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Speculative Attacks and the Dynamics of Exchange Rates

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This paper examines the issue of speculative attacks on the exchange rate in an economy in which. following the attack the government allows the exchange rate to float for a limited period, before repegging it at a higher, sustainable, level. We highlight the tradeoffs between the length of the floating rate period and the timing of the speculative attack, and the implied consequences for the time paths of the exchange rate and consumption. By letting the exchange rate float for an appropriate period, the exchange rate crisis can be delayed for a maximum period of time, although delaying the crisis as long as possible is non-optimal. In some respects our results are qualitatively similar to earlier results based on non-optimizing models, although employing a rational intertemporal optimizing framework substantially enhances our understanding of the determinants of the crises.

Key Words: Speculative attacks; Exchange rates; Dynamcics.
JEL Classification Numbers: F31, F32.

1. INTRODUCTION

The regular recurrence of financial crises throughout the world during the last three decades has ensured that this continues to remain a vibrant topic of research. Formal analysis of a balance of payments crisis was initiated by Krugman (1979).[1] He showed that under a fixed exchange rate regime, domestic credit creation in excess of the growth of money demand leads to a

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[1]Krugman's model was motivated by Salant and Henderson's (1978) analysis of speculative attacks on the government-controlled price of gold.

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gradual loss of government reserves, and ultimately to a speculative attack against the currency. This attack would force the abandonment of the fixed exchange rate and the adoption of a flexible rate regime. He also concluded that the attack would always occur before the central bank would have run out of reserves in the absence of speculation. Soon thereafter, Flood and Garber (1984) employed a similar linear model to determine the time of the collapse of the fixed exchange rate regime.

These early contributions have spawned a substantial literature, extending the original Krugman-Flood-Garber insights in several directions, giving rise to what have become identified as two generations of speculative attack models. A key element of the first generation models is that they involve fixed rules of government behavior, such as maintaining a perfectly fixed exchange rate, and attribute speculative attacks to the inconsistency and unsustainability of such policies.[2] It argues that persistent over-expansionary monetary or fiscal policies, for example, will trigger a speculative attack, and thereby lead to a balance of payment crisis. From a policy-making perspective, these results suggest that it is almost impossible to regain equilibrium from a persistent disequilibrium with a balance-of-payment deficit, by pegging the exchange rate, unless the government also simultaneously fundamentally changes its internal macroeconomic policies (Helpman and Leiderman 1987).

Obstfeld (1986b) presented an alternative view to explain the cause of a speculative attack. He found that even if the level of reserves is sufficient to handle normal pressures on the balance of payments, a speculative attack may still occur as a rational market response. He indicated that a speculative attack could indeed be a self-fulfilling prophesy. Hence consistent macro policies are necessary, but not sufficient, to ensure exchange rate stability (Otker and Pazarbasioglu, 1997). Obstfeld's contribution represents the beginning of the second generation of speculative attack models, the key feature of which is that they focus on endogenizing government policy reactions, relating them to some underlying policy objective. The salient features of these newer models are reviewed by Flood and Marion (1999).

The evolution of these models was motivated in part by changing empirical evidence. The first generation models were motivated largely by the currency crises that occurred in developing countries like Mexico and Argentina during the late 1970s and early 1980s, which typically were the consequence of overly expansive domestic macroeconomic policies. In contrast, the second generation models were stimulated in part by the Euro-

[2]Other key contributions include Flood and Garber (1983), Connolly and Taylor (1984), Obstfeld (1984, 1986a), Calvo (1987), Willman (1988), Edwards (1989), Dellas and Stockman (1993), and Flood Garber, and Kramer (1996). Many of the important earlier contributions to this literature are surveyed by Agenor, Bhandari, and Flood (1992).

pean currency crisis in 1992-1993 and the 1994 Mexican Peso crisis, which differed from the earlier experiences in two respects. In the European case, the attacks seemed unrelated to the economic fundamentals, as predicted by the first-generation models. Moreover, constraints imposed by monetary policies in trading partners often prevented policy makers in the countries experiencing the attacks from using traditional macroeconomic policy instruments to support their exchange rates.

The early studies by Krugman (1979), Flood and Garber (1984), and Calvo (1987) assume that once the speculative attack occurs the central bank withdraws from the foreign exchange market permanently, leaving the exchange rate to float indefinitely. This is a natural benchmark case, particularly for a developed economy having access to well developed financial markets. But the question arises whether or not a purely floating exchange rate is the best regime for a developing economy to adopt. Indeed, in many developing countries balance-of-payment crises often lead to the devaluation of the currency rather than to the abandonment of the fixed exchange rate. There are several reasons for this. First, domestic asset markets for most developing economies are insufficiently developed to insulate them from external shocks. Second, the political structure in developing countries is typically less stable than in developed countries. Governments in those economies usually treat stability as being a prime objective and are willing to intervene to insulate their economies from the outside world for the purposes of protecting the domestic economy. Among economic variables, the exchange rate serves not only as a link between internal and external markets, but also as an important indicator reflecting both political and economic stability. Thus, governments may be reluctant to abandon a fixed exchange rate regime and instead, may prefer a strategy of successive devaluations of the domestic currency.

Thus while policy makers might be forced to give up the fixed exchange rate regime and float the domestic currency temporarily, they may subsequently re-peg the exchange rate at a higher sustainable (devalued) level. Obstfeld (1984) studied this kind of transitional floating exchange rate regime using a linear model.[3] Such an exchange rate regime can be summarized as comprising a three-stage process: an initial fixed exchange rate; a period of a purely floating exchange rate; a final fixed exchange rate, when the exchange rate is re-pegged at a higher, sustainable, level. Thus, the floating exchange rate period can be viewed as a transitional one, linking two periods of fixed exchange rates.

The objective of this paper is to study such a three stage exchange rate regime, focusing on the timing of the crises, the time path of the exchange

[3]Obstfeld (1984) also cites evidence to suggest that this transitional floating regime has been widely adopted among developed countries as well.

rate, and the implied consequences for the time path of consumption. In contrast to Obstfeld (1984), who employed a non-optimizing linear rational expectations structure, our analysis is based on the assumption of fully rational intertemporally optimizing agents, imposing intertemporal viability on the economy; see Turnovsky (1997). But like the early models, the source of the collapse of the fixed exchange rate regime is an unsustainable fiscal policy according to which the government is making continuous transfers in excess of its revenues. The analysis therefore belongs to the first generation of currency crisis models.

The model contains several features that offer a different perspective on the three stage process from that provided by Obstfeld. First, it introduces a role for money via a cash-in-advance constraint, originally introduced by Calvo (1987) into the balance-of-payment crisis model. This relationship provides a link between money demand, inflation, and the government budget. To the extent the domestic currency is depreciating during the floating rate regime, it generates an inflation tax that is a source of government revenue. Second, we assume that the government will revise the fiscal policy when the exchange rate is re-pegged, following the financial crisis, so as to ensure that the (new) re-pegged exchange rate is intertemporally viable. Third, by embedding the analysis within a framework of rational intertemporal optimizing agents satisfying intertemporal budget constraints, the analysis is free of the arbitrariness associated with the linear model, a feature that is particularly important in a context where rationality is central to the issue. Moreover, by deriving the underlying equilibrium from utility maximization, the time path of consumption enables welfare implications to be drawn.

The main conclusions of our approach are to highlight the tradeoffs that exist between the length of the floating rate period and the timing of the speculative attack, and the implications this has for the time paths of the exchange rate and consumption during the transitional floating exchange rate phase. We show how the qualitative behavior of both variables changes as the chosen length of the floating rate period increases through certain critical values. We also show that by letting the exchange rate float for an appropriate period, the exchange rate crisis can be delayed for a maximum period of time, although it cannot be postponed indefinitely. However, delaying the crisis as long as possible is not an optimal strategy. In many respects our results are qualitatively similar to Obstfeld (1984), although the fact that we obtain the equilibrium from an intertemporal optimizing structure is important in enhancing our understanding of the determinants of the crises.

Before proceeding with the analysis, we wish to point out its relevance for the Chinese situation. Since the middle of the 1980s to 1995 China experienced several balance of payments crises. On some occasions foreign reserves dipped down to dangerously low levels. It has been suggested that because Chinese residents cannot hold foreign currency, private speculative attacks cannot be the cause for China's balance of payments crises. However, this argument is not necessarily valid. Since the early 1980s, firms in China having foreign trade business have been allowed to hold foreign currency accounts and have had the right to decide how to use them. Individual foreign currency savings have now grown to over 18 billion dollars, and as a result, individuals may be motivated to speculate in appropriate circumstances. Moreover, since the return of Hong Kong in 1997, there are additional channels for currency flows between China and Hong Kong, which are difficult for the government to eliminate. Finally, China has recently been accepted as a member of the WTO. In a few years the Renminbi will become an international currency available for international trade and then the relevance of this type of analytical framework will increase further.

