{(1, 1, 0)} \rightarrow \mathbf{(1, 3, 0)}; \quad \mathbf{or} \rightarrow \mathbf{(0, 2, 0)} \rightarrow \mathbf{(0, 2, 2)}; \\ & \mathbf{or} \rightarrow \mathbf{(0, 1, 1)} \rightarrow \mathbf{(2, 1, 1)}. \end{aligned}$$

In this subgame, the second seller will earn zero surplus no matter what he chooses. Therefore, he is indifferent among the three choices. Different strategies are used to support the different equilibria later on.

If the subgame is  $(0, 0, 1)$ ,

$$\begin{aligned} &(\mathbf{0}, \mathbf{0}, \mathbf{1}) \rightarrow (\mathbf{1}, \mathbf{0}, \mathbf{1}) \rightarrow (\mathbf{1}, \mathbf{0}, \mathbf{3}); \quad \text{or} \rightarrow (\mathbf{0}, \mathbf{1}, \mathbf{1}) \rightarrow (\mathbf{2}, \mathbf{1}, \mathbf{1}); \\ &\text{or} \rightarrow (\mathbf{0}, \mathbf{0}, \mathbf{2}) \rightarrow (\mathbf{0}, \mathbf{0}, \mathbf{4}). \end{aligned}$$

In this subgame, it is clear that choosing mechanism 3 is the best for seller 2.

Now consider the first seller's choice and the equilibria. We have:

Equilibrium 1: The first seller chooses  $m_2$ , with the second seller choosing  $m_1$  in subgame  $(0, 1, 0)$ . The equilibrium outcome is  $(1, 3, 0)$ . In this equilibrium, the second seller earns zero profit.

Equilibrium 2: The first seller chooses  $m_3$ , with the second seller choosing  $m_2$  or  $m_3$  in subgame  $(0, 1, 0)$ . The equilibrium outcome is  $(0, 0, 4)$ . In this equilibrium, the sellers choose their least preferred mechanism. ■

## REFERENCES

- Bester, Helmut, 1993, Bargaining vs. price competition in markets with quality uncertainty. *American Economic Review* **83**, 278-288.
- Bulow, Jeremy and Paul Klemperer, 1996, Auctions versus negotiations. *American Economic Review* **86**, 180-194.
- Burguet, Roberto and Jozsef Sakovics, 1999, Imperfect competition in auction designs. *International Economic Review* **40**, 231-247.
- Camera, Gabriele and Alain Delacroix, 2001, Bargaining or price posting. Purdue University Department of Economics, *Working Paper*.
- Chen, Yongmin and Robert Rosenthal, 1996, On the use of ceiling-price commitments. *Rand Journal of Economics* **27**, 207-220.
- Chen, Yongmin and Ruqu Wang, A model of competing selling mechanisms. *Economics Letters* Forthcoming.
- Ellison, Glenn, Drew Fudenberg and Markus Mobius, Competing auctions. *Journal of the European Economic Association*.
- Lu, Xiaohua and R. Preston McAfee, 1996, The evolutionary stability of auctions over bargaining. *Games and Economic Behavior* **15**, 228-254.
- McAfee, R. Preston, 1993, Mechanism design by competing sellers. *Econometrica* **61**, 1281-1312.
- Miller, Nolan, Nikita Piankov, and Richard Zeckhauser, 2001, When to haggle. Harvard University, Kennedy School of Government, *Working Paper*.
- Myerson, Roger, 1981, Optimal auction design. *Mathematics of Operations Research* **6**, 58-73.
- Neeman, Zvika and Nir Vulkan, 2002, Markets versus negotiations: The emergence of centralized markets. Boston University, Economics Department, *Working Paper*.

- Peters, Michael, 1994, Equilibrium mechanisms in a decentralized market. *Journal of Economic Theory* **64**, 390-423.
- Riley, John and William Samuelson, 1981, Optimal auctions. *American Economic Review* **71**, 381-392.
- Riley, John and Richard Zeckhauser, 1983, Optimal selling strategies: When to haggle, when to hold firm. *Quarterly Journal of Economics* **98**, 267-289.
- Rosenthal, Robert, 1982, A model in which an increase in the number of sellers leads to a higher price. *Econometrica* **48**, 69-76.
- Wang, Ruqu, 1993, Auctions versus posted-price selling. *American Economic Review* **83**, 838-851.
- Wang, Ruqu, 1995, Bargaining versus posted-price selling. *European Economic Review* **39**, 1747-1764.
- Wernerfelt, Birger, 1994, Selling formats for search goods. *Marketing Science* **13**, 298-309.