The remainder of the paper is arranged as follows: Section 2 sets out the basic analytical framework, while Section 3 derives the timing of the speculative attack. Section 4 briefly discusses the two special cases of (i) a permanently floating exchange rate, and (ii) an immediate discrete devaluation in response to the crisis. Sections 5 and 6 characterize the time paths of two key variables, the exchange rate and consumption. Section 7 briefly considers the issue of the optimal floating period while Section 8 summarizes and suggests some possible extensions. Technical details are relegated to the Appendix.

2. THE ANALYTICAL FRAMEWORK

We consider an economy that produces and consumes a single traded good. The economy is small in both the international goods market and in the world financial market, so that it takes the international commodities price and the world interest rate as given. In the absence of impediments to trade, purchasing power parity (PPP) holds and is expressed as:

$$p = p^* + x \tag{1}$$

where p is the domestic price of the traded good, p^* is the foreign price of the good, and x is the exchange rate, (expressed in terms of domestic currency per units of foreign currency), all expressed in logarithms. For simplicity we assume that the foreign price is constant over time, so that $p^* = 0$, simplifying (1) to

$$p = x \tag{1'}$$

The domestic goods price therefore coincides with the exchange rate and the domestic rate of inflation equals the domestic rate of exchange depreciation.

The economy comprises two agents, an infinitely-lived representative agent and the domestic government, the behavior of whom we shall discuss in turn.

2.1. Consumer Behavior

The agent holds two assets, domestic money, and traded bonds, the real stocks of which expressed in terms of the traded good as numeraire are $m(t)$ and $b(t)$, respectively. The agents' real wealth at time t is thus defined by

$$w(t) = m(t) + b(t) \tag{2}$$

Domestic money is denominated in domestic currency, so that in the presence of domestic inflation its real value declines, yielding the real rate of return $-\dot{x}(t)m(t)$, which is commonly referred to as the "inflation tax". The role for money in this model is introduced via cash-in-advance constraint, originally introduced by Stockman (1981) and Lucas (1982), and applied in a similar model of currency crisis by Calvo (1987). We assume that it holds with equality and specify it in the form

$$m(t) = \alpha c(t) \quad \alpha > 0 \tag{3}$$

thereby tying the real demand (holdings) of money to the level of consumption, $c(t)$.

Bonds are denominated in terms of the traded good and yield the constant real rate of return, r, which the agent in the small economy takes as given. Thus the real wealth of the representative agent accumulates in accordance with

$$\dot{w}(t) = rb(t) + y(t) + g(t) - \dot{x}(t)m(t) - c(t) \tag{4}$$

where $y(t)$ is the exogenously given flow of output produced at time t and $g(t)$is a lump-sum transfer received from the government lump-sum at time t. Using (2), we can rewrite (4) as:

$$\dot{w}(t) = rw(t) + y(t) + g(t) - (r + \dot{x}(t))m(t) - c(t) \tag{5}$$

The agent's objective is to choose his consumption and asset holdings to maximize the present value of the logarithmic utility function

$$U = \int_0^\infty u(c(t))e^{-rt}dt \tag{6}$$

subject to the wealth constraint (2), the cash-in-advance constraint (3), and the accumulation equation (5). In order to yield an interior equilibrium the agent's (constant) rate of time preference must coincide with the world interest rate.

Performing the optimization yields the following standard optimality conditions:

$$u'(c) = \lambda[1 + \alpha(r + \dot{x})] \tag{7a}$$

$$\lambda(t) = \bar{\lambda} \tag{7b}$$

where λ, the Lagrange multiplier associated with the accumulation equation, is the marginal utility of wealth, expressed in terms of the traded good as numeraire. The right side of (7a) is the marginal cost of consumption, which includes the cost of the inflation tax and the foregone interest incurred by holding the cash balance, as required by the cash-in-advance constraint. Thus equation (7a) shows this marginal cost of consumption equals the marginal utility of consumption. Equation (7b) is the familiar "consumption smoothing" condition; with a perfect world capital market, the marginal utility of income must be constant at all times; see Turnovsky (1997, Chapter 2).

In addition, the agent must satisfy the transversality condition

$$\lim_{t\to\infty} we^{-rt} = 0 \tag{8}$$

and solving (5) in conjunction with (8) leads to the agent's intertemporal budget constraint

$$\int_0^\infty [(r + \dot{x})m + c]e^{-rt}dt = w_0 + \int_0^\infty (y + g)e^{-rt}dt \tag{9}$$

This equation asserts that the total value of expenditure equals to the present discounted value of income.

2.2. Government and Fiscal Policy.

The domestic government holds real bonds, $b(t)^G$ and issues non-interest bearing domestic money, which in equilibrium must satisfy the cash-in-advance constraint, (3), where c is derived from utility maximization. The government 's net wealth at time t, $w(t)^G$, is

$$w(t)^G = b(t)^G - m(t) \tag{10}$$

It accumulates wealth from the interest it earns on its holdings of foreign bonds, the inflation tax revenue, though these are offset by the lump-sum transfers to the domestic consumer, yielding

$$\dot{w}(t)^G = rb(t)^G + \dot{x}(t)m(t) - g(t) \tag{11}$$

Substituting (10) into (11) yields:

$$\dot{w}(t)^G = rw(t)^G + (r + \dot{x}(t))m(t) - g(t) \tag{11$'$}$$

Solving (11′) subject to the government's intertemporal solvency condition

$$\lim_{t\to\infty} w(t)^G e^{-rt} = 0 \tag{12}$$

leads to the intertemporal government budget constraint

$$w_0^G + \int_0^\infty [(r + \dot{x})m]e^{-rt}dt = \int_0^\infty ge^{-rt}dt \tag{13}$$

Equation (13) indicates that the present value of government expenditure must equal its initial wealth plus the present value of its revenues.

The total wealth of the economy at time t, $W(t)$, is the sum of consumer and government wealth, and summing (2) and (10) is:

$$W(t) \equiv w(t) + w(t)^G = b(t) + b(t)^G \tag{14}$$

and adding (5) and (11'), the aggregate rate of wealth accumulation is

$$\dot{W}(t) \equiv \dot{w}(t) + \dot{w}(t)^G = rW(t) + y(t) - c(t) \tag{15}$$

Integrating this equation yields the national intertemporal budget constraint:

$$W_0 + \int_0^\infty y(t)e^{-rt}dt = \int_0^\infty c(t)e^{-rt}dt \tag{16}$$

stating that initial wealth, W_0, plus the present value of income must equal lifetime consumption.

2.3. Exchange Rate Policy

The government initially fixes the exchange rate at x_0. As will be shown later, at this level the government loses foreign bonds steadily, forcing it to abandon the fixed exchange rate at some time $T > 0$. We shall assume that the government allows the exchange rate to float freely during the period from T to $T + \tau$. At time $T + \tau$, the exchange rate is again fixed, but this time at a higher level $x_1 > x_0$. The agent (consumer) takes the two fixed exchange rates, x_0, x_1, and the two time periods, T, τ, as given. Since $T > 0$ and is finite, the agent knows that the domestic currency will depreciate in the future and that the exchange rate will again be fixed. In the case where $\tau \to 0$, the exchange rate depreciates immediately from its old to its new level. If $\tau > 0$, then the exchange rate will be flexible for some positive period. The limiting case, $\tau \to \infty$, corresponds to the benchmark considered by Krugman (1979), where the flexible exchange rate introduced at time T continues forever.

In general, the exchange rate regime switches twice; first at time $t = T$, when the flexible rate is first introduced, and again at time $t = T + \tau$, when the exchange rate is re-pegged at its higher level. These regime changes are reflected in the agent's optimality conditions:

$$u'(c(t)) = \begin{cases} \lambda(1 + \alpha r) & 0 \le t < T, \\ \lambda(1 + \alpha(r + \dot{x})) & T \le t < T + \tau, \\ \lambda(1 + \alpha r) & T + \tau < t. \end{cases} \tag{17}$$

Note that in the first and third phases, when the exchange rate is fixed, the marginal utility of consumption, and thus consumption itself are constant, and in fact equal. Noting (17) and the cash-in-advance constraint, the rate of private wealth accumulation during these periods is

$$\dot{w}(t) = \begin{cases} rw(t) + y(t) + g(t) - (1 + \alpha r)c & 0 \le t < T, \\ rw(t) + y(t) + g(t) - (1 + \alpha(r + \dot{x}))c & T \le t < T + \tau, \\ rw(t) + y(t) + g(t) - (1 + \alpha r)c & T + \tau < t. \end{cases} \tag{18}$$

3. THE TIMING OF SPECULATIVE ATTACKS

We now consider the timing of a speculative attack and when a financial crisis occurs. The cause of the speculative attack is due to the fact that the transfer, g, from the government to the representative agent, is ultimately unsustainable, given its rate of asset accumulation.

3.1. Decline in Traded Bond Holdings

To determine the evolution of the government's holdings of traded bonds, we differentiate (11) and combine with (10) to yield:

$$\dot{b}(t)^G = rb(t)^G + \dot{x}(t)m(t) + \dot{m}(t) - g(t) \tag{19}$$

We begin by focusing on the first phase, $0 \le t < T$, during which the exchange rate is pegged at its initial level, x_0, so that $\dot{x}(t) \equiv 0$. From the optimality condition (17) consumption is constant, and from the cash-in-advance constraint, (3), real money balances are constant as well. Thus during this phase, the accumulation equation reduces to:

$$\dot{b}(t)^G = rb(t)^G - g(t) \tag{19'}$$

so that the rate of change of traded bonds held by the government depends upon the relative size of the interest being earned and the transfer granted to the agent. We shall assume that the pre-crisis transfer is constant and is such that

$$g(t) \equiv g > rb_0^G \tag{20}$$

That is, it exceeds the amount of interest earned by the government on its holdings of traded bonds. Substituting (20) into (19′) and solving, we obtain

$$b(t)^G = \frac{g}{r} + \left(b_0^G - \frac{g}{r}\right) e^{rt} \tag{21}$$

from which we infer that the government's holding of traded bonds is steadily declining.

It is clear that this situation cannot continue indefinitely. Either the policy or the exchange rate regime must eventually change. We shall assume that the fiscal policy is fixed and cannot be changed in the short run. Instead, the government chooses to change the exchange rate regime at time $t = T$. During the period $T \leq t \leq T+\tau$ the exchange rate is allowed to float freely, allowing it to depreciate, leading to domestic inflation, and thereby generating an inflation tax for the government. Issuing money provides the government with extra revenue, thereby helping it balance its budget. After time $T+\tau$, the exchange rate is permanently fixed again, but now at the higher level $x_1 > x_0$, with x_1 being set so as to maintain the stock of traded bonds constant. We assume that the agent has perfect information about the foreign bond holdings of the government, the exchange rate, and fiscal policies, so that there are no surprises.

3.2. Solution for the Floating Exchange Rate

The key issue is the determination of T, the instant at which the government abandons the fixed exchange rate and allows it to float. We shall assume that this occurs when the government's holdings of traded bonds decline to a pre-determined level, $\bar{b}^G < b_0^G$. When the exchange rate is flexible, the government is free from any further pressure to decumulate bonds and its holdings of traded bonds remains constant at $\bar{b}^G$. Equation (19) then implies

$$\dot{m}(t) + \dot{x}(t)m(t) = g - r\bar{b}^G \tag{22}$$

We assume that the government maintains the same (constant) level of transfers throughout the period $(0, T+\tau)$. In order to sustain the repegged fixed exchange rate, after time $t > T+\tau$ the transfer level drops to g_1 set so that

$$g_1 = r\bar{b}^G \tag{23}$$

To determine an explicit solution for the time path of the exchange rate during the floating phase, we shall assume that preferences are represented by the logarithmic utility function

$$u(c) = \ln c \tag{24}$$

in which case

$$u'(c) = 1/c \tag{24$'$}$$

Using (24′) together with (17), the agent's rate of wealth accumulation is equal to

$$\dot{w} = rw + y + g - 1/\lambda \tag{25}$$

where λ is constant. This equation holds at all time, and in particular over the first two exchange rate regimes, when $x = x_0$ and x is flexible. Solving this equation in the case where y as well as g is constant and imposing the transversality condition (8), yields

$$y + g + rw_0 = \frac{1}{\lambda} \tag{26}$$

Substituting back into (17) we obtain the following expressions for consumption

$$c(t) = \begin{cases} \dfrac{y + g + rw_0}{(1 + \alpha r)} & 0 \leq t < T, \\ \dfrac{y + g + rw_0}{(1 + \alpha(r + \dot{x}))} & T \leq t < T + \tau, \\ \dfrac{y + g + rw_0}{(1 + \alpha r)} & T + \tau < t \end{cases} \tag{27}$$

As noted earlier, the government will abandon the fixed exchange rate regime at the point where its holdings of foreign bonds drops to $\bar{b}^G$, a policy of which the consumer is aware. During the period $(T \leq t < T + \tau)$, while the exchange rate is floating, condition (22) and the cash-in-advance constraint can be combined to yield

$$\alpha \dot{c}(t) + \alpha c(t) \dot{x}(t) = g - r\bar{b}^G$$

Differentiating the second part of (27) and substituting, leads to the following second-order differential equation describing the evolution of the exchange rate during the floating rate period:

$$\frac{\alpha^2 \ddot{x}(t)}{1 + \alpha(r + \dot{x}(t))} - \alpha \dot{x}(t) + \lambda(g - r\bar{b}^G)\left[1 + \alpha(r + \dot{x}(t))\right] = 0 \tag{28}$$

Since the agent has perfect foresight, future jumps in the exchange rate are ruled out and accordingly we impose the boundary conditions

$$\begin{aligned} x(T) &= x_0; \\ x(T+\tau) &= x_1 \end{aligned} \tag{29}$$

Using these boundary conditions, the solution for the dynamic path of the exchange rate is shown in the Appendix (A.1) to be:

$$x(t) = x_0 + \theta(T-t) + \frac{1}{\beta}\ln\left[1 + \beta\left(1 + \frac{1}{\theta}\dot{x}(T)\right)(1 - e^{\theta(T-t)})\right] \tag{30}$$

where $\theta \equiv (1+\alpha r)/\alpha > 0$ and $\beta \equiv \lambda(g - r\bar{b}^G) - 1$ are both constants. It is straightforward to show that the assumptions we have imposed imply[4]

$$0 > \beta > -1 \tag{31}$$

Setting $t = T + \tau$ in (30) and using the second boundary condition in (29) yields

$$x_1 = x_0 - \theta\tau + \frac{1}{\beta}\ln\left[1 + \beta\left(1 + \frac{1}{\theta}\dot{x}(T)\right)(1 - e^{-\theta\tau})\right] \tag{32}$$

which enables the rate of exchange depreciation (inflation rate) at the instant the exchange rate begins to float, $\dot{x}(T)$, to be expressed as the following function of τ

$$\dot{x}(T) = \theta\frac{e^{\beta(x_1 - x_0) + \theta\beta\tau} - 1 - \beta(1 - e^{-\theta\tau})}{\beta(1 - e^{-\theta\tau})} \tag{32$'$}$$

Condition (32) provides a partial relationship between the re-pegged higher exchange rate, x_1 and the duration of the floating exchange rate regime. *Given* the time of the speculative attack, T, and the re-pegged value of the exchange rate, x_1, which is a government controlled parameter, equation (32) determines the required period, τ for the exchange rate to float.

3.3. Timing of the Devaluation

The next issue concerns the determination of T, as a function of $g, \bar{b}^G$, and τ. In other words, precisely when will speculative attacks occur? To address this question we need to elaborate upon the nature of the speculative attack itself.

[4]The inequality $\beta < 0$ is obtained from the following. Using (26) we have $\beta = \frac{g - r\bar{b}^G}{y+g+rw_0} - 1 = -\frac{y + r\bar{b}^G + rw_0}{y+g+rw_0} < 0$. The condition $\beta > -1$ follows from the condition $g > rb_0^G > r\bar{b}^G$.

When individuals attack the exchange rate, they exchange the domestic money they hold for the traded bonds of the government. This causes a drop in the government's holdings of traded bonds. The severity of an attack can be measured by the rate of decrease of the government's traded bond/reserve holding. As the government is losing traded bonds, the attackers are losing domestic money. We shall let $m(T^-)$ and $m(T^+)$, respectively, denote the attackers' cash holdings just before, and just after, time T, where $m(T^-) > m(T^+)$. The swap of assets that this involves is described by

$$m(T^-) - m(T^+) = b(T^-)^G - \bar{b}^G \tag{33}$$

where $b(T^-)^G \geq \bar{b}^G$ is the government's bond holdings just before the attack at time T. Recall the solution for the government's stock of bonds (21) which holds over the period $0 \leq t < T$. Setting $t = T^-$ in this equation, we immediately see that

$$b(T^-)^G = \frac{g}{r} + \left(b_0^G - \frac{g}{r}\right) e^{rT} \tag{34}$$

where by continuity, $e^{-rT^-} = e^{-rT}$. Using the cash-in-advance constraint (3), (17) and (27) imply

$$\begin{aligned} m(T^-) - m(T^+) &= \alpha(c(T^-) - c(T^+)) \\ &= \alpha^2 \frac{\dot{x}(T)}{\lambda(1+\alpha r)(1+\alpha(r+\dot{x}(T)))} \end{aligned} \tag{35}$$

Furthermore, combining (33), (34), and (35), we can solve for the rate of exchange depreciation (inflation rate) at time $t = T$, $\dot{x}(T)$,

$$\dot{x}(T) = \frac{\eta\lambda\theta^2}{r - \eta\lambda\theta} \tag{36}$$

where θ is defined above, and η is defined by

$$\eta \equiv (rb_0 - g)e^{rT} - (r\bar{b}^G - g) \tag{37}$$

Equation (36), (together with the definition in (37)) determines the initial rate of exchange depreciation in terms of the policy instruments and their values at time T, the time of the speculative attack. From (36) and (37) we immediately see that

$$\operatorname{sgn}\left(\frac{\partial \dot{x}(T)}{\partial T}\right) = \operatorname{sgn}(rb_0 - g) < 0 \tag{38}$$

which implies that the longer the exchange rate remains fixed and the more bonds the government is losing, the smaller is the exchange of assets at the time of attack, T, and thus the smaller is the rate of exchange depreciation at that time.

Equating (36) to (32′) we may solve for η, in terms of the following function of τ

$$\eta = \frac{(e^{\beta(x_1-x_0)+\theta\beta\tau} - 1 - \beta + \beta e^{-\theta\tau})r}{\lambda\theta(e^{\beta(x_1-x_0)+\theta\beta\tau} - 1)} \tag{39}$$

In addition, equating (37) and (39) we derive the following expression for the timing of the speculative attack:

$$\begin{aligned} & T(\tau; g, b_0^G, \bar{b}^G, x_1 - x_0) \\ = \; & \frac{1}{r}\ln\left(\frac{\lambda\theta(g - r\bar{b}^G) + r\left(\frac{(1-e^{-\theta\tau})\beta}{e^{\beta(x_1-x_0)+\theta\beta\tau}-1} - 1\right)}{\lambda\theta(g - rb_0^G)}\right) \end{aligned} \tag{40}$$

Equation (40) shows that the timing of a financial crisis is a function of the government transfer, the initial and the minimum government's foreign bond holdings, the two fixed exchange rates, as well as the length of the transitional floating exchange rate period, τ.

We show in the Appendix (A.2) that the timing of the devaluation has the following qualitative properties:

1. $\partial T/\partial g < 0$, i.e. the larger the initial government transfer, the earlier the crisis will occur.
2. $\partial T/\partial b_0^G > 0$ and $\partial T/\partial \bar{b}^G < 0$, i.e the larger the initial stock of bonds held by the government and the lower the reserve limit is set, the later the crisis will occur.
3. $\partial T/\partial(x_1 - x_0) < 0$, i.e. the greater the amount of devaluation set by the government, the earlier the crisis will occur.
4. Increasing τ initially increases T, which reaches a maximum, $T = T^*$, at $\tau = \tau^*$.

The first three results are straightforward, but the non-monotonic relationship between the time of the crisis and the subsequent length of the transitional floating exchange is not so apparent. The reason for it involves the adjustment of the rate of exchange depreciation at that time (which from (38) is inversely related to T), and we will return to this after our discussion of the behavior of $\dot{x}(T)$.[5]

[5] These results are generally similar to those obtained by Obstfeld (1984).

4. TWO POLAR CASES

Our objective is to characterize the dynamic adjustment path for the exchange rate, but before doing so it is useful to use the present, rather general model to analyze two special cases, both of which have received attention in the literature:

(i) The case where the exchange rate floats permanently after the crisis occurs at time T, obtained by letting $\tau \to \infty$ and,

(ii) The case where the government immediately sets the exchange rate at the higher level at time T, obtained by setting $\tau = 0$.

4.1. Permanently floating Exchange Rate after Crisis

Letting $\tau \to \infty$ in (40), this expression reduces to

$$\lim_{t\to\infty} T(\tau) \equiv T(\infty) = \frac{1}{r}(\ln(g - r\bar{b}^G) - \ln(1+\alpha r)(g - rb_0^G)) \qquad (41)$$

Using the cash-in-advance constraint, (41) implies $T(\infty) \lesseqgtr 0$ according to whether

$$\frac{m_1}{c} \equiv \alpha \gtreqless \frac{b_0^G - \bar{b}^G}{g - rb_0^G}$$

where m_1 is the real cash holdings of the consumer in the fixed exchange rate regime, x_0.

There are two cases to consider. In the first case $T(\infty) < 0$, so that the speculative attack will occur instantaneously at time t= 0. This will occur because, on the one hand, speculators see the profit opportunity by buying foreign bonds from the government just before devaluation. On the other hand, the speculators have sufficient domestic resources to purchase enough foreign bonds from the government to drive b^G down to the critical level, $\bar{b}^G$, forcing the government to devalue the domestic currency immediately.

If the inequality is reversed, $T(\infty) > 0$. This means that at time $t = 0$, speculators have insufficient domestic money to reduce the government's foreign bond holdings to the critical level $\bar{b}^G$. Over time, the government is losing foreign bonds, and less money will be needed for a successful speculative attack. The government will be forced to devalue as soon as speculators have just enough domestic resources to reduce b^G to $\bar{b}^G$.

In either case, the limiting solution (41) implies that even if the government declares that the exchange rate will be left to float indefinitely after the crisis, a crisis will nevertheless occur. We can further show that a crisis will occur at the same date, determined by (41), if instead the government announces that it will permit the exchange rate to float for τ' periods, after which it will re-peg the exchange rate at its new fixed rate x_1 provides that

τ' is set by[6]

$$\tau' = -\frac{\beta}{\theta(1+\beta)}(x_1 - x_0) > 0 \tag{42}$$

We can therefore identify the time in (41) as $T(\tau'; x_1 - x_0) \equiv T(\tau')$. It is easy to check that $T(\tau)$, in (40), is monotonically increasing in τ at $\tau = \tau'$. This means that $\tau' < \tau^*$, the point at which $T(\tau)$ reaches its maximum value (i.e. where $dT/d\tau|_{\tau=\tau^*} = 0$). We summarize the above results as follows:

(i) Even if the government declares that the exchange rate will be left to float freely and permanently should its holdings of traded bonds drop to the critical level $\bar{b}^G$, the speculative attack will still occur at time $t = T(\tau')$, which may even be immediately.

(ii) The timing of speculative attacks and the financial crisis is the same as if the government declares that the exchange rate will be allowed to float freely for a period of τ', after which it be re-pegged to x_1.

(iii) There exists τ^* such that $T(\tau^*) \geq T(\tau)$ for all τ

(iv) For all $\tau' < \tau < \tau^*$, $T(\tau) > T(\tau')$.

Intuitively this means that in a given macroeconomic environment, the crisis will inevitably occur, regardless of whether the fixed exchange rate is to be given up temporarily or permanently. By choosing $\tau = \tau^*$, the policy maker can delay the crisis for as long as possible, but not avoid it.

4.2. Discrete Devaluation

We now turn to the second polar case, $\tau = 0$, where the government devalues the exchange rate immediately from x_0 to x_1 at time T, when the crisis occurs. Normally, given perfect foresight and no impediments to trading assets, the only time when the discrete jump in the exchange rate can occur would be at $T = 0$. Otherwise, the agent would perfectly anticipate the potential to make a infinite future return, advancing the pressure on the exchange rate. However, the cash in advance constraint restricts this possibility. Letting $\tau \to 0$ in (40) yields:

$$\lim_{t\to 0} T(\tau) \equiv T(0) = \frac{1}{r}\ln\left(\frac{\lambda\theta(r\bar{b}^G - g) + r}{\lambda\theta(rb_0^G - g)}\right) \tag{43}$$

Analogously to (41), $T(0) \lesseqgtr 0$ according to whether $m_1 \gtreqless b_0^G - \bar{b}_G$. The economic rationale is essentially the same as for $T(\infty)$ given above.

Figure 1 graphs the dependence of the attack date, T, upon the length of the floating period τ. Figure 1.A illustrates the case $m_1 > b_0^G - r\bar{b}^G$. Let

[6] One can interpret (40) as defining a tradeoff between the duration of the floating exchange rate,τ, and the magnitude of the eventual devaluation, $x_1 - x_0$.

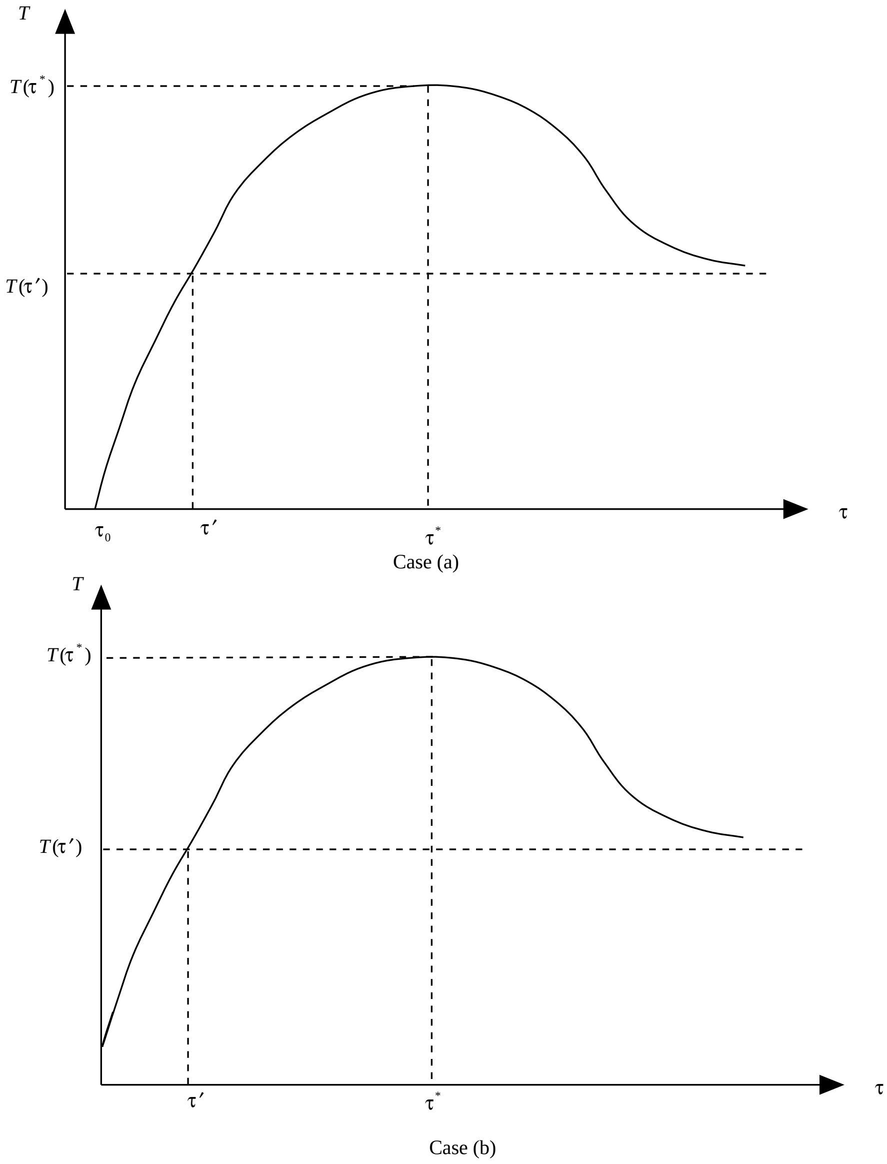

FIG. 1. Timing of Financial Crisis

us define τ_0, where $T(\tau_0) = 0$. For $\tau \leq \tau_0$, devaluation will occur instantaneously at $t = 0$. For $\tau > \tau_0$ devaluation does not occur instantaneously. Figure 1.B illustrates the relationship in the case $m_1 < b_0^G - r\bar{b}^G$, in which the vertical intercept represents the time of speculative attacks.

This figure has the same general characteristics as Figure 1 in Obstfeld (1984), although the key determinants are substantially different. In Obstfeld's case the key parameters determining the dependence of T on τ, including τ^*, pertain to the demand for money and the monetary growth rate. By contrast, in our case they involve the preference parameters, λ, r, as well as the cash in advance parameter, α (included in θ), together with measures of the government's fiscal position (and imbalance).

5. TIME PATH OF THE EXCHANGE RATE DURING THE TRANSITION

We now turn to the general case where $0 < \tau < \infty$ and recall the formal solution, (30), for the exchange rate during the transitional phase. In order to characterize fully the transitional adjustment of the exchange rate, it is important to determine $\dot{x}(t)$ at both $t = T$ and $t = T + \tau$, the initial and terminal points of the floating rate regime, as well as at all intermediate points. Recalling (32′) we have

$$\dot{x}(T) = \theta \frac{e^{\beta(x_1 - x_0) + \theta\beta\tau} - 1 - \beta(1 - e^{-\theta\tau})}{\beta(1 - e^{-\theta\tau})} \tag{32'}$$

In the Appendix (A.3) we establish the following properties of $\dot{x}(T)$:

1. $\dot{x}(T) > 0$ for any τ. In other words, although the exchange rate cannot jump at time T, it will nevertheless undergo a positive rate of exchange depreciation at that initial point.

2. $\partial\dot{x}(T)/\partial(x_1 - x_0) > 0$, i.e. the greater the eventual devaluation set by the government, the greater is the initial rate of exchange depreciation at the time the exchange rate begins to float.

3. Increasing τ from an initially small value reduces the initial increase in the rate of exchange depreciation, $\dot{x}(T)$. However, as τ continues to increase, the initial rate of increase reaches a minimum at $\tau = \tau^*$ (the point at which T reaches its maximum, T^*), after which it again increases.

Differentiating (30) with respect to t yields the rate of exchange depreciation:

$$\dot{x}(t) = -\theta + \frac{\left(1 + \frac{1}{\theta}\dot{x}(T)\right)\theta e^{\theta(T-t)}}{1 + \beta\left(1 + \frac{1}{\theta}\dot{x}(T)\right)(1 - e^{\theta(T-t)})} \tag{44}$$

Since $\dot{x}(T)$ is a function of τ, the inflation rate described by (44) is a function of τ as well as t. Substituting for $\dot{x}(T)$ from (32') and letting

$\tau \to \infty$ in (44) we find that

$$\lim_{\tau\to\infty} \dot{x}(t) = -\theta\left(\frac{1+\beta}{\beta}\right) > 0 \tag{45}$$

In other words, if the policymaker lets the exchange rate float permanently following the crisis, it will depreciate at a constant rate indefinitely.

Evaluating (44) at the terminal point, $T+\tau$, of the floating rate regime, the rate of exchange depreciation at that time is

$$\dot{x}(T+\tau) = -\theta + \frac{\left(1+\frac{1}{\theta}\dot{x}(T)\right)\theta e^{-\theta\tau}}{1+\beta\left(1+\frac{1}{\theta}\dot{x}(T)\right)(1-e^{-\theta\tau})} \tag{46}$$

Substituting (32′) into (46) $\dot{x}(T+\tau)$ can be expressed as a function of τ. The Appendix (A.4) also shows that at all t

$$\operatorname{sgn}(\dot{x}(T+\tau)) = \operatorname{sgn}(dT/d\tau)$$

This result has the following implications:

1. If $\tau = \tau^*$, $dT/d\tau = 0$ and $\dot{x}(T+\tau) = 0$. This means that $x(t)$ reaches x_1, its maximum value, at $t = T+\tau$.
2. If $\tau > \tau^*$, $dT/d\tau < 0$ and $\dot{x}(T+\tau) < 0$. This means that $x(t)$ overshoots its eventual long-run response during the transition, and approaches x_1 at $t = T+\tau$ from above.
3. If $\tau < \tau^*$, $dT/d\tau > 0$ and $\dot{x}(T+\tau) > 0$. This means that $x(t)$ approaches x_1 at $t = T+\tau$ from below.
4. In the special case in which the government chooses to let the exchange rate float permanently after a financial crisis, i.e. as $\tau \to \infty$, we obtain

$$\lim_{\tau\to\infty} \dot{x}(T(\tau)) = -\theta\frac{(1+\beta)}{\beta}$$

Similarly,

$$\lim_{t\to\tau'} \dot{x}(t) = \lim_{\tau\to\infty} \dot{x}(T(\tau)) = -\theta\frac{(1+\beta)}{\beta}$$

It is easy to check, at $\tau' = \beta(x_0-x_1)/[\theta(1+\beta)]$ the inflation rate is the same as $\lim_{\tau\to\infty}\dot{x}(T+\tau)$. Thus, by either choosing the length of the flexible rate period equal to τ' or abandoning the fixed exchange rate forever, the exchange rate depreciates at a constant rate.

5. Equation (44) implies $\operatorname{sgn}((\ddot{x}(t)) = \operatorname{sgn}(\tau'-\tau)$; see Appendix (A.5). Thus, if $\tau < \tau'$, then $\ddot{x}(t) > 0$ and the exchange rate adjustment path is convex. If $\tau > \tau'$, then $\ddot{x}(t) < 0$ and the exchange rate adjustment path is concave.

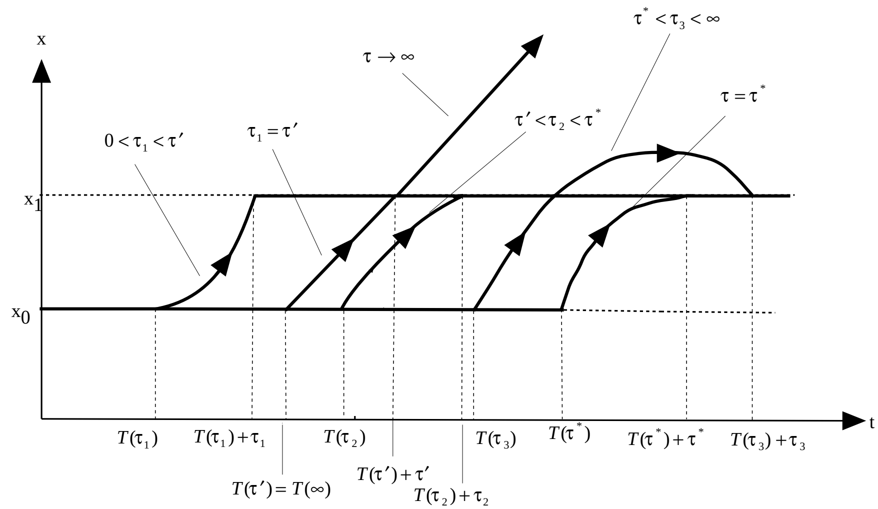

FIG. 2. Time Path of Exchange Rate

Figure 2 illustrates the time paths of the exchange rate during the transitional floating phase, as τ increases from a short period ($\tau < \tau'$) to a permanent float ($\tau \to \infty$). For $\tau < \tau^*$ we see that increasing τ raises $T(\tau)$ and therefore raises the total length of time, $T(\tau) + \tau$, until the exchange rate is re-pegged. For $\tau > \tau^*$, $T(\tau)$ declines, although the total length of time until the new pegged rate is reached still increases.

Looking through these results, we see the following conditions, namely

$$\operatorname{sgn}\left(\frac{dT}{d\tau}\right) = -\operatorname{sgn}\left(\frac{d\dot{x}(T)}{d\tau}\right) = \operatorname{sgn}\left(\dot{x}(T+\tau)\right) = \operatorname{sgn}\left(\tau^* - \tau\right)$$

indicating an intimate relationship between the timing of the speculative attack and the rates of exchange depreciation at the beginning and end of the floating regime phase. The following intuition may be given. Suppose that τ is initially small, certainly less than τ^*. In this case, the exchange rate depreciates from x_0 to x_1 during the floating period, without ever rising above x_1. Since a reduction in the length of the floating rate regime reduces the time available for the transition, the exchange rate must be rising more quickly initially and for this to occur the speculative crisis must occur earlier, i.e. T must decline. On the other hand, if $\tau > \tau^*$, the exchange rate overshoots its long-run response before appreciating toward x_1. A reduction in τ still reduces the time for the transition, but now a delay in the onset of the crisis reduces the extent of the initial depreciation, thereby enabling the necessary appreciation to occur during the latter phase of the transition.

6. CONSUMPTION ADJUSTMENT PATH

The behavior of the exchange rate is reflected in the time path for consumption. Using equations (27) recalling that λ is constant, we can conveniently summarize consumption by

$$c(t) = \begin{cases} \dfrac{1}{\lambda\alpha\theta} \equiv c_0 & 0 \le t < T \\ \dfrac{1}{\lambda\alpha(\theta + \dot{x}(t))} & T \le t < T + \tau \\ \dfrac{1}{\lambda\alpha\theta} \equiv c_0 & T + \tau < t \end{cases} \tag{47}$$

From (47) we see that the time path for consumption involves three phases. Prior to the exchange crisis, when the exchange rate is fixed at x_0, consumption is constant at c_0. During the transitional period when the exchange rate is flexible, consumption varies with $\dot{x}$ and thus is responsive to the choice of floating rate period, τ. At time $T + \tau$, when the exchange rate is re-pegged at the higher level, x_1, consumption reverts back to its initial level c_0, where it remains thereafter.

We shall therefore focus our attention on the adjustment path of $c(t)$ during the period that the exchange rate is flexible, showing how it mirrors the time path of the rate of exchange depreciation, $\dot{x}(t)$. Thus from (47) we see the following:

$$c(T) = \frac{1}{\lambda\alpha[\theta + \dot{x}(T)]} \tag{48a}$$

$$\dot{c}(t) = -\frac{\ddot{x}(t)}{\lambda\alpha[\theta + \dot{x}(t)]^2} \quad T \le t \le T + \tau \tag{48b}$$

$$c(T+\tau) = \frac{1}{\lambda\alpha[\theta + \dot{x}(T+\tau)]} \tag{48c}$$

Since $\dot{x}(T) > 0$, we immediately see that consumption always undergoes a discrete decline at time T, reflecting the increase in $\dot{x}(T)$ at that time. Six different cases, depending upon the length of the transitional phase τ, can be identified. These cases are described below and are illustrated in the six panels of Figure 3.

1. If $\tau < \tau'$, so that $\ddot{x}(t) > 0$, consumption decreases monotonically during the period of the flexible exchange rate.

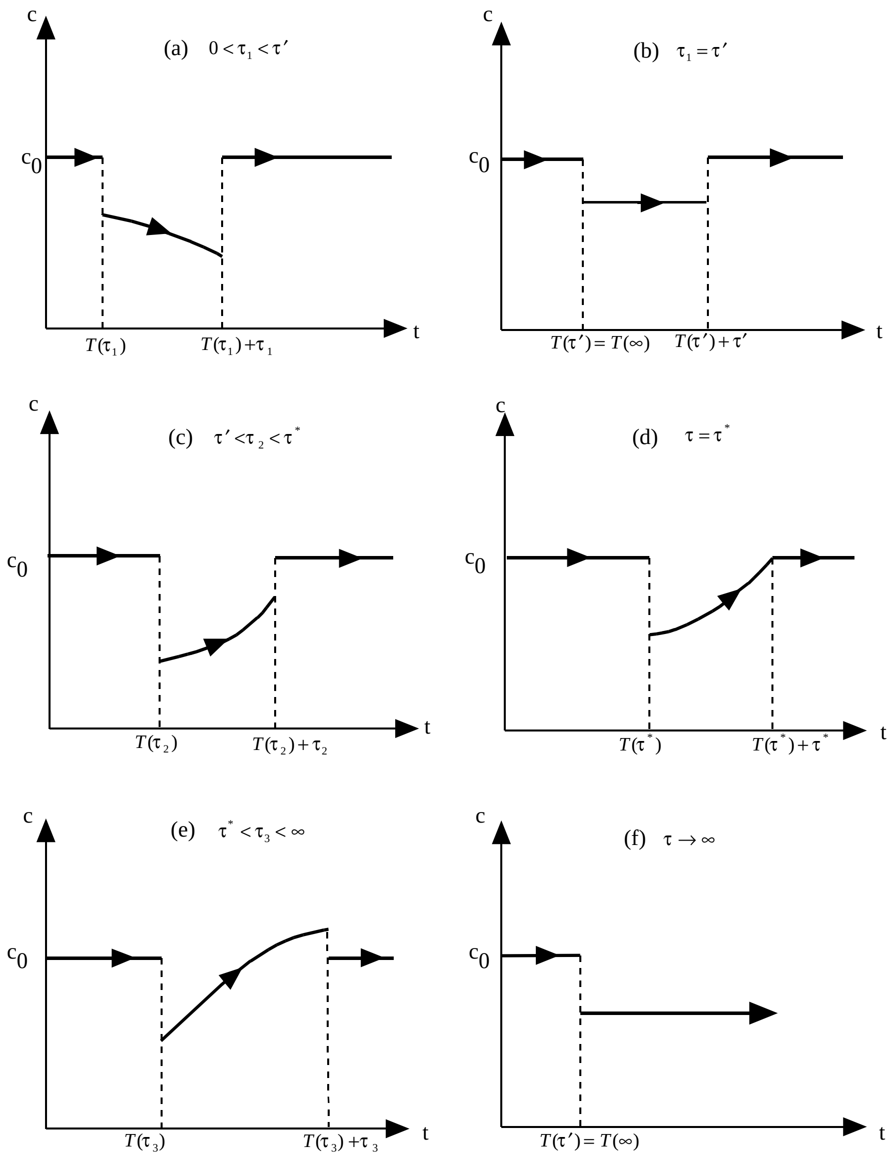

FIG. 3. Time Path of Consumption

2. If $\tau = \tau'$, so that $\ddot{x}(t) = 0$, consumption is higher than when $\tau < \tau'$, and remains constant over the duration of the flexible exchange rate.

3. If $\tau' < \tau < \tau^*$, so that $\ddot{x}(t) < 0$, consumption increases over the period the exchange rate is flexible, but it reaches its pre-crisis level only after the exchange rate is re-pegged at its new level, x_1.

4. If $\tau = \tau^*$, consumption increases over the period the exchange rate is flexible, and reaches its pre-crisis level continuously when the exchange rate is re-pegged at its new level,x_1.

5. If $\tau^* < \tau < \infty$, consumption increases during the transitional period and exceeds its pre-crisis level, at the end of the flexible rate regime, $T + \tau$, before it drops back to its initial level c_0. This is because the exchange rate is appreciating, just prior to being re-pegged.

6. If $\tau \to \infty$, the exchange rate is left flexible, permanently. Because domestic currency depreciates at a constant rate, the inflation tax is constant, too. This causes consumption to drop at time $T$and remain there permanently thereafter.

With the logarithmic utility function, the time path for consumption translates to implications for instantaneous welfare. Thus, for example, for $0 < \tau < \tau'$ welfare is declining over time while the exchange rate is depreciating. Also, in the case where $\tau^* < \tau < \infty$, welfare actually increases above its long-run level during the latter states of the transition.

7. OPTIMAL FLOATING PERIOD

An important implication of the analysis, evident from Figures 2 and 3, is that by choosing τ, the length of transitional floating rate period, the policy maker can influence the behavior of the exchange rate after the attack, the time path of consumption over that period, and therefore welfare. Indeed, it is straightforward to formulate the choice of the optimal length of the floating rate regime.

Specifically, we shall assume that the policy maker's objective is to choose τ to maximize the welfare of the representative agent. For notational convenience, we shall let

$$1 + \frac{1}{\theta}\dot{x}(T) \equiv \varphi(T)$$

in which case the optimization problem can be stated as being to choose τ to maximize

$$W(T(\tau), \tau) = \int_T^{T+\tau} \ln c(t) e^{-rt} dt \tag{49a}$$

subject to the equilibrium consumption condition

$$c(t) = \frac{1}{\lambda\alpha[\theta + \alpha\dot{x}(t;T)]} \tag{49b}$$

where

$$\dot{x}(t;T) = -\theta + \frac{\varphi(T)\theta e^{\theta(T-t)}}{1+\beta\varphi(\tau)(1-e^{\theta(T-t)})} \tag{49c}$$

$$\varphi(T) = \theta\frac{e^{\beta(x_1-x_0)+\theta\beta\tau}-1}{\beta(1-e^{-\theta\tau})} \tag{49d}$$

In writing (49), we have noted that welfare depends upon τ both directly, and indirectly through the determination of the crisis date, T.

Substituting (49b) into (49a) and taking logarithms we may express the objective function as:

$$W(T,\tau) = -\int_T^{T+\tau}[\ln\lambda\alpha + \ln(\theta+\dot{x}(t,T))]e^{-rt}dt$$

The optimal choice of floating period is therefore obtained by setting

$$\frac{dW}{d\tau} \equiv \frac{\partial W}{\partial\tau} + \frac{\partial W}{\partial T}\frac{dT}{d\tau} = 0$$

where $dT/d\tau$ is obtained from (40) and an explicit expression for which is provided in the Appendix (A.11e). Thus, we obtain the formal optimality condition:

$$\begin{aligned} &- [\ln\lambda\alpha + \ln(\theta+\dot{x}(T+\tau,T)]\left[1+\frac{dT}{d\tau}\right]e^{-r(T+\tau)} \\ &+ [\ln\lambda\alpha + \ln(\theta+\dot{x}(T,T)]\frac{dT}{d\tau}e^{-rT} \\ &- \frac{dT}{d\tau}\int_T^{T+\tau}\frac{\partial\dot{x}/\partial T}{\theta+\dot{x}(t,T)}e^{-rt}dt = 0 \end{aligned} \tag{50}$$

In principle, equation (50), in conjunction with (49c), (49d), provides a relationship that determines the optimal period for which the exchange rate should be floated.

Rather than pursue this, we shall compare the optimal $\tau \equiv \tau_{opt}$, with τ^* that maximizes the delay of the crisis. We have seen that $\tau = \tau^*$ is associated with $dT/d\tau = \dot{x}(T+\tau,T) = 0$, in which case we have

$$\left.\frac{\partial W}{\partial\tau}\right|_{\tau=\tau^*} = -\ln(\lambda\alpha\theta)e^{-r(T^*+\tau^*)} = \ln c(T+\tau)e^{-r(T^*+\tau^*)}$$

The first point to observe is that since $\partial W/\partial\tau\big|_{\tau=\tau^*} \neq 0$, setting $\tau = \tau^*$ is not welfare-maximizing. Indeed, if $\ln c(T+\tau) > 0$, then $\partial W/\partial\tau\big|_{\tau=\tau^*} > 0$, implying that $\tau_{opt} > \tau^*$. Because of arbitrary units, we cannot assert $\ln c(T+\tau) > 0$. However, Figure 3 strongly supports the notion that welfare losses over the floating period will be minimized by setting $\tau_{opt} > \tau^*$.

8. CONCLUSIONS

In this paper, we have examined the issue of speculative attacks on the exchange rate in an economy in which, following the attack the government chooses to allow the exchange rate to float for a limited period, before re-pegging it at a higher, sustainable level. In contrast to Obstfeld (1984) who originally addressed this problem using an ad hoc linear model, we have employed an intertemporal optimizing framework, in which intertemporal solvency conditions play a crucial role.

While our results share many of the characteristics of those originally obtained by Obstfeld, the fact that they generally continue to apply in a more rigorous intertemporal setting is important. In addition, key features of our results can be related to more basic parameters, thereby enhancing our understanding of the causes of balance of payments crises. For example, unlike Obstfeld, we found the timing of a balance-of-payment crises is critically affected by whether or not the individual holds enough domestic cash to deplete the foreign reserve in the central bank. In addition to government transfers, initial and minimum government bond holdings and the magnitude of the ultimate devaluation are all determinants of the time of crisis. The timing is also strongly affected by the length of the floating regime interval, τ, though not in a straightforward way. Critical values of τ that identify changes in behavior are determined in part by preference parameters, rather than only by characteristics of the aggregate demand for and supply of money, as in Obstfeld. Finally, by adopting a utility function based on consumption, we not only consider the implications for the time path of consumption, but can provide the basis for undertaking welfare analysis. In this respect, our conclusion that to delay the attack for as long as possible is not optimal is quite significant.

We conclude by suggesting two extensions. First, the assumption of the logarithmic utility function is very advantageous in that it permits us to obtain a tractable closed-form solution for the time path of the exchange rate during the flexible rate phase. But the logarithmic utility function is restrictive and the extent to which we can generalize it and still retain tractability is worth exploring. Second, speculative attacks are identified with risk, an element completely absent from our analysis. Modifying the framework to include risk in an intertemporal setting would be a challenging, but worthwhile, extension.

APPENDIX A

This Appendix proves some of the results cited in the text.

A.1. DYNAMIC TIME PATH OF THE EXCHANGE RATE

From equation (28) we have the second order differential equation:

$$\alpha^2 \frac{\ddot{x}}{[1+\alpha(r+\dot{x})]} - \alpha\dot{x} + (g - r\bar{b}^G)[\lambda(1+\alpha(r+\dot{x}))] = 0 \tag{A.1}$$

which can be rewritten as:

$$\alpha \frac{d}{dt} \ln\left(\frac{1}{1+\alpha(r+\dot{x})}\right) - \alpha\dot{x} = -(g - r\bar{b}^G)[\lambda(1+\alpha(r+\dot{x}))] \tag{A.2}$$

Integrating both sides of the equation from T to t, we obtain:

$$\begin{aligned} &\alpha \int_T^t \left(\frac{d}{dt} \ln\left(\frac{1}{1+\alpha(r+\dot{x})}\right) - \dot{x}\right) dt \\ = &-(g - r\bar{b}^G)\left[\lambda \int_T^t (1+\alpha(r+\dot{x}))dt\right] \end{aligned} \tag{A.3}$$

the solution to which is:

$$\begin{aligned} \ln\left(\frac{1+\alpha(r+\dot{x}(T))}{1+\alpha(r+\dot{x}(t))}\right) &= \lambda(g - r\bar{b}^G)\frac{(1+\alpha r)}{\alpha}(t-T) \\ &+ [(g - r\bar{b}^G)\lambda - 1](x(t) - x(T)) \end{aligned} \tag{A.4}$$

Defining $\beta \equiv \lambda(g - r\bar{b}^G) - 1$ and $\theta \equiv (1+\alpha r)/\alpha$, we may rewrite (A.4) as

$$\ln\left(\frac{1+\alpha(r+\dot{x}(T))}{1+\alpha(r+\dot{x}(t))}\right) = \theta(\beta+1)(t-T) + \beta(x(t) - x(T)) \tag{A.5}$$

and taking exponentials, yields:

$$(\theta + \dot{x}(T)) = \left(e^{(\beta+1)\theta(t-T)} e^{\beta(x(t)-x(T))}\right)(\theta + \dot{x}(t)) \tag{A.6}$$

Letting $z = e^{\beta(x(t)-x(T))}$, equation (A.6) can be transformed to:

$$\frac{1}{\beta}\dot{z} + \theta z = (\theta + \dot{x}(T))e^{-\theta(\beta+1)(t-T)} \tag{A.7}$$

which may be solved to yield

$$ze^{\theta\beta t} = -\frac{\beta}{\theta}(\theta + \dot{x}(T))e^{(\beta+1)\theta T}e^{-\theta t} + C \tag{A.8}$$

where C is a constant. By setting $t = T$ in (A.8), and using the fact that $z(T) = 1$, we find

$$C = \left[1 + \frac{\beta}{\theta}(\theta + \dot{x}(T))\right] e^{\theta\beta T} \tag{A.9}$$

and substituting (A.9) into (A.8) yields the solution:

$$z = \left[1 + \frac{\beta}{\theta}(\theta + \dot{x}(T))(1 - e^{\theta(T-t)})\right] e^{\theta\beta(T-t)} \tag{A.10}$$

Combining (A.10) with the definition of z, together with the boundary condition, $x(T) = x_0$, leads to the solution, (30), for the exchange rate.

A.2. DERIVATIVES OF T

Differentiating (40) with respect to g, $b_0, \bar{b}_G, (x_1 - x_0)$ we obtain:

$$\frac{\partial T}{\partial g} = \frac{e^{-rT} - 1}{r(g - rb_0^G)} < 0 \tag{A.11a}$$

$$\frac{\partial T}{\partial b_0^G} = \frac{1}{(g - rb_0^G)} > 0 \tag{A.11b}$$

$$\frac{\partial T}{\partial \bar{b}^G} = -\frac{e^{-rT}}{(g - rb_0^G)} < 0 \tag{A.11c}$$

$$\frac{\partial T}{\partial (x_1 - x_0)} = \frac{e^{-rT}\beta^2(e^{-\theta\tau} - 1)e^{\beta(x_1 - x_0) + \theta\beta\tau}}{\lambda\theta(g - rb_0^G)(e^{\beta(x_1 - x_0) + \theta\beta\tau} - 1)} < 0 \tag{A.11d}$$

$$\frac{\partial T}{\partial \tau} = -\frac{\beta e^{-rT}\Omega(\tau)}{\lambda(g - rb_0^G)(e^{\beta(x_1 - x_0) + \theta\beta\tau})^2} \tag{A.11e}$$

where

$$\Omega(\tau) \equiv \left[e^{-\theta\tau} - (1 + \beta)e^{\beta(x_1 - x_0) + \theta(\beta - 1)\tau} + \beta e^{\beta(x_1 - x_0) + \theta\beta\tau}\right]$$

and we recall that $(g - rb_0^G) > 0$.

Letting $\tau \equiv \tau^*$denote the point where $\Omega(\tau) = 0$, so that $\partial T/\partial\tau = 0$, it is straightforward to show that $\partial^2 T/\partial\tau^2\big|_{\tau=\tau^*} < 0$, implying that T is maximized at $\tau \equiv \tau^*$.

A.3. DETERMINATION OF $\dot{X}(T)$

Note that because T is a function of τ, $\dot{x}(T)$ is also a function of τ, as can be seen by recalling (32′) repeated here for convenience as:

$$\dot{x}(T(\tau)) = -\frac{\theta}{\beta(1-e^{-\theta\tau})}\left[(1+\beta) - \beta e^{-\theta\tau} - e^{\beta(x_1-x_0)+\theta\beta\tau}\right] \tag{A.12}$$

Note that $-\theta/\left[\beta(1-e^{-\theta\tau})\right] > 0$ for any $\tau > 0$, and let us define

$$h(\tau) \equiv (1+\beta) - \beta e^{-\theta\tau} - e^{\beta(x_1-x_0)+\theta\beta\tau}$$

Letting $\tau \to 0, \infty$, we have $h(0) = -e^{-\theta(x_1-x_0)} > 0, \lim_{t\to\infty} h(\tau) = 1+\beta > 0$, implying that $\dot{x}(T(0) > 0, \quad \lim_{t\to\infty} \dot{x}((T)) > 0$. Taking the derivative of $h(\tau)$, yields

$$h'(\tau) \equiv \theta\beta(e^{-\theta\tau} - e^{\beta(x_1-x_0)+\theta\beta\tau})$$

and define $\hat{\tau}$ such that $h'(\hat{\tau}) \equiv 0$, namely

$$\hat{\tau} = -\frac{\beta(x_1-x_0)}{\theta(1+\beta)} \equiv \tau'(\text{as defined by (42)})$$

Furthermore,

$$h''(\hat{\tau}) \equiv -\theta^2\beta(1+\beta)e^{-\theta\hat{\tau}} > 0$$

implying that $h(\tau)$ achieves a minimum at $\tau = \hat{\tau}$. Substituting $\hat{\tau}$ into $h(\tau)$, we find $h(\hat{\tau}) = (1+\beta)(1-e^{-\theta\hat{\tau}}) > 0$, implying that $h(\tau) > 0$ for all τ and hence that $\dot{x}(T(\tau)) > 0$ for all τ.

A.4. DETERMINATION OF $\dot{X}(T+\tau)$

Taking the time derivative of (30), the rate of exchange depreciation (inflation) at any arbitrary time t can be expressed as

$$\dot{x}(t) = -\theta + \frac{\theta\,(1+\dot{x}(T)/\theta)\,e^{\theta(T-t)}}{1+\beta\,(1+\dot{x}(T)/\theta)\,(1-e^{\theta(T-t)})} \tag{A.13}$$

Setting $t = T+\tau$ in (A.13) yields

$$\dot{x}(T+\tau) = -\theta + \frac{(1+\dot{x}(T)/\theta)\theta e^{-\theta\tau}}{1+\beta(1+\dot{x}(T)/\theta)(1-e^{-\theta\tau})} \tag{A.14}$$

Substituting (32’) into (A.14) and rearranging terms yields

$$\dot{x}(T+\tau) = -\frac{\Omega(\tau)\theta}{\beta(1-e^{-\theta\tau})e^{\beta(x_1-x_0)+\theta\beta\tau}} \tag{A.15}$$

where $\Omega(\tau)$ is defined above. Thus we find

$$\operatorname{sgn}[\dot{x}(T+\tau)] = \operatorname{sgn}[\partial T/\partial\tau]. \tag{A.16}$$

A.5. DETERMINATION OF THE CURVATURE OF THE TIME PATH OF THE EXCHANGE RATE

Taking the time derivative of $\dot{x}(t)$ in (A.13), yields:

$$\ddot{x}(t) = -\frac{\theta^2\,(1+\dot{x}(T)/\theta)\,(1+\beta\,[1+\dot{x}(T)/\theta])\,e^{\theta(T-\tau)}}{\left[1+\beta\,(1+\dot{x}(T)/\theta)\,(1-e^{\theta(T-\tau)})\right]^2}$$

and using (32') this may be expressed in terms of τ by

$$\begin{aligned}\ddot{x}(t) \;&=\; -\frac{\theta^2\,(1+\dot{x}(T)/\theta)\,e^{\theta(T-\tau)}}{\left[1+\beta\,(1+\dot{x}(T)/\theta)\,(1-e^{\theta(T-\tau)})\right]^2(1-e^{-\theta\tau})}\\ &\quad\times\left(e^{\beta(x_1-x_0)+\theta\beta\tau}-e^{-\theta\tau}\right)\end{aligned} \tag{A.17}$$

Hence,

$$\begin{aligned}\operatorname{sgn}(\ddot{x}(t)) \;&=\; -\operatorname{sgn}\left(e^{\beta(x_1-x_0)+\theta\beta\tau}-e^{-\theta\tau}\right)\\ &=\; -\operatorname{sgn}\left(\beta(x_1-x_0)+\theta(\beta+1)\tau\right)\end{aligned} \tag{A.18}$$

That is,

$$\operatorname{sgn}(\ddot{x}(t)) = \operatorname{sgn}\left(e^{\beta(x_1-x_0)+\theta\beta\tau}-e^{-\theta\tau}\right) = \operatorname{sgn}(\tau'-\tau) \tag{A.19}$$

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